An Induce-on-Boundary Magnetostatic Solver for Grid-Based Ferrofluids

Xingyu Ni*[1], Ruicheng Wang*[1], Bin Wang†[2], Baoquan Chen†[1]

[1] Peking University [2] Beijing Institute for General Artificial Intelligence

* Joint first authors † Corresponding authors

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Introduction

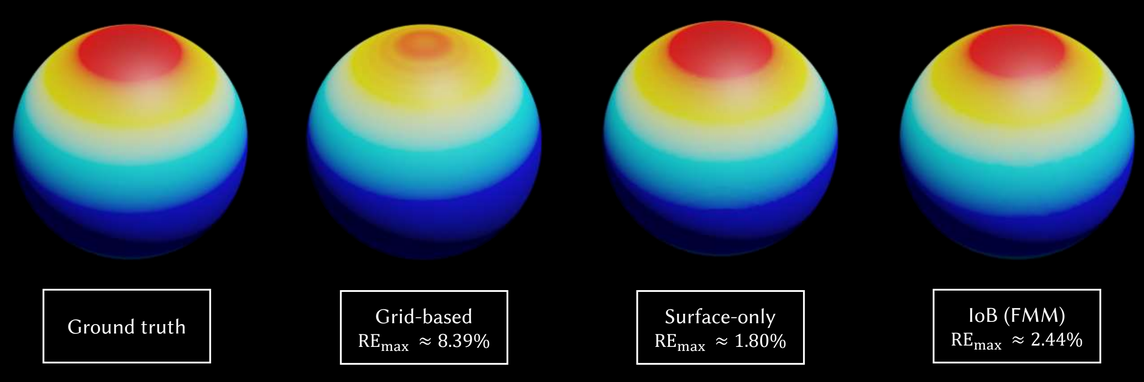
This paper introduces a novel Induce-on-Boundary (IoB) solver designed to address the magnetostatic governing equations of ferrofluids. The IoB solver offers a lightweight, fast, and accurate solution for calculating magnetic fields. It holds minimal computational complexities with a straightforward Fixed-Point Iterations solver, achieving state-of-the-art performance among existing methods. We show its effectiveness and compatibility by integrating it into the grid-based fluid and rigid-body simulation pipelines.

Method	Force type	Physical accuracy		Linear solver	Complexity
		Sharp interface	BC at infinity		
SPH	Volumetric	No	Yes	CG Squared	$O(N\sqrt{N})$
Grid-based	Interfacial	No	No	GMGPCG	$O(N\sqrt{N})$
Surface-only	Interfacial	Yes	Yes	GMRES	$O(N^2)$
IoB (ours)	Interfacial	Yes	Yes	FPI	$O(N)$

For SPH methods, $N_p \approx N\sqrt{N}$, where N_p denotes the number of SPH particles; for the grid-based method, $n^3 \approx N\sqrt{N}$, where n is the grid resolution.

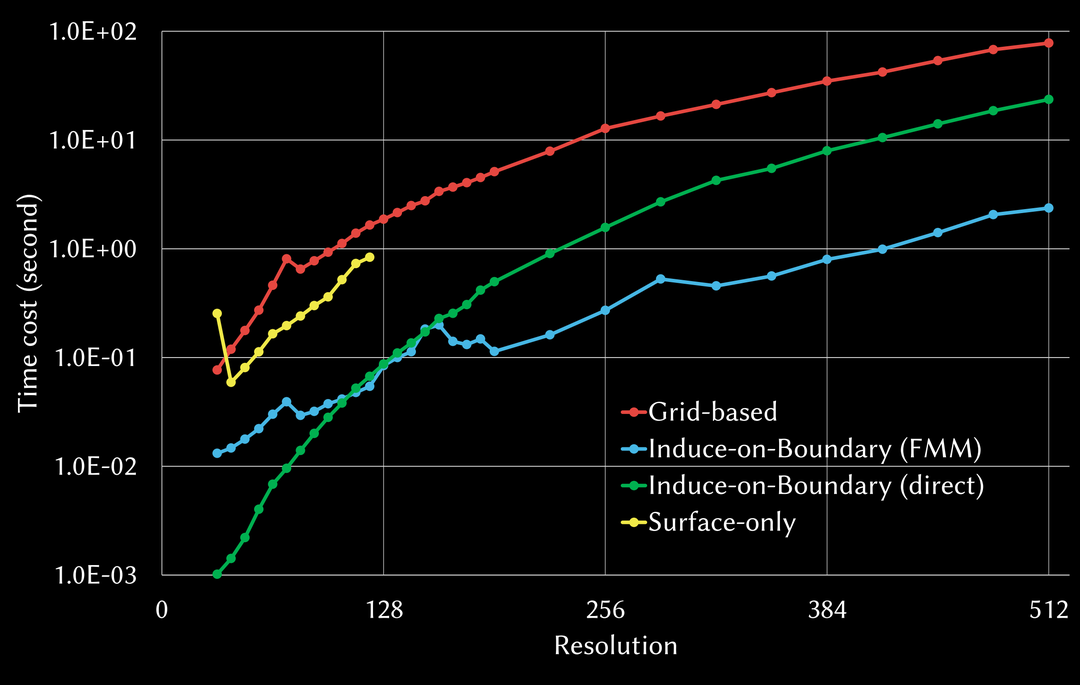
Validation

Accuracy



Ground truth | Grid-based $RE_{max} \approx 8.39\%$ | Surface-only $RE_{max} \approx 1.80\%$ | IoB (FMM) $RE_{max} \approx 2.44\%$

Performance



Method Overview

$$\left.\frac{\partial \psi}{\partial n}\right|_{+} + \chi \boldsymbol{H}_{\text{app}} \cdot \boldsymbol{n} = (1+\chi)\left.\frac{\partial \psi}{\partial n}\right|_{-} \quad \text{At interface } (\partial\Omega)$$

$$\psi = 0 \quad \text{At infinity}$$

$$\psi(\boldsymbol{x}) = \iint_{\partial\Omega} G(\boldsymbol{x},\boldsymbol{y})\phi(\boldsymbol{y})\mathrm{d}A_y \qquad G(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{4\pi}\frac{1}{\|\boldsymbol{x}-\boldsymbol{y}\|}$$

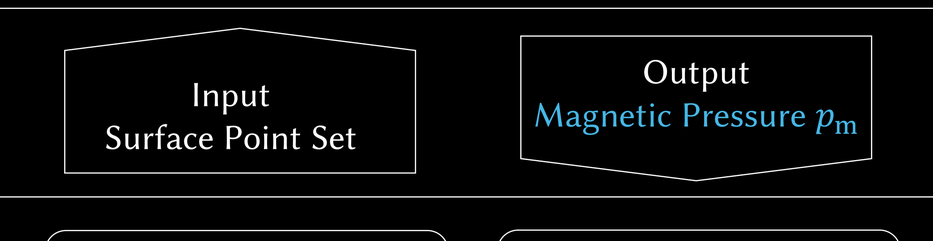
Boundary Conditions ⟩ Single-Layer Potential

$$\frac{2+\chi}{2\chi}\phi(\boldsymbol{x}) = -\iint_{\partial\Omega}\frac{\partial G}{\partial n_x}(\boldsymbol{x},\boldsymbol{y})\phi(\boldsymbol{y})\mathrm{d}A_y + \boldsymbol{H}_{\text{app}}(\boldsymbol{x})\cdot\boldsymbol{n}$$

Unknowns — Indirect Induction — Direct Source

Boundary Integral Equation

(Fredholm integral equation of the second kind)



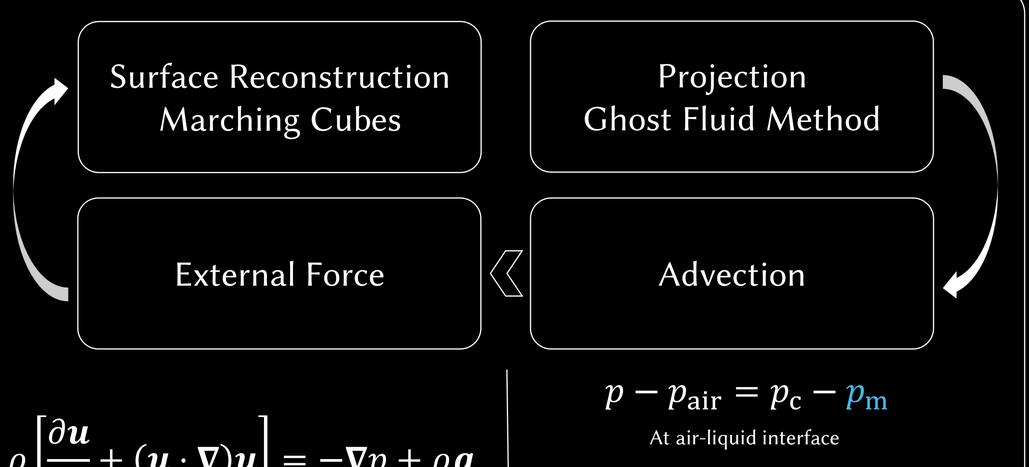
$$\frac{2+\chi}{2\chi}\phi_i = -\sum_{j=1}^{N}\frac{\partial G}{\partial n_i}(\boldsymbol{x}_i,\boldsymbol{x}_j)\phi_j\Delta A_j + \boldsymbol{H}_{\text{app}}(\boldsymbol{x}_i)\cdot\boldsymbol{n}_i$$

Linear Solver - Fixed Point Iterations

- Accelerated with Fast Multipole Method (FMM) – $O(N^2) \rightarrow O(N)$.
- A progress of magnetization and induction – Induce-on-Boundary.
- Unconditionally convergent for any χ.

Input: Surface Point Set

Output: Magnetic Pressure p_{m}



$$\rho\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right] = -\nabla p + \rho\boldsymbol{g}$$

$$p - p_{\text{air}} = p_{\text{c}} - p_{\text{m}} \quad \text{At air-liquid interface}$$

$$\boldsymbol{u}\cdot\boldsymbol{n} = \boldsymbol{u}_{\text{solid}}\cdot\boldsymbol{n} \quad \text{At solid-liquid interface}$$

Grid-based Fluid Simulation Pipeline

Modeling — Discretization — Numerical Solver — Coupling

Results