

Model-Based Optimization for Effective and Reliable Decision-Making

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Model-Based Optimization for Effective and Reliable Decision-Making

Optimization originated as an advanced mathematical technique, but it has become an accessible and widely used decision-making tool. A key factor in the spread of successful optimization applications has been the adoption of a model-based approach: A domain expert or operations analyst focuses on modeling the problem of interest, while the computation of a solution is left to general-purpose, off-the-shelf solvers; powerful yet intuitive modeling software manages the difficulties of translating

between the human modeler's formulation and the solver software's needs. This talk introduces *model-based optimization* by contrasting it to a method-based approach that relies on customized implementation of rules and algorithms. Model-based implementations are illustrated using the AMPL modeling language and popular solvers. The presentation concludes by surveying the variety of modeling languages and solvers available for model-based optimization today.

Dr. Fourer has over 40 years' experience in studying, creating, and applying large-scale optimization software. In collaboration with colleagues in Computing Science Research at Bell Laboratories, he initiated the design and development of AMPL, which has become one of the most widely used software systems for modeling and analyzing optimization problems, with users in hundreds of universities, research

institutes, and corporations worldwide; he is also author of a popular book on AMPL. Additionally, he has been a key contributor to the NEOS Server project and other efforts to make optimization services available over the Internet, and has supported development of open-source software for operations research through his service on the board of the COIN-OR Foundation.

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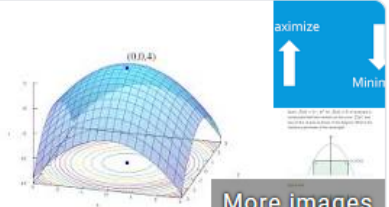
Dictionary

Search for a word

op·ti·mi·za·tion
 / ˌɑptəməˈzāSHən, ˌæptəˌmīˈzāSHən/
noun
 the action of making the best or most effective use of a situation or resource.
 "companies interested in the optimization of the business"

Translations, word origin, and more definitions

Feedback



More images

Mathematical optimization

In mathematics, computer science and operations research, mathematical optimization or mathematical programming is the selection of a best element from some set of available alternatives. [Wikipedia](#)

Videos

W Mathematical optimization - Wik x +

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Mathematical optimization

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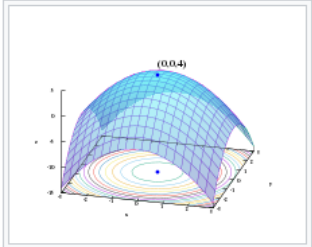
"Mathematical programming" redirects here. For the peer-reviewed journal, see [Mathematical Programming](#).
"Optimization" and "Optimum" redirect here. For other uses, see [Optimization \(disambiguation\)](#) and [Optimum \(disambiguation\)](#).

Mathematical optimization (alternatively spelled *optimisation*) or **mathematical programming** is the selection of a best element (with regard to some criterion) from some set of available alternatives.^[1] Optimization problems of sorts arise in all quantitative disciplines from [computer science](#) and [engineering](#) to [operations research](#) and [economics](#), and the development of solution methods has been of interest in [mathematics](#) for centuries.^[2]

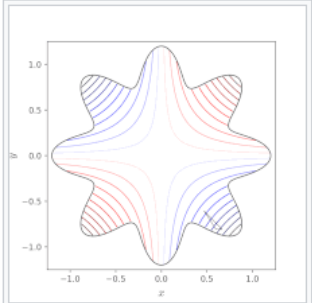
In the simplest case, an [optimization problem](#) consists of [maximizing](#) or [minimizing](#) a real function by systematically choosing [input](#) values from within an allowed set and computing the [value](#) of the function. The generalization of optimization theory and techniques to other formulations constitutes a large area of [applied mathematics](#). More generally, optimization includes finding "best available" values of some objective function given a defined [domain](#) (or input), including a variety of different types of objective functions and different types of domains.

Contents [hide]

- Optimization problems
- Notation
 - Minimum and maximum value of a function
 - Optimal input arguments
- History
- Major subfields
 - Multi-objective optimization
 - Multi-modal optimization
- Classification of critical points and extrema
 - Feasibility problem
 - Existence
 - Necessary conditions for optimality
 - Sufficient conditions for optimality
 - Sensitivity and continuity of optima
 - Calculus of optimization
- Computational optimization techniques



Graph of a paraboloid given by $z = f(x, y) = -(x^2 + y^2) + 4$. The global maximum at $(x, y, z) = (0, 0, 4)$ is indicated by a blue dot.



Nelder-Mead minimum search of Simionescu's function. Simplex vertices are ordered by their value, with 1 having the lowest (best) value.

Mathematical optimization (alternatively spelled *optimisation*) or **mathematical programming** is the selection of a best element (with regard to some criterion) from some set of available alternatives.^[1]

Optimization problems of sorts arise in all quantitative disciplines from computer science and engineering to operations research and economics, and the development of solution methods has been of interest in mathematics for centuries.^[2]

In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. The generalization of optimization theory and techniques to other formulations constitutes a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective function given a defined domain (or input), including a variety of different types of objective functions and different types of domains.

Optimization in Practice

Given a recurring need to make many interrelated decisions

- ❖ Purchases, production and shipment amounts, assignments, . . .

Consistently make highly desirable choices

By applying ideas from mathematical optimization

- ❖ Ways of describing problems (*formulations*)
- ❖ Ways of solving problems (*algorithms*)

Outline

Optimization

- ❖ The optimization modeling cycle
- ❖ Model-based vs. method-based approaches

Model-based optimization

- ❖ Modeling language vs. programming language approaches

Algebraic modeling languages

- ❖ Declarative vs. executable approaches
- ❖ Completed example in AMPL

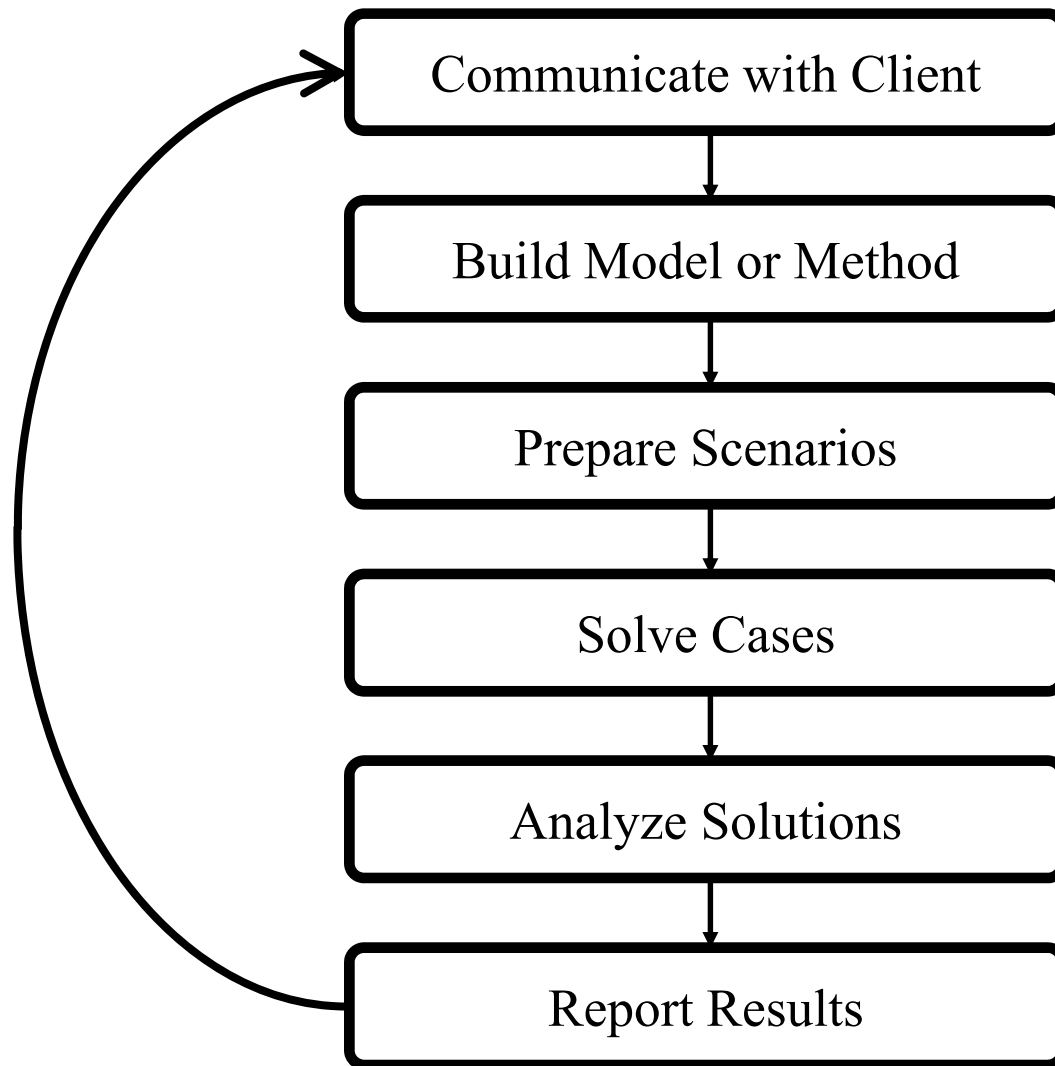
Solvers

- ❖ Linear/quadratic, nonlinear, global, constraint-based

Applications

- ❖ Range of AMPL users
- ❖ Case studies

The Optimization Modeling Cycle



The Optimization Modeling Cycle

Goals for the project manager

- ❖ Repeat the cycle quickly and *reliably*
 - * Get results before client loses interest
- ❖ Deploy *effectively* for application

Goals for optimization software

- ❖ Fast prototyping
- ❖ Easy integration (with decision systems)
- ❖ Successful long-term maintenance

Where is the Work in Optimization?

It depends on the approach that you take

Method-based approach

- ❖ Programming a method (algorithm) for computing solutions

Model-based approach

- ❖ Formulating a description (model) of the desired solutions

Which should you prefer?

- ❖ For simple problems, any approach can be easy
- ❖ *But real optimization problems have complications . . .*

Example:

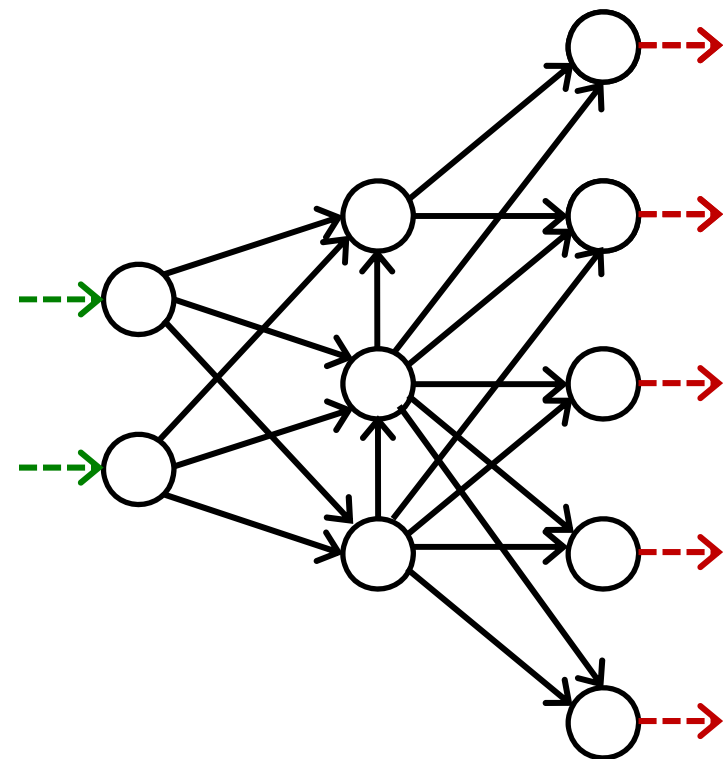
Multi-Product Optimal Network Flow

Motivation

- ❖ Ship products efficiently to meet demands

Context

- ❖ a transportation network
 - * nodes ○ representing cities
 - * arcs → representing roads
- ❖ supplies → at nodes
- ❖ demands → at nodes
- ❖ capacities on arcs
- ❖ shipping costs on arcs



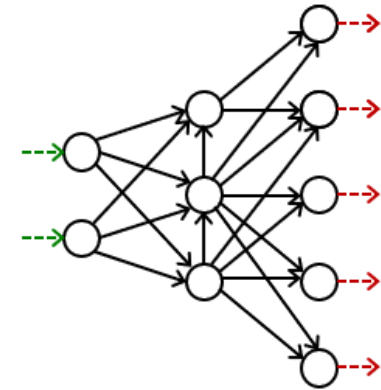
Multi-Product Network Flow

Decide

- ❖ how much of each product to ship on each arc

So that

- ❖ shipping costs are kept low
- ❖ shipments on each arc respect capacity of the arc
- ❖ supplies, demands, and shipments are in balance at each node



Two approaches . . .

Multi-Product Flow

Method-Based Approach

Program an algorithm to build a shipping plan

Order-driven

- ❖ Develop rules for how each order should be met
 - * Given some demand and given available capacity, determine where to ship it from and which route to use
- ❖ Fill orders one by one
 - * Decrement capacity as each one is filled

Route-driven

- ❖ Repeat until all demands are met
 - * Choose a shipping route and a product
 - * Add as much flow as possible of that product along that route without exceeding supply, demand, or capacity

Program refinements to get better results

Multi-Product Flow

Method-Based Refinements

Develop rules for choosing good routes

- ❖ Generate batches of routes
- ❖ Apply routes in some systematic order

Improve the initial solution

- ❖ *Local optimization*: swaps and other simple improvements
- ❖ *Local-search metaheuristics*:
simulated annealing, tabu search, GRASP
- ❖ *Population-based metaheuristics*:
evolutionary methods, particle swarm optimization

Multi-Product Flow

Model-Based Approach

Formulate a minimum shipping cost model

- ❖ Define decision variables X_{pij} to represent amount of product p shipped from node i to node j
- ❖ Specify allowed shipment amounts by writing equations that the variables must satisfy
- ❖ Write total shipping cost as a summation over the variables

*Send to a **solver** that computes optimal solutions*

- ❖ Handles broad problem classes efficiently
 - * Ex: Linear objective & equations, continuous or integer variables
- ❖ Recognizes and exploits special cases
- ❖ Available ready to run, without programming

Multi-Product Flow

Model-Based Formulation

Given

P set of products

N set of network nodes

$A \subseteq N \times N$ set of arcs connecting nodes

and

u_{ij} capacity of arc from i to j , for each $(i, j) \in A$

s_{pj} supply/demand of product p at node j , for each $p \in P, j \in N$
> 0 implies supply, < 0 implies demand

c_{pij} cost per unit to ship product p on arc (i, j) ,
for each $p \in P, (i, j) \in A$

Model-Based Formulation (*cont'd*)

Determine

X_{pij} amount of commodity p to be shipped from node i to node j ,
for each $p \in P$, $(i, j) \in A$

to minimize

$$\sum_{p \in P} \sum_{(i, j) \in A} c_{pij} X_{pij}$$

total cost of shipping

subject to

$$\sum_{p \in P} X_{pij} \leq u_{ij}, \text{ for all } (i, j) \in A$$

total shipped on each arc must not exceed capacity

$$\sum_{(i, j) \in A} X_{pij} + s_{pj} = \sum_{(j, i) \in A} X_{pji}, \text{ for all } p \in P, j \in N$$

shipments in plus supply/demand must equal shipments out

Example:

Complications in Multi-Product Flow

Additional restrictions imposed by the user

- ❖ Cost has fixed and variable parts
 - * Each arc incurs a cost if it is *used* for shipping
- ❖ Shipments cannot be too small
- ❖ Not too many arcs can be used

Additional data for the problem

- d_{ij} fixed cost for using the arc from i to j , for each $(i, j) \in A$
- m smallest total that may be shipped on any arc used
- n largest number of arcs that may be used

Complications

Method-Based (*cont'd*)

What has to be done?

- ❖ Revise or re-think the solution approach
- ❖ Update or re-implement the algorithm

What are the challenges?

- ❖ In this example,
 - * Good routes are harder to identify
 - * Improvements are harder to find
- ❖ In general,
 - * Even small changes to a problem can necessitate major changes to the method and its implementation
 - * Each problem change requires more programming

... and problem changes are frequent!

Complications

Model-Based (*cont'd*)

What has to be done?

- ❖ Update the objective expression
- ❖ Formulate additional constraint equations
- ❖ Send back to the solver

What are the challenges?

- ❖ In this example,
 - * New variables and expressions to represent fixed costs
 - * New constraints to impose shipment and arc-use limits
- ❖ In general,
 - * The formulation tends to get more complicated
 - * A new solver type or solver options may be needed

*... but it's easier to update formulations than methods
... and a few solver types handle most cases*

Complications

Model-Based Formulation (*revised*)

Determine

X_{pij} amount of commodity p to be shipped on arc (i, j) ,
for each $p \in P$, $(i, j) \in A$

Y_{ij} 1 if any amount is shipped from node i to node j ,
0 otherwise, for each $(i, j) \in A$

to minimize

$$\sum_{p \in P} \sum_{(i,j) \in A} c_{pij} X_{pij} + \sum_{(i,j) \in A} d_{ij} Y_{ij}$$

total cost of shipments

Complications

Model-Based Formulation (*revised*)

Subject to

$$\sum_{p \in P} X_{pij} \leq u_{ij} Y_{ij}, \quad \text{for all } (i, j) \in A$$

when the arc from node i to node j is used for shipping, total shipments must not exceed capacity, and Y_{ij} must be 1

$$\sum_{(i,j) \in A} X_{pij} + s_{pj} = \sum_{(j,i) \in A} X_{pji}, \quad \text{for all } p \in P, j \in N$$

shipments in plus supply/demand must equal shipments out

$$\sum_{p \in P} X_{pij} \geq m Y_{ij}, \quad \text{for all } (i, j) \in A$$

when the arc from node i to node j is used for shipping, total shipments from i to j must be at least m

$$\sum_{(i,j) \in A} Y_{ij} \leq n$$

At most n arcs can be used

Method-Based Remains Popular for . . .

Applications of heuristic methods

- ❖ Simple heuristics
 - * Greedy algorithms, local improvement methods
- ❖ Metaheuristics
 - * Evolutionary methods, simulated annealing, tabu search, GRASP, . . .

Situations hard to formulate mathematically

- ❖ Difficult combinatorial constraints
- ❖ Black-box objectives and constraints

Very large, intensive applications

- ❖ Routing huge fleets of delivery trucks
- ❖ Finding shortest routes in mapping apps
- ❖ Training huge neural networks

. . . and it appeals to programmers

Model-Based Has Become Common in . . .

Diverse industries

- ❖ Manufacturing, distribution, supply-chain management
- ❖ Air and rail operations, trucking, delivery services
- ❖ Medicine, medical services
- ❖ Refining, electric power flow, gas pipelines, hydropower
- ❖ Finance, e-commerce, . . .

Model-Based Has Become Common in . . .

Diverse industries

Diverse fields

- ❖ Operations research & management science
- ❖ Business analytics
- ❖ Engineering & science
- ❖ Economics & finance

Model-Based Has Become Standard for . . .

Diverse industries

Diverse fields

Diverse kinds of users

- ❖ Anyone who took an “optimization” class
- ❖ Anyone else with a technical background
- ❖ Newcomers to optimization

These have in common . . .

- ❖ Users inclined toward modeling; focus is
 - * more on *what* should be solved
 - * less on *how* it should be solved
- ❖ Good algebraic formulations for off-the-shelf solvers
- ❖ Emphasis on fast prototyping *and* long-term maintenance

Where is the Work in Model-Based Optimization?

Translating between two forms of the problem

- ❖ **Modeler's form**
 - * Mathematical description, easy for people to work with
- ❖ **Solver's form**
 - * Explicit data structure, easy for solvers to compute with

Programming language approach

- ❖ Write a **program** to generate the solver's form

Modeling language approach

- ❖ Write the **model formulation**
in a language that a computer can read and translate

Programming Language Approach

Write a program to generate the solver's form

- ❖ Read data and compute objective & constraint coefficients
- ❖ Send the solver the data structures it needs
- ❖ Receive solution data structure for viewing or processing

Some attractions

- ❖ Ease of embedding into larger systems
- ❖ Access to advanced solver features

Serious disadvantages

- ❖ Difficult environment for modeling
 - * program does not resemble the modeler's form
 - * model is not separate from data
- ❖ Very slow modeling cycle
 - * hard to check the program for correctness
 - * hard to distinguish modeling from programming errors

Modeling Language Approach

Use a computer language to describe the modeler's form

- ❖ Write your model
- ❖ Prepare data for the model
- ❖ Let the computer translate to & from the solver's form

Limited drawbacks

- ❖ Need to learn a new language
- ❖ Incur overhead in translation
- ❖ Make formulations clearer and hence easier to steal?

Great advantages

- ❖ Faster modeling cycles
- ❖ More reliable modeling
- ❖ More maintainable applications

Algebraic Modeling Languages

Most popular today

- ❖ Computer language based on *algebraic* formulations as seen in our model-based examples

Executable approach

- ❖ Create an algebraic modeling language inside a general-purpose programming language
- ❖ Redefine operators like + and <= to return constraint objects rather than simple values

Declarative approach

- ❖ Design a language specifically for optimization modeling
- ❖ Extend with basic programming concepts: loops, tests, assignments
- ❖ Access from popular programming languages via APIs

Example:

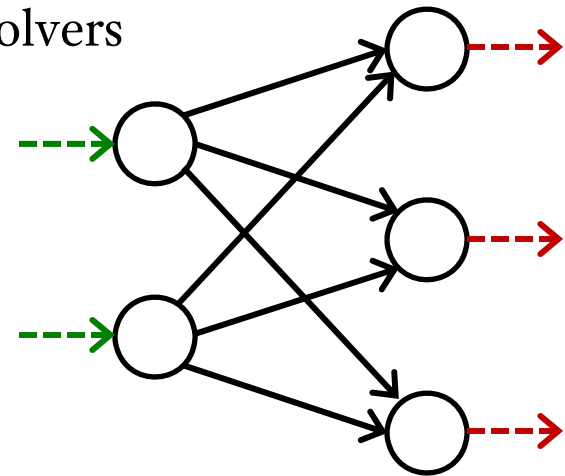
Multi-Product Optimal Network Flow

Executable approach:  *gurobipy*

- ❖ Based on the Python programming language
- ❖ Generates problems for the Gurobi solver

Declarative approach:  **AMPL**

- ❖ Based on algebraic notation (like our sample formulation)
- ❖ Designed specifically for optimization
- ❖ Generates problems for Gurobi and other solvers



Formulation: Data

Given

P set of products

N set of network nodes

$A \subseteq N \times N$ set of arcs connecting nodes

and

u_{ij} capacity of arc from i to j , for each $(i, j) \in A$

s_{pj} supply/demand of product p at node j , for each $p \in P, j \in N$
> 0 implies supply, < 0 implies demand

c_{pij} cost per unit to ship product p on arc (i, j) ,
for each $p \in P, (i, j) \in A$

Statements: Data

gurobipy

- ❖ Assign values to Python lists and dictionaries

```
products = ['Pencils', 'Pens']
nodes = ['Detroit', 'Denver',
         'Boston', 'New York', 'Seattle']
arcs, capacity = multidict({
    ('Detroit', 'Boston'): 100,
    ('Detroit', 'New York'): 80,
    ('Detroit', 'Seattle'): 120,
    ('Denver', 'Boston'): 120,
    ('Denver', 'New York'): 120,
    ('Denver', 'Seattle'): 120 })
```

- ❖ Provide data later in a separate file



AMPL

- ❖ Define symbolic model sets and parameters

```
set PRODUCTS;
set NODES;

set ARCS within {NODES,NODES};
param capacity {ARCS} >= 0;
```

```
set PRODUCTS := Pencils Pens ;

set NODES := Detroit Denver
            Boston 'New York' Seattle ;

param: ARCS: capacity:
        Boston 'New York' Seattle :=
Detroit   100      80      120
Denver    120     120     120 ;
```

Statements: Data (*cont'd*)

gurobipy

```
inflow = {  
    ('Pencils', 'Detroit'): 50,  
    ('Pencils', 'Denver'): 60,  
    ('Pencils', 'Boston'): -50,  
    ('Pencils', 'New York'): -50,  
    ('Pencils', 'Seattle'): -10,  
    ('Pens', 'Detroit'): 60,  
    ('Pens', 'Denver'): 40,  
    ('Pens', 'Boston'): -40,  
    ('Pens', 'New York'): -30,  
    ('Pens', 'Seattle'): -30 }
```

AMPL

```
param inflow {COMMODITIES, NODES};
```

```
param inflow (tr):  
    Pencils Pens :=  
    Detroit    50    60  
    Denver    60    40  
    Boston   -50   -40  
    'New York' -50  -30  
    Seattle  -10  -30 ;
```

Statements: Data (*cont'd*)

gurobipy

```
cost = {  
    ('Pencils', 'Detroit', 'Boston'): 10,  
    ('Pencils', 'Detroit', 'New York'): 20,  
    ('Pencils', 'Detroit', 'Seattle'): 60,  
    ('Pencils', 'Denver', 'Boston'): 40,  
    ('Pencils', 'Denver', 'New York'): 40,  
    ('Pencils', 'Denver', 'Seattle'): 30,  
    ('Pens', 'Detroit', 'Boston'): 20,  
    ('Pens', 'Detroit', 'New York'): 20,  
    ('Pens', 'Detroit', 'Seattle'): 80,  
    ('Pens', 'Denver', 'Boston'): 60,  
    ('Pens', 'Denver', 'New York'): 70,  
    ('Pens', 'Denver', 'Seattle'): 30 }
```

Multi-Product Flow

Statements: Data (*cont'd*)

AMPL

```
param cost {COMMODITIES,ARCS} >= 0;
```

```
param cost  
[Pencils,*,*] (tr) Detroit Denver :=  
  Boston          10      40  
  'New York'      20      40  
  Seattle         60      30  
  
[Pens,*,*]      (tr) Detroit Denver :=  
  Boston          20      60  
  'New York'      20      70  
  Seattle         80      30 ;
```

Multi-Product Flow

Formulation: Model

Determine

X_{pij} amount of commodity p to be shipped from node i to node j ,
for each $p \in P$, $(i, j) \in A$

to minimize

$$\sum_{p \in P} \sum_{(i,j) \in A} c_{pij} X_{pij}$$

total cost of shipping

subject to

$$\sum_{p \in P} X_{pij} \leq u_{ij}, \text{ for all } (i, j) \in A$$

total shipped on each arc must not exceed capacity

$$\sum_{(i,j) \in A} X_{pij} + s_{pj} = \sum_{(j,i) \in A} X_{pji}, \text{ for all } p \in P, j \in N$$

shipments in plus supply/demand must equal shipments out

Statements: Model

gurobipy

```
m = Model('netflow')  
flow = m.addVars(products, arcs, obj=cost, name="flow")  
m.addConstrs(  
    (flow.sum('*',i,j) <= capacity[i,j] for i,j in arcs), "cap")  
m.addConstrs(  
    (flow.sum(p,'*',j) + inflow[p,j] == flow.sum(p,j,'*')  
     for p in products for j in nodes), "node")
```

alternatives

```
for i,j in arcs:  
    m.addConstr(sum(flow[p,i,j] for p in products) <= capacity[i,j],  
                "cap[%s,%s]" % (i,j))  
m.addConstrs(  
    (quicksum(flow[p,i,j] for i,j in arcs.select('*',j)) + inflow[p,j] ==  
     quicksum(flow[p,j,k] for j,k in arcs.select(j,'*'))  
     for p in products for j in nodes), "node")
```

(Note on Summations)

gurobipy quicksum

```
m.addConstrs(  
    (quicksum(flow[p,i,j] for i,j in arcs.select('*',j)) + inflow[p,j] ==  
     quicksum(flow[p,j,k] for j,k in arcs.select(j,'*'))  
     for p in commodities for j in nodes), "node")
```

quicksum (data)

A version of the Python `sum` function that is much more efficient for building large Gurobi expressions (`LinExpr` or `QuadExpr` objects). The function takes a list of terms as its argument.

Note that while `quicksum` is much faster than `sum`, it isn't the fastest approach for building a large expression. Use `addTerms` or the `LinExpr()` constructor if you want the quickest possible expression construction.

Statements: Model (*cont'd*)

AMPL

```
var Flow {PRODUCTS,ARCS} >= 0;

minimize TotalCost:
    sum {p in PRODUCTS, (i,j) in ARCS} cost[p,i,j] * Flow[p,i,j];

subject to Capacity {(i,j) in ARCS}:
    sum {p in PRODUCTS} Flow[p,i,j] <= capacity[i,j];

subject to Conservation {p in PRODUCTS, j in NODES}:
    sum {(i,j) in ARCS} Flow[p,i,j] + inflow[p,j] =
    sum {(j,i) in ARCS} Flow[p,j,i];
```

$$\sum_{(i,j) \in A} X_{pij} + s_{pj} = \sum_{(j,i) \in A} X_{pji}, \text{ for all } p \in P, j \in N$$

Multi-Product Flow

Solution

gurobipy

```
m.optimize()

if m.status == GRB.Status.OPTIMAL:
    solution = m.getAttr('x', flow)
    for p in products:
        print('\nOptimal flows for %s:' % p)
        for i,j in arcs:
            if solution[p,i,j] > 0:
                print('%s -> %s: %g' % (i, j, solution[p,i,j]))
```

Solved in 0 iterations and 0.00 seconds

Optimal objective 5.500000000e+03

Optimal flows for Pencils:

Detroit -> Boston: 50

Denver -> New York: 50

Denver -> Seattle: 10

Optimal flows for Pens: ...

Multi-Product Flow

Solution (*cont'd*)

AMPL

```
ampl: model netflow.mod;
ampl: data netflow.dat;

option solver gurobi;
ampl: solve;

Gurobi 8.1.0: optimal solution; objective 5500
2 simplex iterations

ampl: display Flow;

Flow [Pencils,*,*]
:      Boston 'New York' Seattle :=
Denver    0      50      10
Detroit   50      0       0

[Pens,*,*]
:      Boston 'New York' Seattle :=
Denver    10      0      30
Detroit   30     30      0
;
```

Multi-Product Flow

Solution (*cont'd*)

AMPL

```
ampl: model netflow.mod;
ampl: data netflow.dat;

option solver cplex;
ampl: solve;

CPLEX 12.9.0.0: optimal solution; objective 5500
0 dual simplex iterations (0 in phase I)

ampl: display Flow;

Flow [Pencils,*,*]
:      Boston 'New York' Seattle :=
Denver    0      50      10
Detroit   50      0       0

[Pens,*,*]
:      Boston 'New York' Seattle :=
Denver    10      0      30
Detroit   30     30      0
;
```

Integration with Applications

gurobipy

- ❖ Everything can be developed in **Python**
 - * Extensive data, visualization, deployment tools available
- ❖ Limited modeling features also in **C++, C#, Java**

AMPL

- ❖ Modeling language extended with loops, tests, assignments
- ❖ Application programming interfaces (APIs) for calling AMPL from **C++, C#, Java, MATLAB, Python, R**
 - * Efficient methods for data interchange

Integration with Solvers

gurobipy

- ❖ Works closely with the Gurobi solver:
callbacks during optimization, fast re-solves after problem changes
- ❖ Supports Gurobi's extended expressions:
min/max, and/or, if-then-else

AMPL

- ❖ Supports all popular solvers
- ❖ Extends to general nonlinear and logic expressions
 - * Connects to nonlinear function libraries and user-defined functions
 - * Automatically computes nonlinear function derivatives
 - * Connects to global optimization and constraint programming solvers

Multi-Product Flow

Complications

Easily accommodated

- ❖ Add variables to the model
- ❖ Add a term to the objective
- ❖ Update one constraint and add two
- ❖ Send to the same solver

See live example . . .

Survey

Algebraic Modeling Language Software

Solver-specific

- ❖ Associated with popular commercial solvers
 - * IBM CPLEX, Gurobi, FICO Xpress
- ❖ Executable and declarative alternatives

Solver-independent

- ❖ Support multiple solvers and solver types
- ❖ Commercial options are mainly declarative
 - * AIMMS, AMPL, GAMS
 - * include APIs for popular programming languages
- ❖ Open-source options are mainly executable
 - * CVX/MATLAB, FLOPC++/C++, JuMP/Julia, Pyomo/Python, YALMIP/MATLAB,

Survey

Solver Software

Off-the-shelf solvers for broad problem classes

- ❖ Based on optimal algorithms
- ❖ Implemented as complex methods + heuristics
- ❖ Adapted to special cases

Survey

Solver Software

Off-the-shelf solvers for broad problem classes

Many difficult problems solved regularly

- ❖ Millions of variables and constraints
- ❖ Hard problems of 10-20 years ago are now easy

Survey

Solver Software

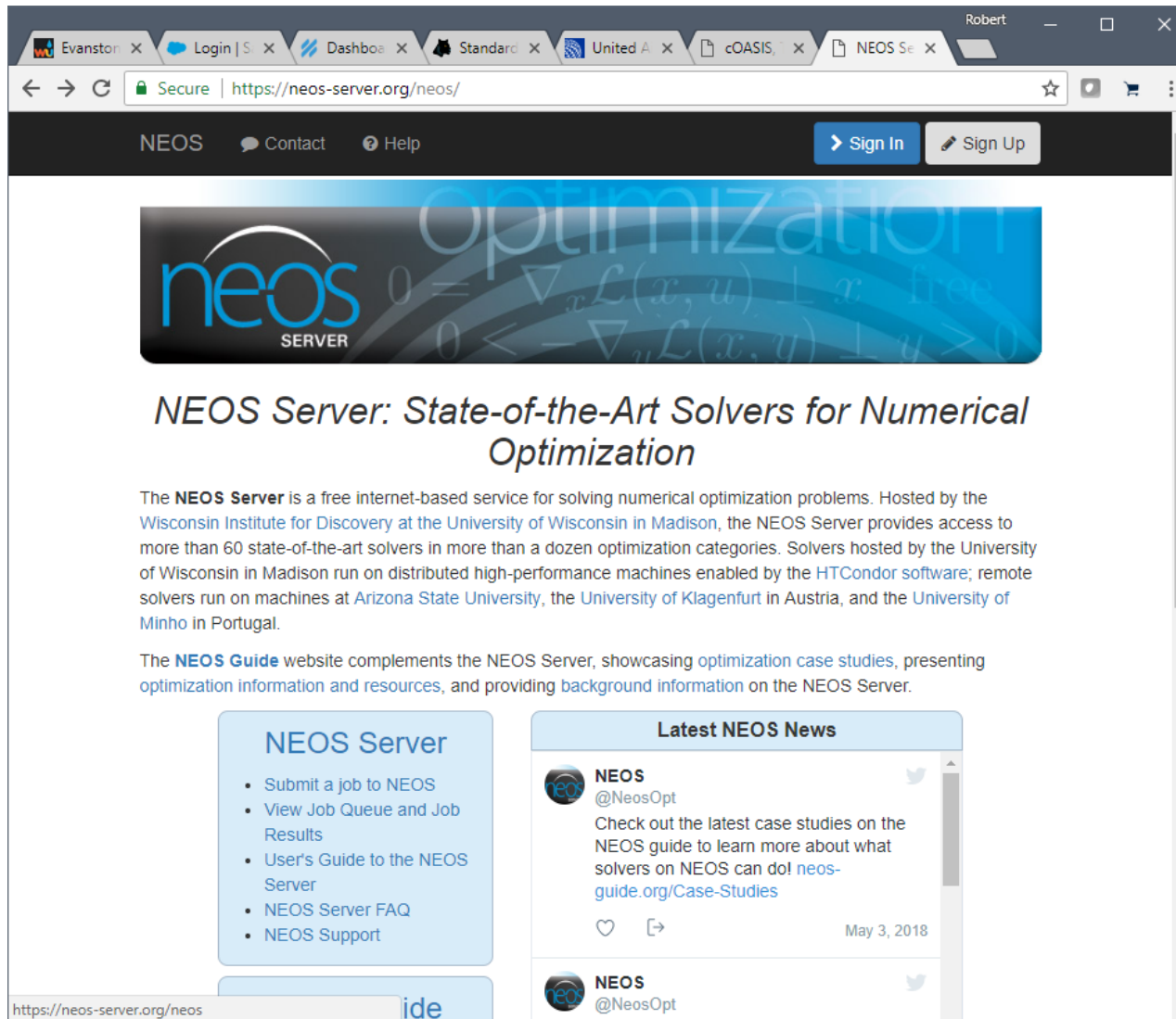
Off-the-shelf solvers for broad problem classes

Many difficult problems solved regularly

Commercial + open source examples

- ❖ “Linear/Quadratic”:
CPLEX, Gurobi, Xpress, MOSEK + SCIP, CBC, MIPCL
- ❖ “Nonlinear”:
CONOPT, Knitro, LOQO, MINOS, SNOPT + Ipopt, Bonmin
- ❖ “Global”:
BARON, LINDO Global + Couenne
- ❖ “Constraint”:
IBM ILOG CP + Gecode, JaCoP

Curious? Try Them Out on NEOS!



The screenshot shows a web browser window with the URL <https://neos-server.org/neos/>. The browser tabs include "Evanston", "Login | S...", "Dashboa...", "Standard...", "United A...", "cOASIS...", and "NEOS Se...". The page features a navigation bar with "NEOS", "Contact", "Help", "Sign In", and "Sign Up" buttons. The main content area has a blue header with the "neos SERVER" logo and the word "Optimization" in a large, semi-transparent font. Below the header, the text reads: "NEOS Server: State-of-the-Art Solvers for Numerical Optimization". A paragraph follows: "The **NEOS Server** is a free internet-based service for solving numerical optimization problems. Hosted by the [Wisconsin Institute for Discovery at the University of Wisconsin in Madison](#), the NEOS Server provides access to more than 60 state-of-the-art solvers in more than a dozen optimization categories. Solvers hosted by the University of Wisconsin in Madison run on distributed high-performance machines enabled by the [HTCondor](#) software; remote solvers run on machines at [Arizona State University](#), the [University of Klagenfurt](#) in Austria, and the [University of Minho](#) in Portugal." Another paragraph states: "The **NEOS Guide** website complements the NEOS Server, showcasing [optimization case studies](#), presenting optimization information and resources, and providing [background information](#) on the NEOS Server." At the bottom, there are two sections: "NEOS Server" with a list of links: "Submit a job to NEOS", "View Job Queue and Job Results", "User's Guide to the NEOS Server", "NEOS Server FAQ", and "NEOS Support"; and "Latest NEOS News" with a tweet from @NeosOpt dated May 3, 2018, promoting case studies on the NEOS guide.

Solver & Language Listing

The screenshot shows a web browser window with the URL <https://neos-server.org/neos/solvers/index.html>. The page features a navigation bar with "NEOS", "Contact", "Help", "Sign In", and "Sign Up" buttons. The main content is a list of optimization categories, each with a plus or minus sign to expand or collapse the list of solvers and languages. The "Mixed Integer Linear Programming" category is expanded, showing a list of solvers and their supported input languages.

- Linear Programming +
- Mathematical Programs with Equilibrium Constraints +
- Mixed Integer Linear Programming -**
 - Cbc [AMPL Input][GAMS Input][MPS Input]
 - CPLEX [AMPL Input][GAMS Input][LP Input][MPS Input][NL Input]
 - feaspump [AMPL Input][CPLEX Input][MPS Input]
 - FICO-Xpress [AMPL Input][GAMS Input][MOSEL Input][MPS Input][NL Input]
 - Gurobi [AMPL Input][GAMS Input][LP Input][MPS Input][NL Input]
 - MINTO [AMPL Input]
 - MOSEK [AMPL Input][GAMS Input][LP Input][MPS Input][NL Input]
 - proxy [CPLEX Input][MPS Input]
 - qsopt_ex [AMPL Input][LP Input][MPS Input]
 - scip [AMPL Input][CPLEX Input][GAMS Input][MPS Input][OSIL Input][ZIMPL Input]
 - SYMPHONY [MPS Input]
- Mixed Integer Nonlinearly Constrained Optimization +
- Mixed-Integer Optimal Control Problems +
- Nondifferentiable Optimization +
- Nonlinearly Constrained Optimization -**
 - ANTIGONE [GAMS Input]
 - CONOPT [AMPL Input][GAMS Input]
 - filter [AMPL Input]
 - Ipopt [AMPL Input][GAMS Input][NL Input]
 - Knitro [AMPL Input][GAMS Input]
 - LANCELOT [AMPL Input]

Applications

Range of AMPL users

Case studies

- ❖ Power grid management
- ❖ Passenger flow management
- ❖ Sales representative assignment

Range of AMPL Users

Energy and Utilities

- ❖ power networks, gas pipelines, hydroelectric power, water distribution

Industry

- ❖ mining, steel, chemicals, oil refining, forestry and paper
- ❖ cars & trucks, paper products, processed foods

Transportation

- ❖ airlines, trucking, package delivery

Services

- ❖ supply chain, hospitals & medicine, construction management

Communications

- ❖ telecommunications, professional networking, file hosting

Finance

- ❖ software tools, investment management, commodity management

Advanced Technologies

- ❖ artificial intelligence, distributed computing, biotechnology

Case: ABB

Power Grid Management

ABB GridView - Market Analysis (Ener: X)

ABB Asea Brown Boveri Ltd. [CH] | <https://new.abb.com/enterprise-software/energy-portfolio-man...>

ABB HOME > OFFERINGS > ENTERPRISE SOFTWARE > ENERGY PORTFOLIO MANAGEMENT > MARKET ANALYSIS > GRIDVIEW GLOBAL SITE

GridView

For studies within the Western Electric Coordinating Council territory, GridView provides an industry-accepted simulation approach. The advanced analysis methodology combines generation, transmission, loads, fuels, and market economics into one integrated framework to deliver location dependent market indicators, transmission system utilization measures and power system reliability and market performance indices. It provides invaluable information for both generation and transmission planning, operational decision making and risk management.

GridView uses state-of-the-art modeling technology to simulate security-constrained unit commitment and economic dispatch. It produces unit commitment and economic dispatch that respect the physical laws of power flow and transmission reliability requirements. As such, the generation dispatch and market clearing price are feasible market solutions within real power transmission networks.

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Case: ABB

Power Grid Management

GridView

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Power Grid Management

Situation

- ❖ A power grid operator providing electrical service
- ❖ Two kinds of decisions
 - * *Unit commitment*: When to turn power plants on and off
 - * *Network flow*: How to transmit power over the grid to meet demand

Goal

- ❖ Simulate optimal decisions to support planning
 - * Transmission network expansion
 - * Plant addition and retirement
 - * Integration of renewable energy sources

Evaluation

Approaches considered

- ❖ C++ for entire GridView system
- ❖ Modeling language for optimization, C++ for user interfaces

Choice of AMPL

- ❖ *Ease of modeling*
 - * ABB can formulate complex and powerful models
 - * Customers can understand the AMPL formulations
 - * Customers can customize models for their particular situations
- ❖ *Ease of embedding*
 - * AMPL has an API (application programming interface) for C++
 - * ABB can easily build AMPL into the GridView product

Power Grid Management

Formulation (*data*)

Production data

- ❖ Power generation units
 - * Location
 - * Fuel, design, age, capacity
 - * Ramp-up and ramp-down times
- ❖ Renewable energy sources

Transmission network data

- ❖ Nodes: units, sources, substations, customers
 - * Supply at plants and other sources
 - * Demand at customers
- ❖ Arcs: power lines
 - * Transmission capacities

Cost data

Formulation (*variables*)

Decision variables

- ❖ For each unit, in each time period
 - * On or off (discrete)
 - * Level of output (continuous)
- ❖ For each *critical path* through the grid, in each time period
 - * Capacity

Formulation (*model*)

Objectives

- ❖ For short-term operation management
 - * Minimize total operating costs
- ❖ For long-term investment planning
 - * Minimize total operating and investment costs

Constraints

- ❖ Balance of supply and demand
- ❖ Capacity restriction on power lines
- ❖ Ramp-up and ramp-down times
- ❖ Contingencies for generation and transmission

Power Grid Management

Implementation

Development

- ❖ Prototype at University of Tennessee, Knoxville
- ❖ Full AMPL implementation by 3 analysts at ABB

Optimization

- ❖ Mixed-integer linear solver such as Gurobi
- ❖ Millions of variables
- ❖ Tens of thousands of integer variables
- ❖ 10 minutes to solve

Deployment

- ❖ 30+ customer companies
- ❖ Hundreds of customer-side users

Case: MTR HK / Strategis Partners

Passenger Flow Management

The screenshot displays the MTR HK Tourist Home website. The browser address bar shows the URL www.mtr.com.hk/en/customer/tourist/index.php. The page features the MTR logo and a '40' anniversary banner with the slogan 'Connecting Together'. Navigation links include 'Tourist Home', 'Tickets and Fares', 'Buy Tickets', 'Shops and Malls', and 'Services and Facilities'. A 'TRAIN TRIP PLANNER' sidebar is visible on the left, with tabs for 'Route Suggestion' and 'Ticket Suggestion'. The 'From:' and 'To:' fields are set to 'Please Select Category' and '--Input or select--' respectively. A 'Search' button is located at the bottom of the sidebar. The main content area features a large 'WELCOME TO HONG KONG' heading, followed by a paragraph: 'You are never far from town with the Airport Express – the fastest link to the city. All of the territory is within your reach with metro lines covering most parts of Hong Kong. It's easy and convenient to explore the city with the MTR. You can also travel beyond Hong Kong with cross-boundary services to cities in Mainland China.' Below the text is an image of three MTR trains (Airport Express) in silver and blue, with the MTR logo on their fronts.

Passenger Flow Management



Passenger Flow Management

Situation

- ❖ Large public train operator
- ❖ 2 million passengers in 2 hours each weekday afternoon
- ❖ Arrivals exceed capacity
 - * More passengers arrive on a platform than a train can handle
- ❖ Measures are in place that can limit entry to station & platforms

Goal

- ❖ Decide where and when to implement passenger-limiting measures
- ❖ Balance platform use throughout the system

Evaluation

Approaches

- ❖ *Old*: Best guesses of experience managers
- ❖ *New*: Modeling language for optimization, R to manipulate input data and display results

Choice of AMPL

- ❖ *Ease of use*
 - * Convenient model syntax
 - * Speed of processing
- ❖ *Ease of embedding*
 - * AMPL is easily built into an R application, using AMPL's API (application programming interface) for R

Formulation (*data and variables*)

Data

- ❖ Design of the train network
- ❖ Passenger entry and intended exit stations
 - * Supplied by the ticketing system
- ❖ Platform capacities

Decision variables

- ❖ For each time interval, at each station, for each train service:
 - * How many passengers to allow in to the platform
 - * How many passengers to expect out at the platform

Formulation (*objective and constraints*)

Objective

- ❖ Minimize aggregate passenger travel times across the network

Constraints

- ❖ Train travel times
- ❖ Train capacities
- ❖ Station concourse capacities

Passenger Flow Management Implementation

The screenshot shows a web browser window with the URL www.strategispartners.com.au. The page features the company logo, a search bar, and a navigation menu. The main content area highlights 'Client Services' and 'Decision Analytics' with a sub-headline 'Turn data into insight for decision making with our methods of decision analytics.' and a 'Read More' button. A large graphic on the right depicts a person's head filled with circuitry and data icons.

Strategis Partners

Search for:

Client Services Industry Practices Whats New About Us Downloads

Follow Us: [in](#) [RSS](#) [Twitter](#)

Client Services

Decision Analytics

Turn data into insight for decision making with our methods of decision analytics.

Algorithms are playing an increasingly important role in business strategy. Most strategic decisions require the analysis of thousands of alternative patterns when working out how to improve concept-to-product-to-market performance.

[Read More](#)

Passenger Flow Management

Implementation

Development

- ❖ **Strategis Partners** consulting firm
- ❖ AMPL and R implementation by 2 analysts

Optimization

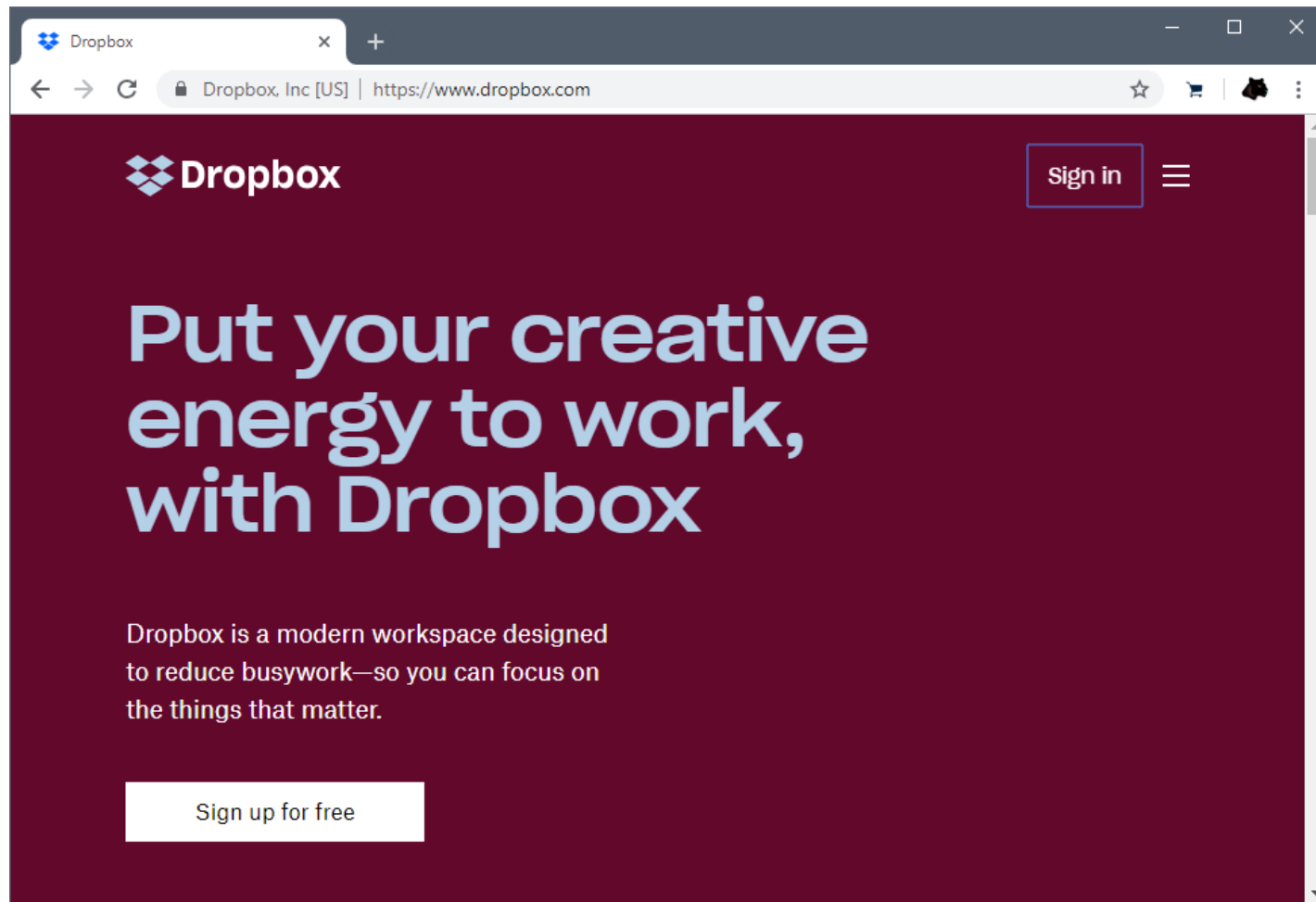
- ❖ Free open-source mixed-integer linear solver
- ❖ 15 core stations
- ❖ 250,000 variables and constraints
- ❖ 20 minutes to solve

Deployment

- ❖ 2 users at MTR HK run as needed
- ❖ Extensions and enhancements planned
 - * using machine learning to forecast passenger flows
 - * expanding to more stations

Case: Dropbox

Sales Representative Assignment



Sales Representative Assignment

Situation

- ❖ Cloud storage provider
- ❖ Over 500 million users upload 1.2 billion files every day
- ❖ Tens of thousands of large business customer accounts
- ❖ Hundreds of sales representatives worldwide
 - * enough to cover most but not all accounts

Goal

- ❖ Assign accounts to representatives
 - * Assign each representative a similar number and quality of accounts
 - * Give priority to assigning higher quality accounts

Sales Rep Assignment

Evaluation

Approaches considered

- ❖ Manual system
- ❖ Spreadsheet-based solvers
- ❖ Automated system using model-based optimization

Choice of AMPL

- ❖ Ease of use
- ❖ Speed
- ❖ Reliability
- ❖ Ability to handle large problems

Formulation (*data and variables*)

Data

- ❖ Quality *score* for each customer account
 - * predicted revenue increase if contacted by a representative
- ❖ Location of each representative

Decision variables

- ❖ For each account i and representative j ,
 - $X_{ij} = 1$ if account i is assigned to representative j
 - $X_{ij} = 0$ otherwise

Formulation (*objective and constraints*)

Objective

- ❖ Maximize total score of all assigned accounts

Constraints

- ❖ At most 15% variance between representatives in . . .
 - * number of accounts assigned
 - * quality of accounts assigned
- ❖ Assigned accounts must be near the representative's location
- ❖ All subaccounts of a business must have the same representative

Sales Rep Assignment

Implementation

Development

- ❖ Implementation by 3 analysts at Dropbox

Optimization

- ❖ Mixed-integer linear solver
- ❖ 10,000 zero-one variables
- ❖ 3-6 hours to solve for largest region

Deployment

- ❖ 5-10 sales leaders are direct users
- ❖ AMPL is embedded in Dropbox's systems
 - * Customer data is extracted from *Salesforce*
 - * Customer scores are computed using the *scikit-learn* Python toolbox
 - * An AMPL script reads the file of score data
 - * Results from optimization are written to an *Excel* spreadsheet

