Data-driven optimisation for constrained trajectories

July 2021

Florent Dewez

01

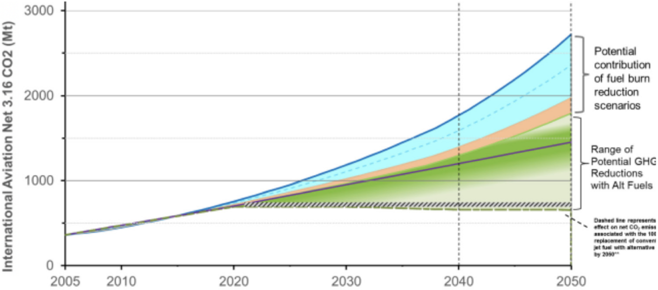
Motivation

The air traffic

Source: `http://flightradar24.com`

- 4,300,000,000 passengers – 38,000,000 flights (2018)
- 24,000 airliners
- 72 airliners take off each minute

Evolution of CO_2 emissions



Source: ICAO - Environmental report 2016

- 650,000,000 tonnes of CO_2 (2018)
- 3% of the total CO_2 emissions
- Paris-New York $\simeq$ 1 000 kg CO_2/passenger

- Doubling of air traffic each 15 years (without any pandemic)

Enhance Aircraft Performance and Optimisation through utilisation of Artificial Intelligence

- Led by Safety Line and funded by Clean Sky 2
- Scientific partnership with Inria (Modal team*)
- From November 2018 to October 2020

* Benjamin Guedj, Arthur Talpaert, Vincent Vandewalle and Florent Dewez

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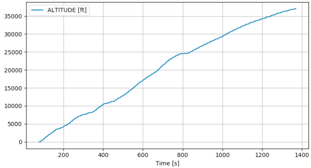
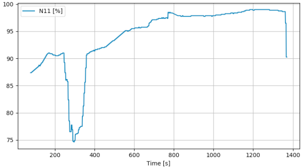
Many hundred of parameters recorded at each second by

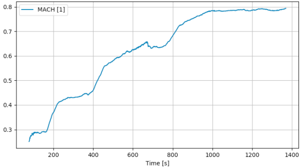
- the black boxes for analysis in case of crash;
- the Quick Access Recorder to improve flight safety and efficiency.

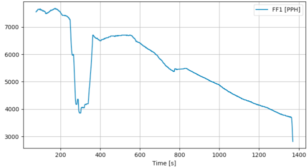
Key element: Flight data

	ALTITUDE	AOA	FF1	FF2	GS	GW	IAS	LEVEL_OFF	MACH	N11	N12	PITCH	SAT
1200	34243.0	1.06640625	4144.0	4144.0	450.0	144080.0	271.75	1	0.7859999999999999	98.875	99.0	3.69140625	-53.4654464917
1201	34252.0	1.06640625	4144.0	4144.0	450.0	144080.0	271.75	1	0.7859999999999999	98.875	99.0	3.69140625	-53.4654464917
1202	34262.0	1.06640625	4144.0	4128.0	450.0	144080.0	271.75	1	0.787	98.875	99.0	3.69140625	-53.5255699099
1203	34271.0	1.06640625	4144.0	4128.0	450.0	144080.0	272.0	1	0.787	98.875	98.875	3.69140625	-53.5255699099
1204	34280.0	1.06640625	4144.0	4128.0	450.0	144080.0	271.75	1	0.787	98.875	98.875	3.69140625	-53.5255699099
1205	34289.0	1.06640625	4144.0	4128.0	450.0	144080.0	271.75	1	0.787	98.875	98.875	3.69140625	-53.748629457
1206	34299.0	1.06640625	4128.0	4112.0	450.0	144080.0	271.75	1	0.7879999999999999	98.875	98.875	3.69140625	-53.80873524850001
1207	34308.0	1.06640625	4128.0	4128.0	450.0	144080.0	271.75	1	0.7879999999999999	99.0	99.0	3.69140625	-53.80873524850001
1208	34317.0	1.06640625	4128.0	4112.0	450.0	144080.0	271.5	1	0.7879999999999999	99.0	99.0	3.69140625	-53.80873524850001
1209	34327.0	1.06640625	4128.0	4112.0	450.0	144080.0	271.5	1	0.7879999999999999	99.0	99.0	3.69140625	-54.0317336876



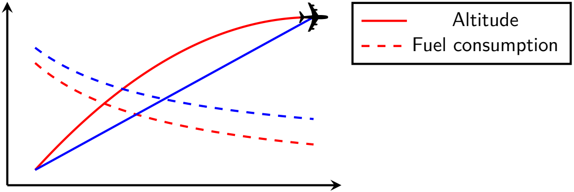






Trajectory optimisation

Find a trajectory (altitude, speed,...) being as fuel-efficient as possible and acceptable by aeronautic experts.



Goal

Exploit flight data to propose a new and efficient trajectory optimisation methodology.

Let $y = (x, u)$ be a trajectory with states x and controls u.

- Cost function: Total fuel consumption, traveled distance,...

$$\mathrm{TFC}(y) = \int_0^T \mathrm{FF}\big(y(t)\big)\, dt$$

- Constraints: Flight domain, initial and final conditions,...
- Dynamics: $\dot{y}(t) = g\big(t, y(t)\big)$

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- Dynamics: $\dot{y}(t) = g\big(t, y(t)\big) + \varepsilon(t)$

$\longrightarrow$ System identification†: $\dot{y}(t) = \hat{g}\big(t, y(t)\big)$

† C. Rommel, F. Bonnans, P. Martinon, B. Gregorutti. *Aircraft Dynamics Identification for Optimal Control*. 7th EUCASS, 2017.

This approach requires two steps:

- Identification: not always straightforward
- Optimisation: potentially affected by the statistical errors

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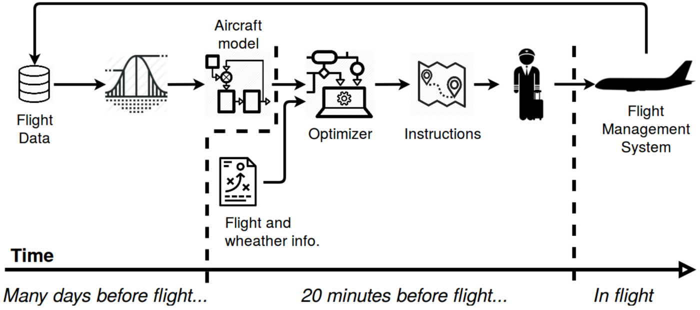
- Identification: not always straightforward
- Optimisation: potentially affected by the statistical errors

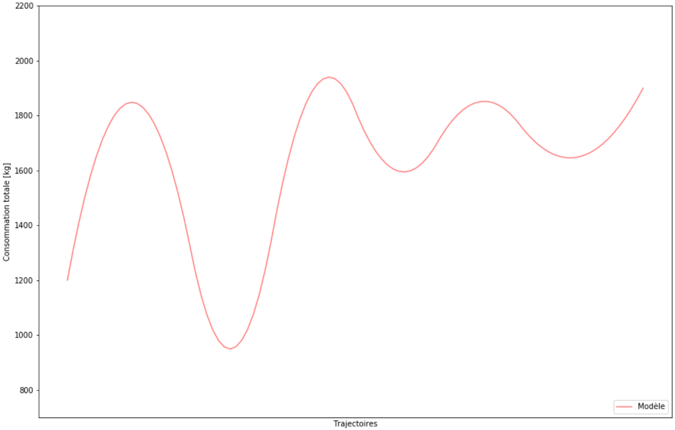
Sometimes unacceptable for applications !



02

Generic methodology

Visualisation of the idea



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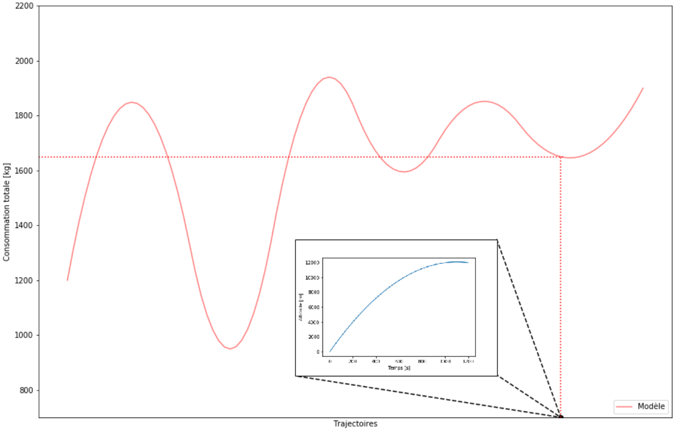

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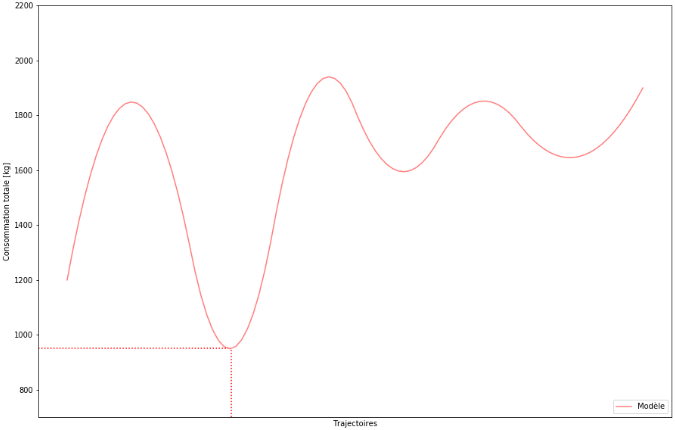

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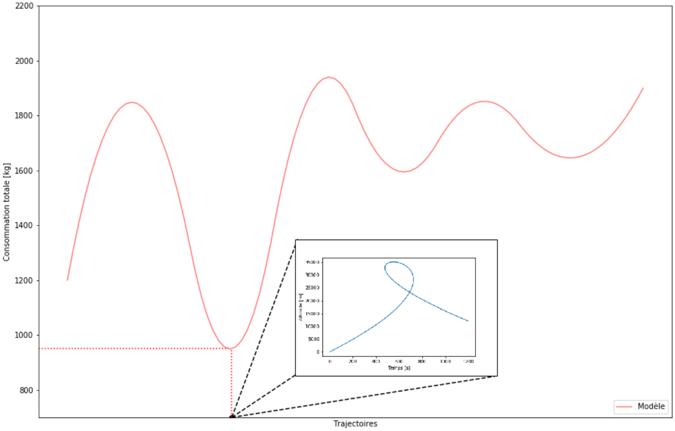
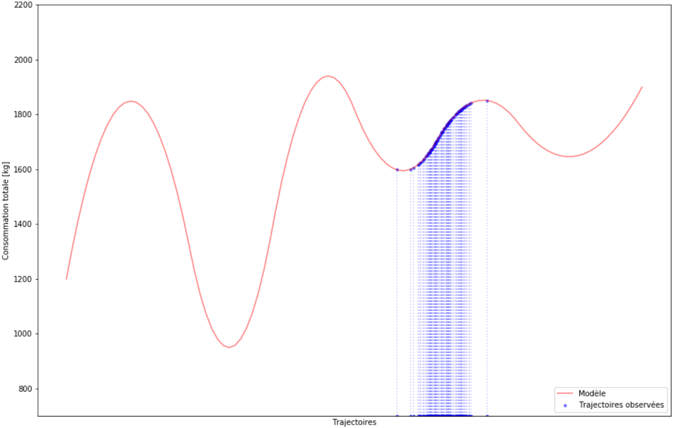


Inria

Visualisation of the idea



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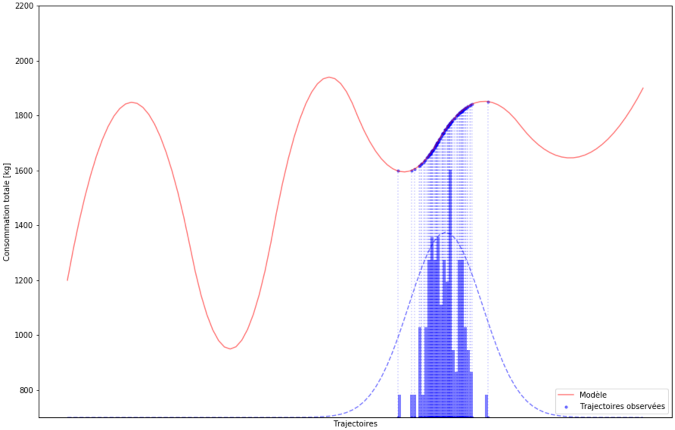

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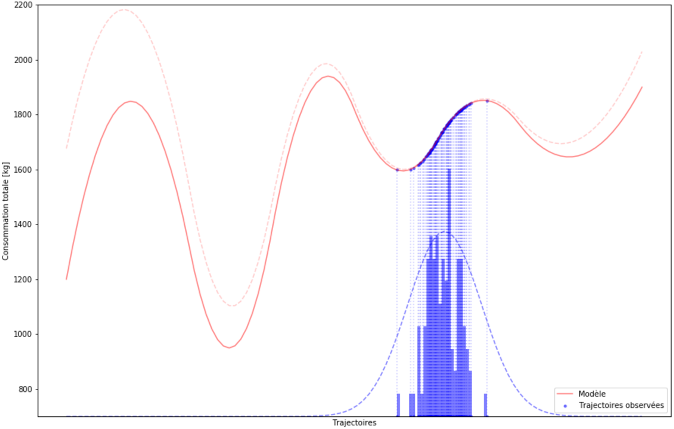

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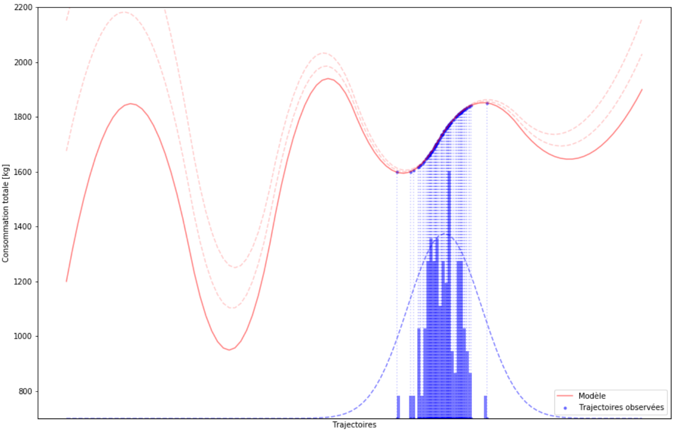

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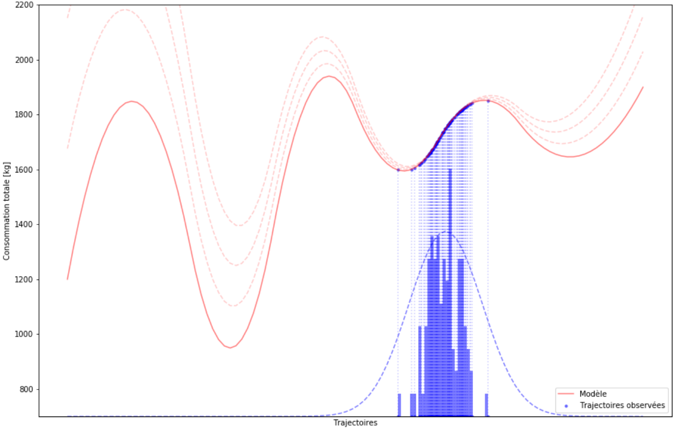

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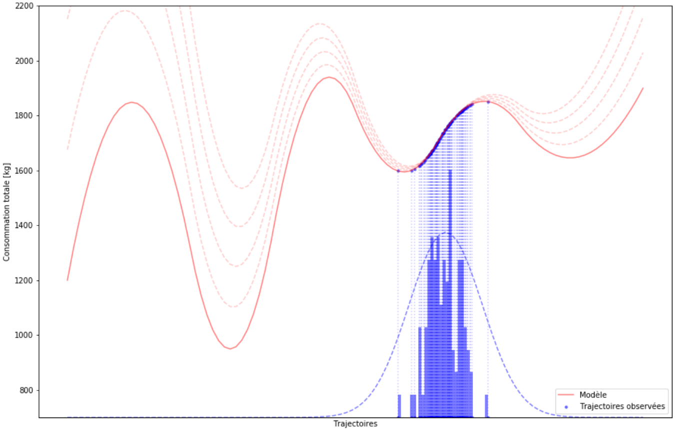

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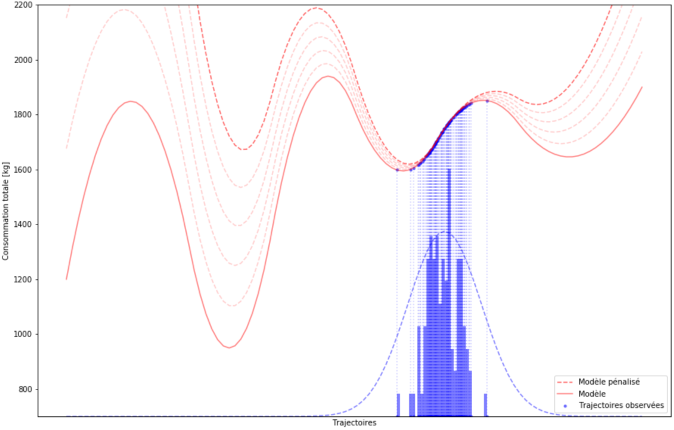

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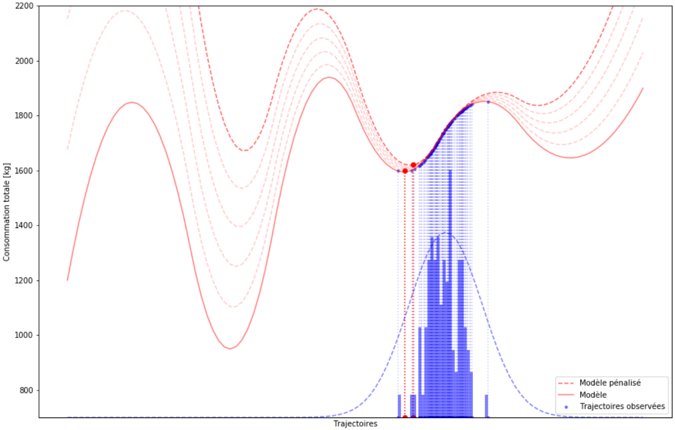

- Trajectory: $y = \left(y^{(1)}, \ldots, y^{(D)}\right) \in C\left([0, T], \mathbb{R}^D\right)$

- Endpoints conditions:

$$y \in \mathcal{D}(y_0, y_T) \qquad \Longleftrightarrow \qquad \begin{cases} y(0) = y_0 \\ y(T) = y_T \end{cases}$$

- Additionnal constraints:

$$y \in \mathcal{G} \qquad \Longleftrightarrow \qquad \forall\, \ell = 1, \ldots, L \quad g_\ell\big(y(t)\big) \leqslant 0$$

- Cost function: $F : C\left([0, T], \mathbb{R}^D\right) \longrightarrow \mathbb{R}$

- Reference trajectories: $\mathbf{Y}_R := \{y_{R_1}, \ldots, y_{R_I}\} \subset \mathcal{D}(y_0, y_T) \cap \mathcal{G}$

Reference trajectories

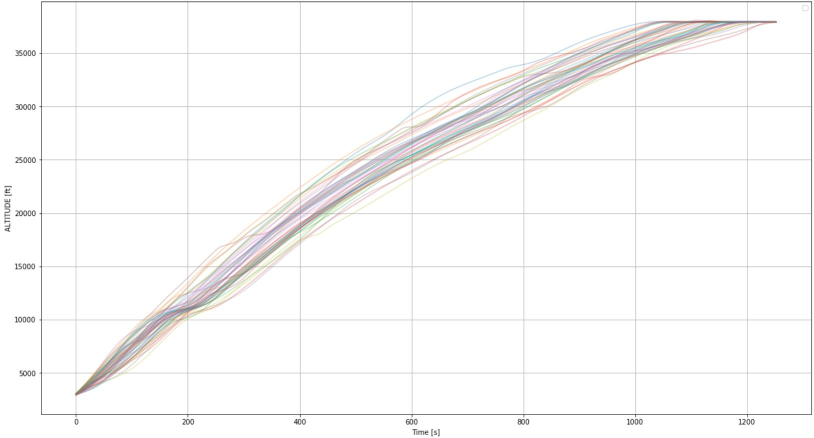

Figure: 48 reference climbs with the same initial and final states – Here the altitude is displayed

Inria

Aim

Find an optimised and constrained trajectory with a realistic pattern without involving the dynamics of the system !

$$\min_{y} F(y) \quad \text{s.t.} \quad \begin{cases} y \in \mathcal{G} \cap \mathcal{D}(y_0, y_T) \\ \cancel{\dot{y}(t) = \hat{g}(t, y(t))} \end{cases}$$

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Key point

Use differently the data to derive a new problem of type

$$\min_{y} \Big\{ F(y) + \kappa \, \mathrm{pen}_{\mathbf{Y}_R}(y) \Big\} \quad \text{s.t.} \;\; y \in \mathcal{G} \cap \mathcal{D}(y_0, y_T)$$

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Consider a statistical framework and interpret the optimised trajectory as a Maximum A Posteriori

1. Finite representation of trajectories

   Trajectory $=$ Vector in $\mathbb{R}^K$

2. Distribution of reference trajectories

   Likelihood function $\longrightarrow$ Penalised term

3. A priori distribution localised on low cost trajectories

   Prior distribution $\longrightarrow$ Initial cost

4. Bayes's rule application

   Posterior $\longrightarrow$ Penalised cost

5. Computation of the posterior mode

   Maximum A Posteriori $=$ Optimised trajectory

- Basis of L^2: $\{\varphi_k\}_k$, each element being continuous
- Finite dimensional space: $\mathcal{Y} := \prod_{d=1}^{D} \operatorname{span} \{\varphi_k\}_{k=1}^{K_d}$
- Finite representation: $y^{(d)}(t) = \sum_{k=1}^{K_d} c_k^{(d)} \varphi_k(t)$
- Equivalence: $\Phi y = c \iff c = \Phi^{-1} y$, with

$$c = \left(c_1^{(1)}, \ldots, c_{K_1}^{(1)},\ c_1^{(2)}, \ldots, c_{K_2}^{(2)}, \ldots,\ c_1^{(D)}, \ldots, c_{K_D}^{(D)} \right)^T \in \mathbb{R}^K$$

- Endpoints conditions: There exists a matrix $A \in \mathbb{R}^{2D \times K}$ such that

$$y \in \mathcal{Y} \cap \mathcal{D}(y_0, y_T) \iff Ac = \begin{pmatrix} y_0 \\ y_T \end{pmatrix}$$

- Hypothesis: The reference trajectories are noisy observations of an efficient trajectory $y = \Phi^{-1}c \in \mathcal{G} \cap \mathcal{D}(y_0, y_T)$

- Choice for the noise: Centered Gaussian with intensity depending on the reference trajectory

- Additional property: The noise should not modify the endpoints

- Modelling: There exists $c \in \mathbb{R}^K$ such that

$$\begin{cases} c_{R_i} = c + \varepsilon_i \\ \varepsilon_i \sim \mathcal{N}(0_{\mathbb{R}^K}, \Sigma_i) \ , \quad \text{with } \Sigma_i = \frac{1}{2\omega_i}\Sigma \\ \varepsilon_i \in \ker A \end{cases}$$

Distribution of $\mathbf{Y}_R$

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⚠ The covariance matrix Σ is here singular !

- Advantage: Gaussian hypothesis indicates simply localisation and degrees of freedom (interpretability !)
- Bonus: Provides also some constraints from the correlations
  $\rightsquigarrow$ Use the fact that the matrices Σ and $A^T A$ are simultaneously diagonalisable to change the basis

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- Equivalent modelling: We can find an explicit basis V such that

$$\begin{cases} \widetilde{c}_{R_i,1} = \widetilde{c}_1 + \widetilde{\varepsilon}_{i,1} \\ \widetilde{\varepsilon}_{i,1} \sim \mathcal{N}\left(0_{\mathbb{R}^\sigma}, \dfrac{1}{2\omega_i}\Lambda_\Sigma\right) \\ V_2^T c_{R_i} = \widetilde{c}_2 \\ V_3^T A^\dagger (y_0 \quad y_T)^T = \widetilde{c}_3 \end{cases}$$

with $\widetilde{c} = V^T c = (V_1 \ V_2 \ V_3)^T c$ and Λ_Σ diagonal and non-singular

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$$\begin{cases} \widetilde{c}_{R_i,1} = \widetilde{c}_1 + \widetilde{\varepsilon}_{i,1} & \longrightarrow \text{Constraints-free} \\ \widetilde{\varepsilon}_{i,1} \sim \mathcal{N}\left(0_{\mathbb{R}^\sigma}, \dfrac{1}{2\omega_i}\Lambda_\Sigma\right) & \\ V_2^T c_{R_i} = \widetilde{c}_2 & \longrightarrow \text{Constraints contained in data} \\ V_3^T A^\dagger (y_0 \quad y_T)^T = \widetilde{c}_3 & \longrightarrow \text{Endpoints constraints} \end{cases}$$

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\end{cases}
$$

with $\widetilde{c} = V^T c = (V_1 \ V_2 \ V_3)^T c$ and Λ_Σ diagonal and non-singular

$$\longrightarrow \quad \mathcal{V} := \left\{ c \in \mathbb{R}^K \;\middle|\; \widetilde{c}_2 = V_2\, c_{R_i} \,,\; \widetilde{c}_3 = V_3^T A^\dagger (y_0 \quad y_T)^T \right\}$$

- A priori knowledge: Efficient trajectories with respect to the cost F are the most likely ones
- Reminder: A trajectory depends only on the component $\widetilde{c}_1$
- Restricted cost function: Let $\widetilde{F}$ be the restriction to $\mathcal{V}$ of the cost $\check{F} = F \circ \Phi^{-1}$ defined on $\mathbb{R}^K$
- Prior: For $\kappa > 0$,

$$u(\widetilde{c}_1) \propto \exp\Big(- \kappa^{-1} \widetilde{F}(\widetilde{c}_1) \Big)$$

- Posterior: By Bayes's rule, we have

$$u(\widetilde{c}_1 \,|\, \widetilde{c}_{R_1,1}, \ldots, \widetilde{c}_{R_I,1}) \propto u(\widetilde{c}_{R_1,1}, \ldots, \widetilde{c}_{R_I,1} \,|\, \widetilde{c}_1)\, u(\widetilde{c}_1)$$

- Likelihood: Independence assumption and preceding modelling give

$$\begin{aligned} u(\widetilde{c}_{R_1,1}, \ldots, \widetilde{c}_{R_I,1} \,|\, \widetilde{c}_1) &= \prod_{i=1}^{N} u(\widetilde{c}_{R_i,1} \,|\, \widetilde{c}_1) \\ &\propto \prod_{i=1}^{N} \exp\Big(- \omega_i \left(\widetilde{c}_1 - \widetilde{c}_{R_i,1}\right)^T \Lambda_{\Sigma,1}^{-1} \left(\widetilde{c}_1 - \widetilde{c}_{R_i,1}\right)\Big) \end{aligned}$$

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- Maximum A Posteriori: Take the negative of the logarithm

$$\begin{cases} \widetilde{c}_1^\star \in \underset{\widetilde{c}_1 \in \mathbb{R}^\sigma}{\arg\min}\, \widetilde{F}(\widetilde{c}_1) + \kappa \displaystyle\sum_{i=1}^{N} \omega_i \left(\widetilde{c}_1 - \widetilde{c}_{R_i,1}\right)^T \Lambda_{\Sigma,1}^{-1} \left(\widetilde{c}_1 - \widetilde{c}_{R_i,1}\right) \\ \widetilde{c}_2^\star = V_2^T\, c_{R_i} \\ \widetilde{c}_3^\star = V_3^T A^\dagger (y_0 \quad y_T)^T \end{cases}$$

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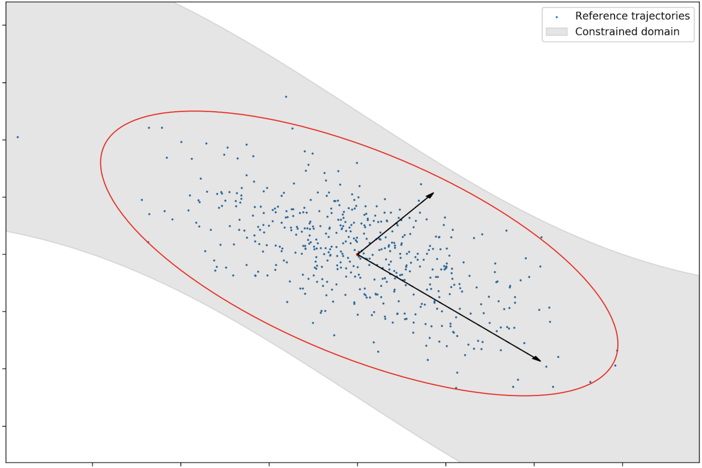
- Maximum A Posteriori: Take the negative of the logarithm and change the basis back to obtain

$$c^\star \in \operatorname*{arg\,min}_{c \in \mathcal{V}} \check{F}(c) + \kappa \sum_{i=1}^{N} \omega_i \left(c - c_{R_i}\right)^T \Sigma^\dagger (c - c_{R_i})$$

Search space visualisation



- Additional constraints: The solution $y_\kappa^\star = \Phi^{-1} c_\kappa^\star$ should belong to the set $\mathcal{G}$! Not explicitly taken into account in

$$c_\kappa^\star \in \underset{c \in \mathcal{V}}{\arg\min}\, \check{F}(c) + \kappa \sum_{i=1}^{N} \omega_i \left(c - c_{R_i}\right)^T \Sigma^\dagger \left(c - c_{R_i}\right)$$

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- Observation: In the limit cases, we have

	$\kappa = 0$	$\kappa = +\infty$
$y_\kappa^\star \in \mathcal{G}$	✗	✓
$\min \check{F}$	✓	✗

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$\min \check{F}$	✓	✗

- Idea: Tune κ to find a compromise between optimisation and constraints
  $\longrightarrow$ Iterative approach (linear search, binary search,...)

- Hypothesis: Quadratic instantaneous cost[†]

$$F(y) = \int_0^T y(t)^T Q y(t) + w^T y(t) + r \, dt$$

- Quadratic optimisation problem:

$$c^\star \in \underset{c \in \mathcal{V}}{\arg\min}\, c^T \left(\check{Q} + \kappa \, \Sigma^\dagger \right) c + \left(\check{w} - 2\,\kappa \sum_{i=1}^{N} \omega_i \, \Sigma^\dagger c_{R_i} \right)^T c$$

- Convex optimisation problem: The above problem is convex if

$$\kappa \geqslant -\lambda_{min}\left(V_1^T \check{Q} V_1 \right) \lambda_{max}(\Sigma)$$

† F. Dewez, B. Guedj, V. Vandewalle. *From industry-wide parameters to aircraft-centric on-flight inference: improving aeronautics performance prediction with machine learning*. Data-Centric Engineering, 2020.

- **Py**thon **Ro**ute **t**rajectory **o**ptimise**r**
- Preceding generic method developed in `Python` to be used in a wide range of fields
- Based on well-known librairies (`SciPy`, `NumPy`, `sklearn`,...)
- Developed on GitHub: https://github.com/bguedj/pyrotor

03

Application: Minimisation of aircraft fuel consumption during the climb phase

- Setting: Climbing phase of an aircraft in a vertical plane
- Cost function: Total fuel consumption

$$\mathrm{TFC}(y) := \int_0^T \widehat{\mathrm{FF}}\big(y(t)\big)\,dt$$

where FF is the fuel flow

- Trajectory: Altitude h, Mach M, engines power $\mathrm{N1}$

$$\forall\, t \in [0, T] \qquad y(t) := \big(h(t), \mathrm{M}(t), \mathrm{N1}(t)\big)$$

- Endpoints conditions:
  1. Altitude: from 3,000 ft to 38,000 ft;
  2. Mach: from 0.3 to 0.78.
- Additional constraints:
  1. Rate of climb smaller than 3,600 ft/min;
  2. Mach smaller than 0.82.

- Data: 2,162 recorded flights; 48 satisfy endpoints conditions (used to estimate Σ)
- Fuel flow: Fitted to the climb data by a quadratic model ($\sim 500,000$ observations); error: 1.73 %
- Weights ω_i: Chosen so that only the five most fuel-efficient trajectories are involved in the penalised term
- Climb duration: Given by the first time where final endpoints are reached by the optimised flight
- Basis: Legendre polynomials for each state; overall dimension $= 20$

Visualisation

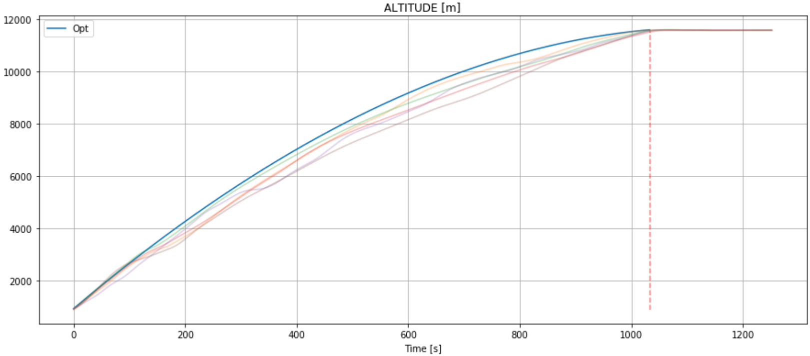

Figure: Altitude

- Fuel savings = 260 ± 86 kg (min = 72 kg, max = 394 kg)
- Percentages = 17 ± 5 % (min = 5%, max = 23%)
- Execution time = 4 ± 0.1 s (Intel Core i7 6 cores, 2.2 GHz)

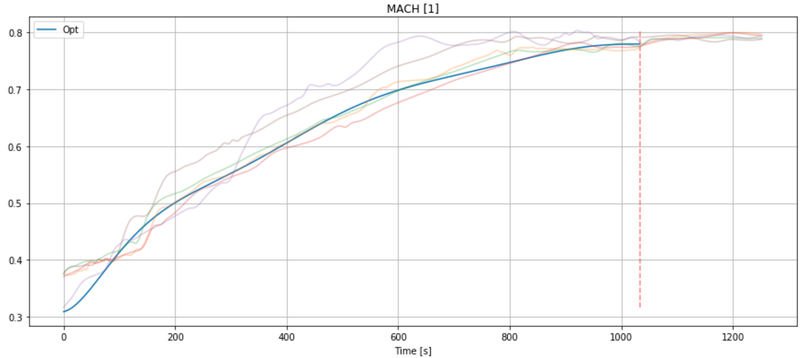

Figure: Mach number

- Fuel savings = 260 ± 86 kg (min = 72 kg, max = 394 kg)
- Percentages = 17 ± 5 % (min = 5%, max = 23%)
- Execution time = 4 ± 0.1 s

Visualisation

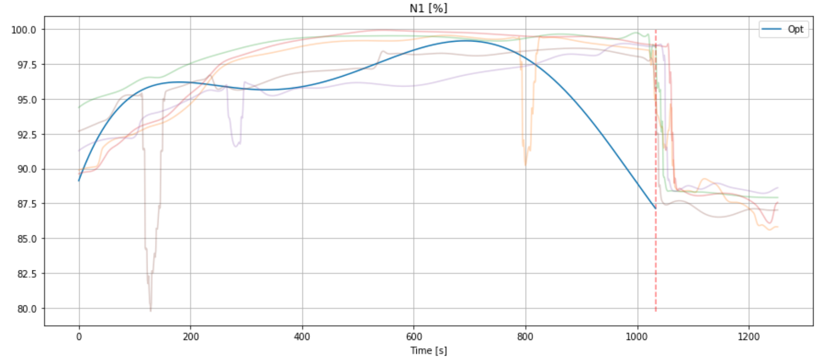

Figure: Engines power

- Fuel savings $= 260 \pm 86$ kg (min $= 72$ kg, max $= 394$ kg)
- Percentages $= 17 \pm 5$ % (min $= 5\%$, max $= 23\%$)
- Execution time $= 4 \pm 0.1$ s

04

On the horizon

Outlook

Improvements/Extensions

- Different modellings for the reference trajectories
- Clustering step applied to the reference trajectories
- Combination with optimal control methods†
- Other kinds of optimisation problems

† C. Rommel, F.C. Bonnans, P. Martinon, B. Gregorutti. *Gaussian mixture penalty for trajectory optimization problems*. Journal of Guidance, Control, and Dynamics 42 (8), 1857–1862, 2019.

Improvements/Extensions

- Different modellings for the reference trajectories
- Clustering step applied to the reference trajectories
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- Other kinds of optimisation problems

New applications

- Health: optimal dose-response, patient path,...
- Sailing: routing, best control for a manoeuvre,...
  $\rightarrow$ Exploratory project with the Fédération Française de Voile

† C. Rommel, F.C. Bonnans, P. Martinon, B. Gregorutti. *Gaussian mixture penalty for trajectory optimization problems*. Journal of Guidance, Control, and Dynamics 42 (8), 1857–1862, 2019.

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