Semidefinite relaxations for certifying robustness to adversarial examples

Jacob Steinhardt

Percy Liang

Aditi Raghunathan

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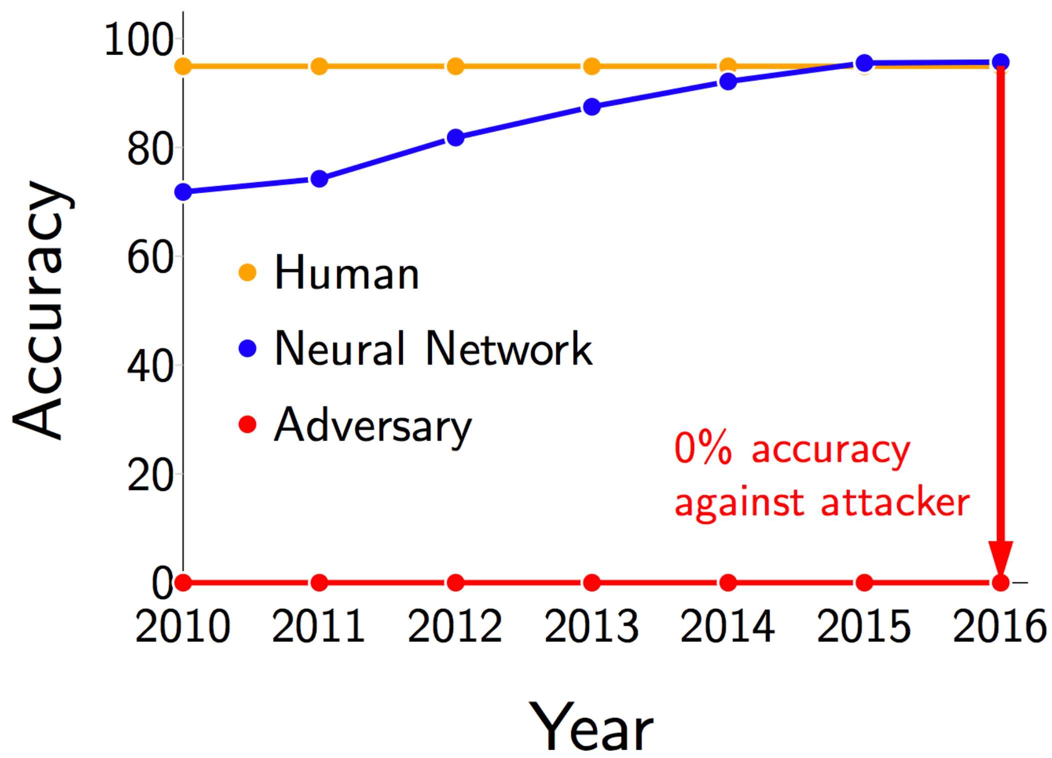

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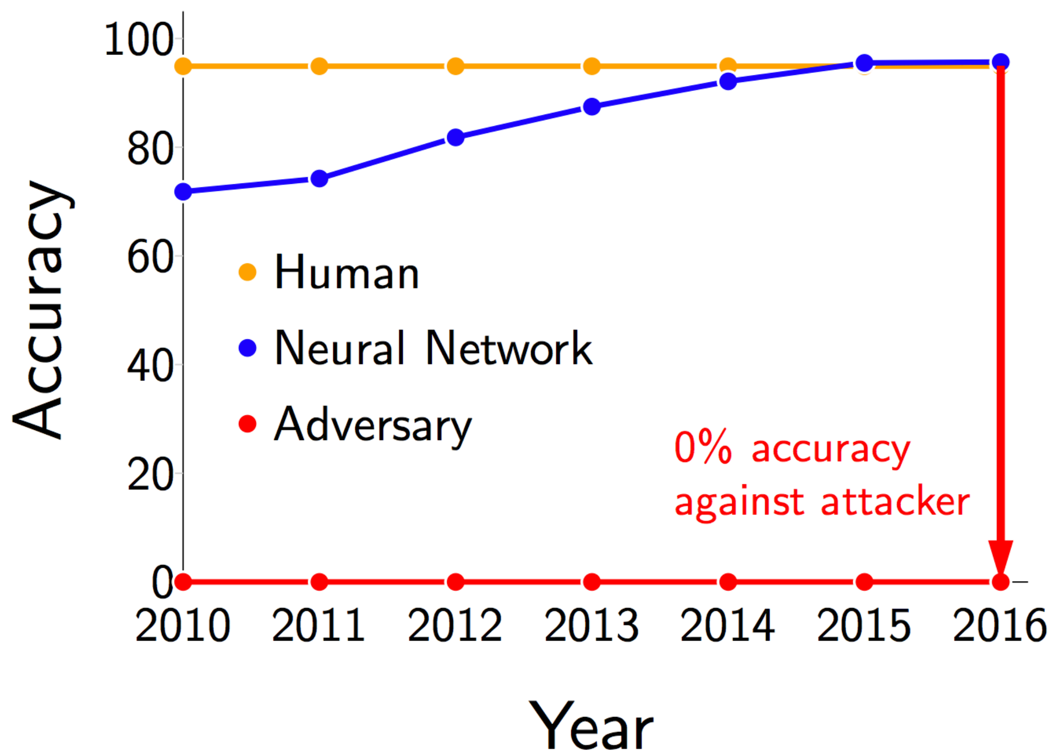

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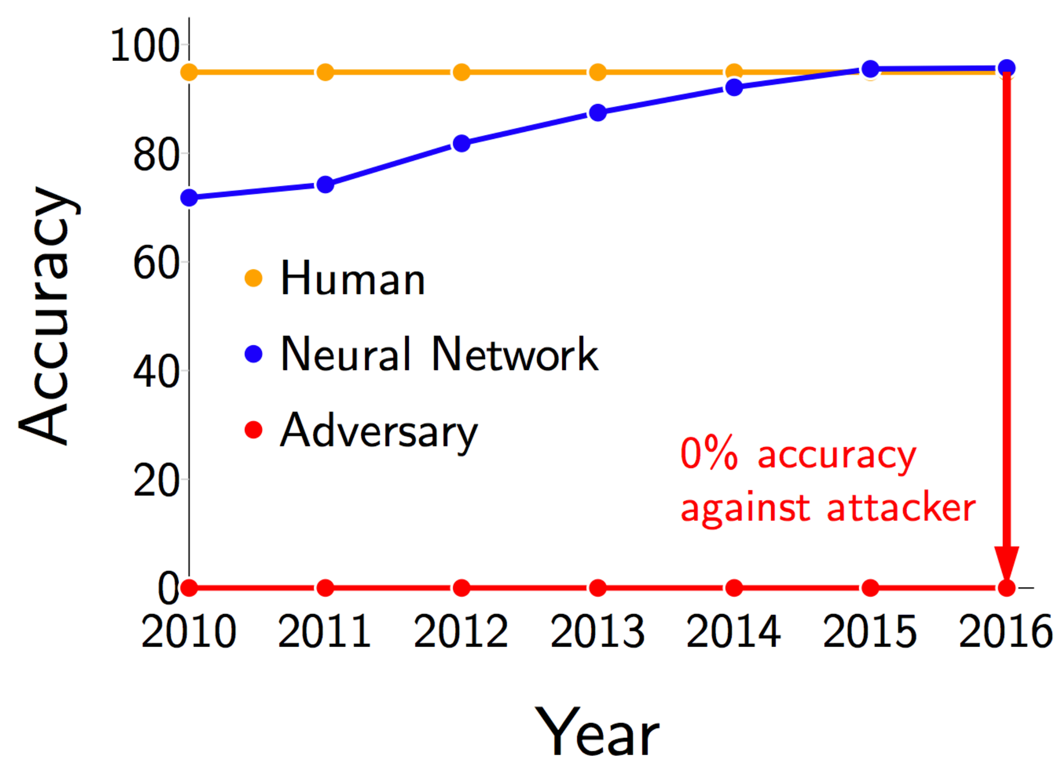

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- Focus on **adversarial examples** — manipulation of test inputs

Adversarial Examples

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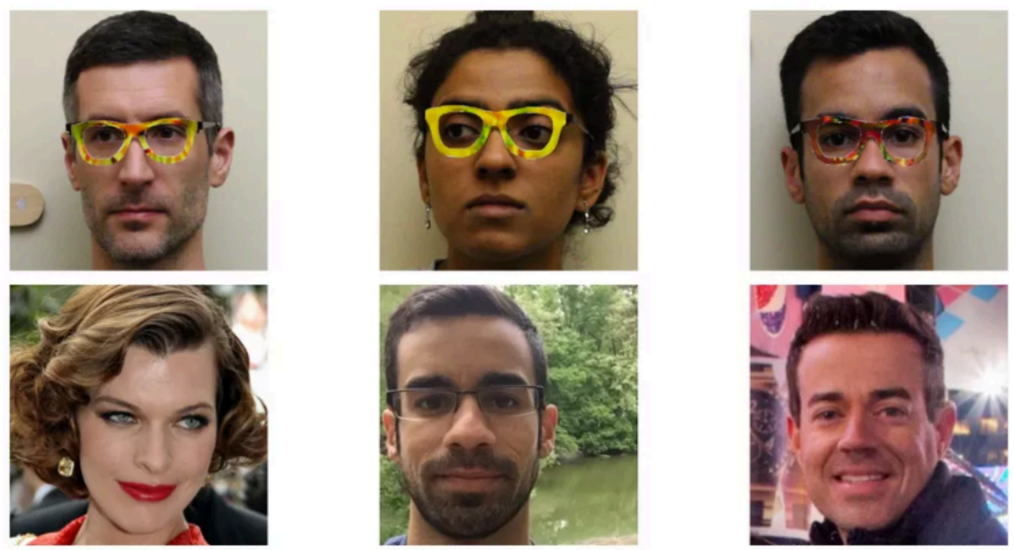

Glasses → Impersonation

[Sharif et al. 2016]

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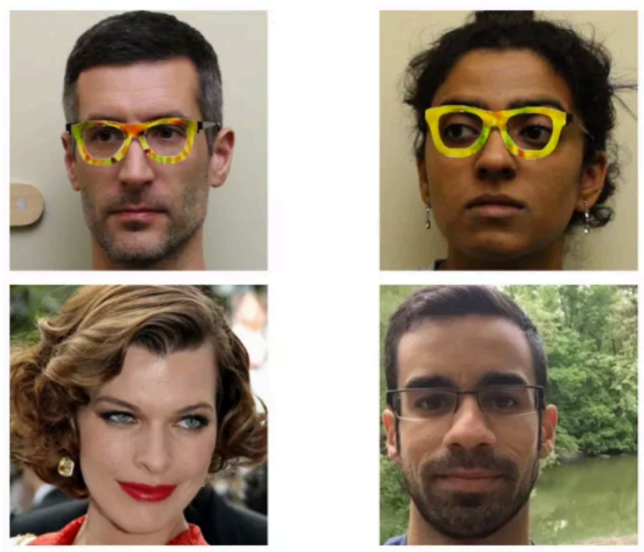

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Stop + sticker →Yield

[Evtimov et al. 2017]

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3D Turtle → Rifle

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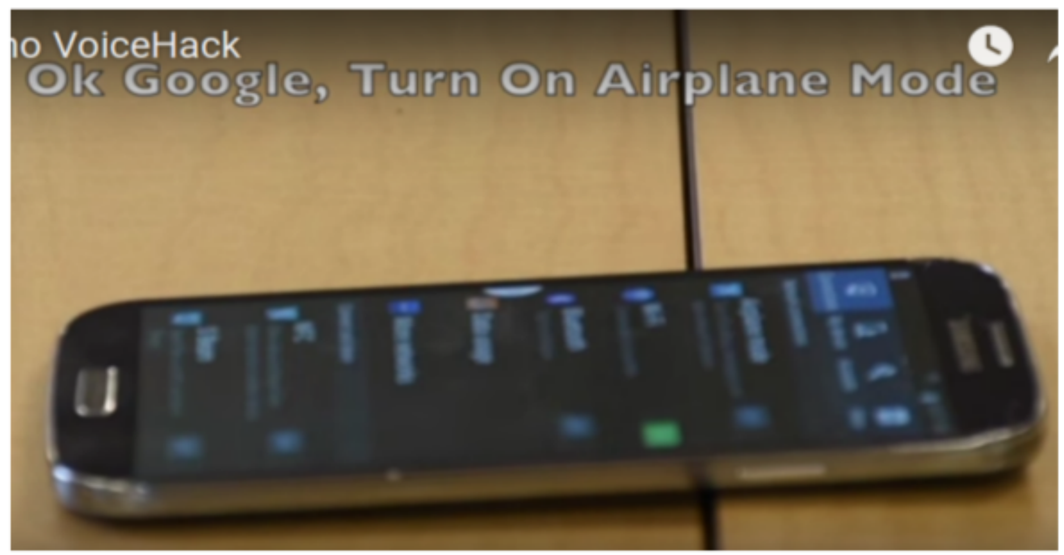

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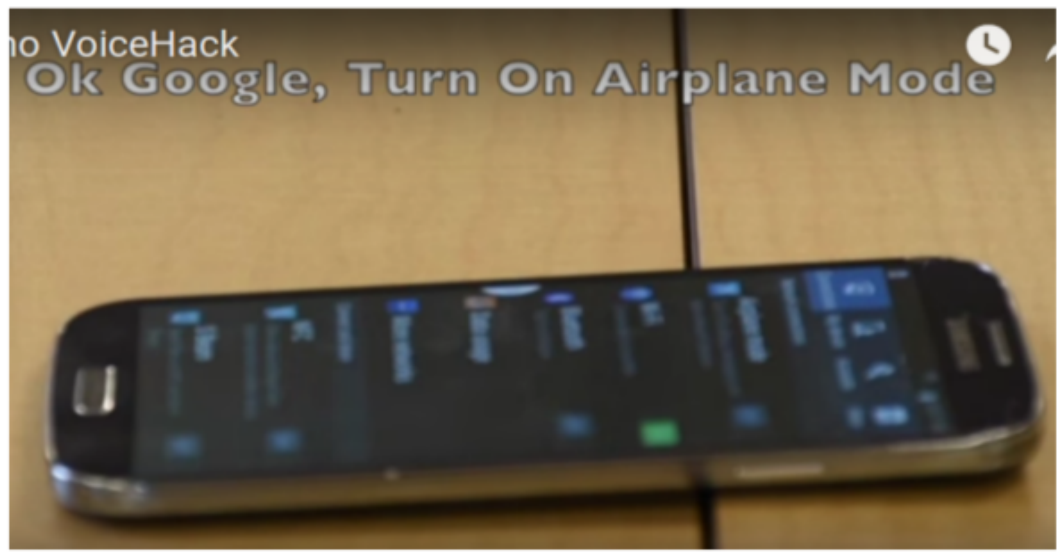

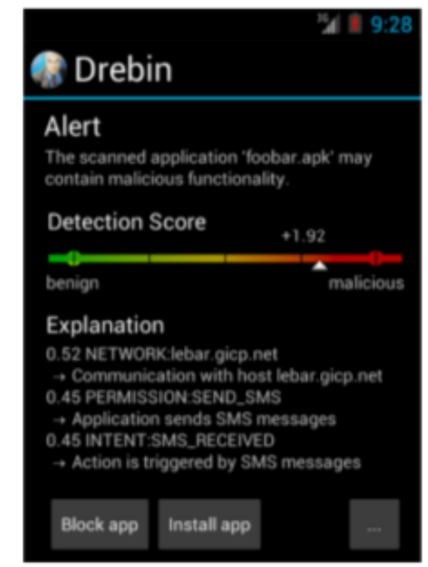

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Malware → Benign

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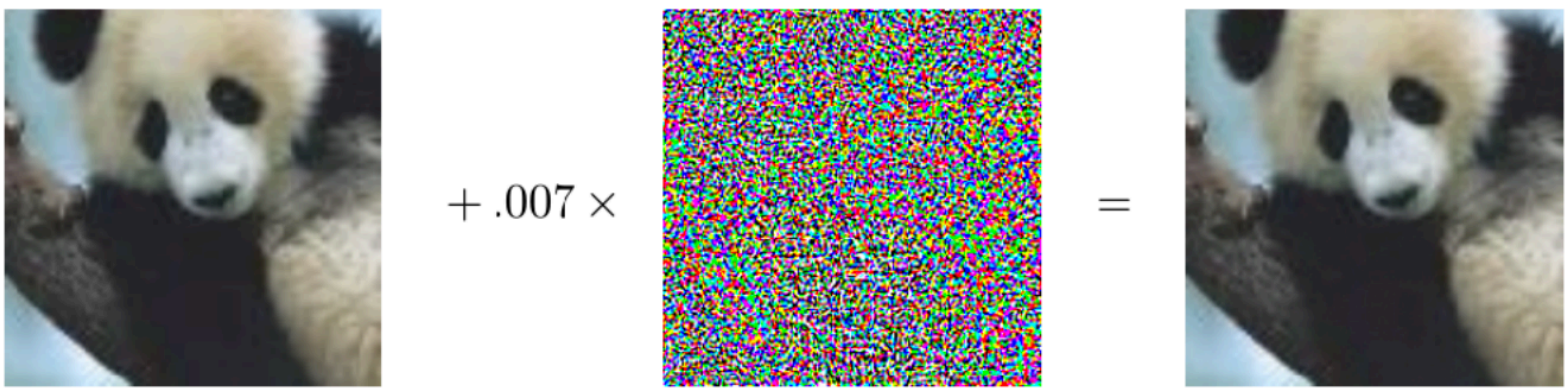

Panda

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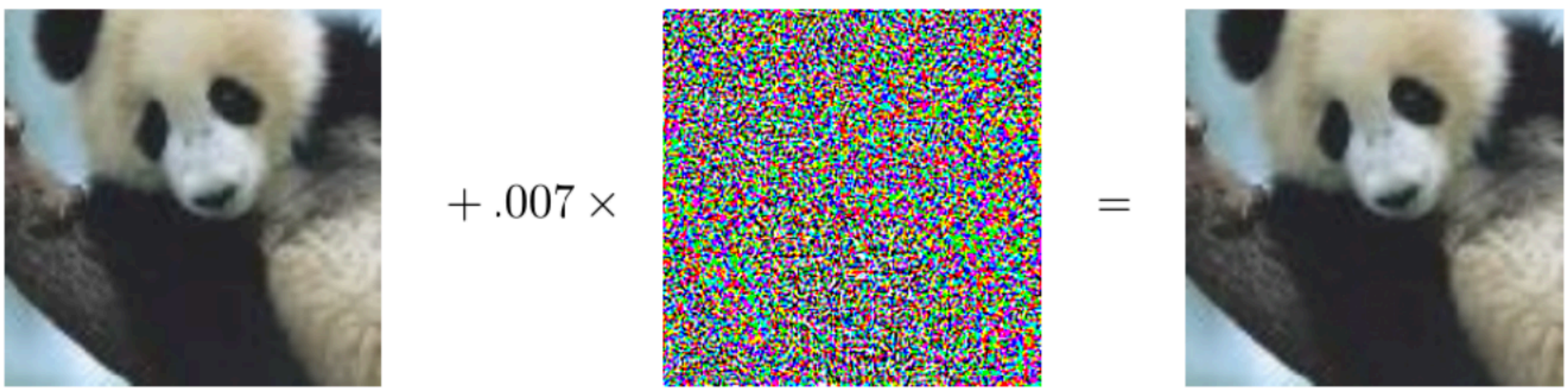

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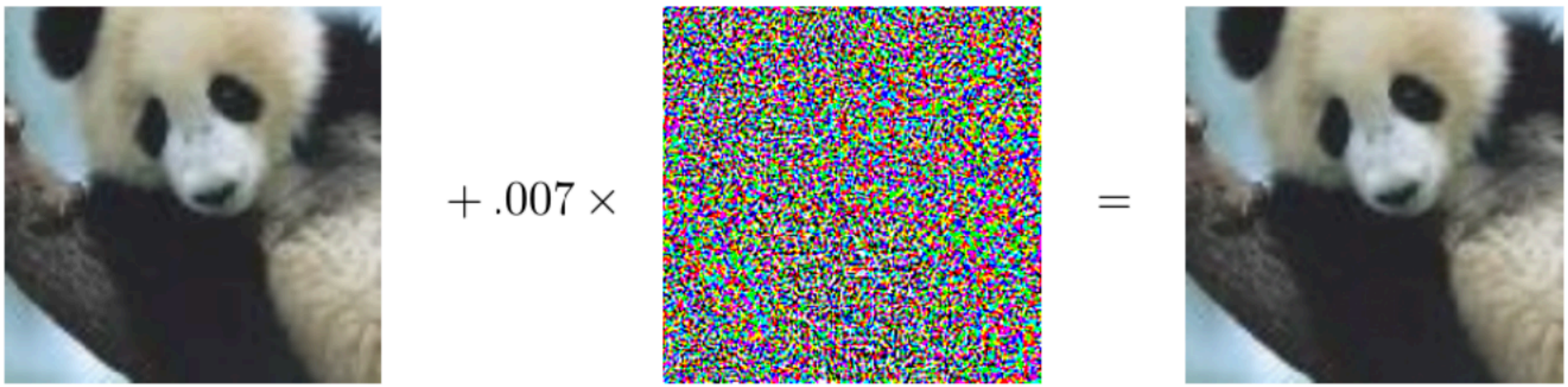

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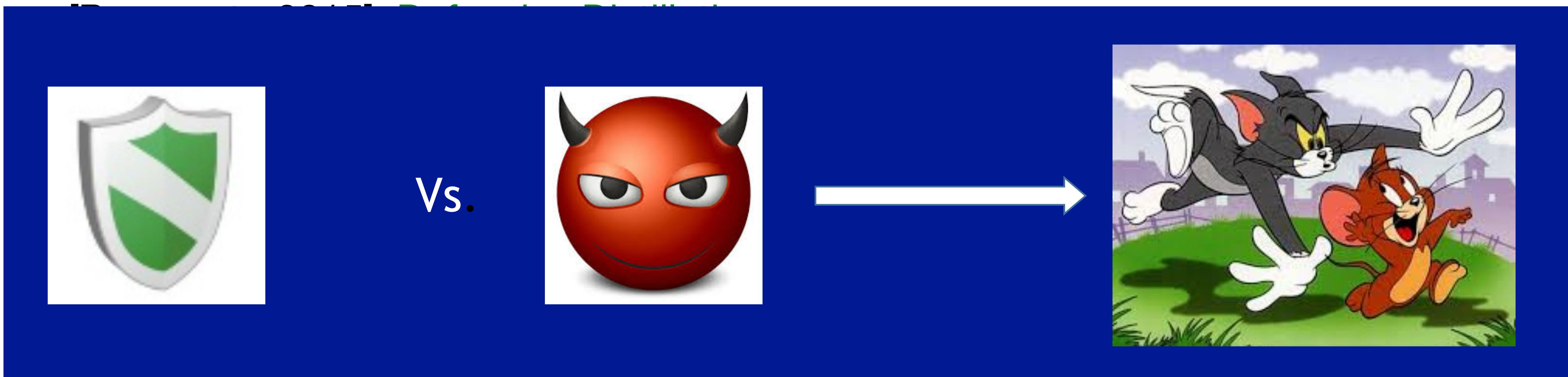

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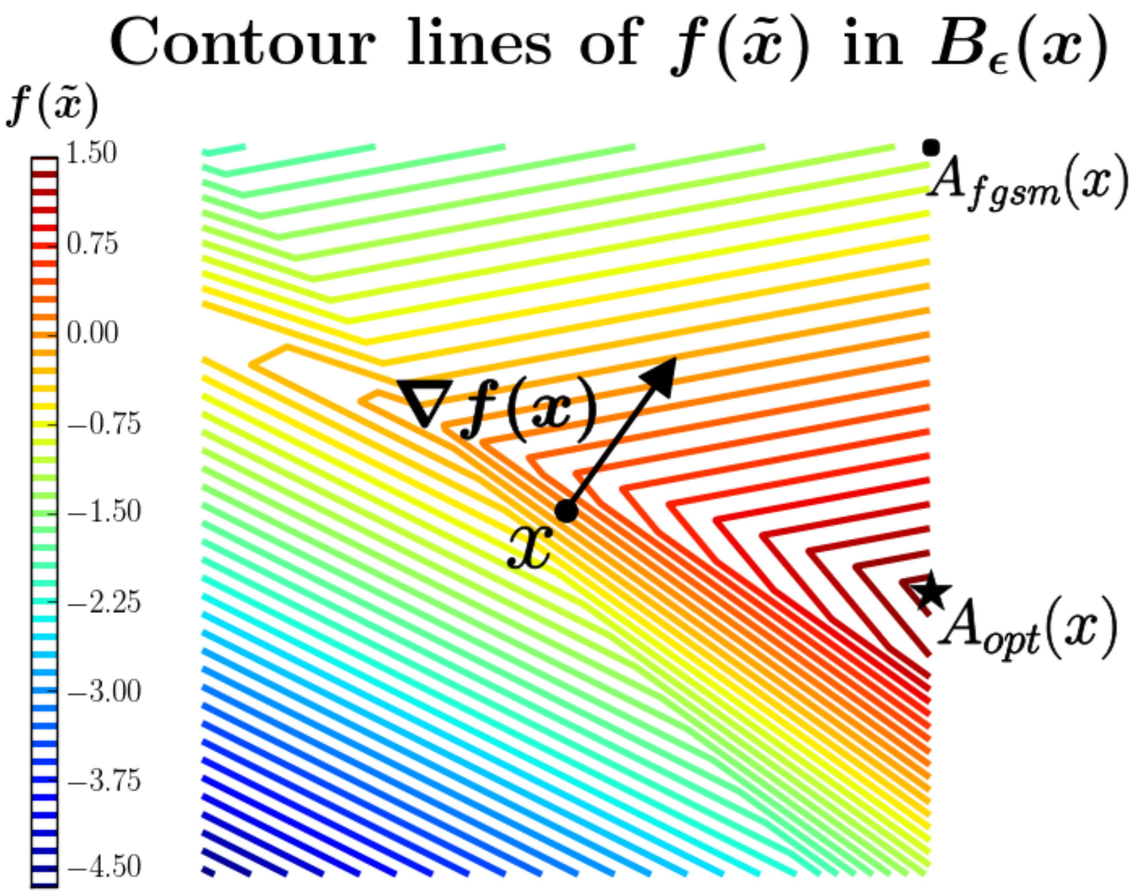

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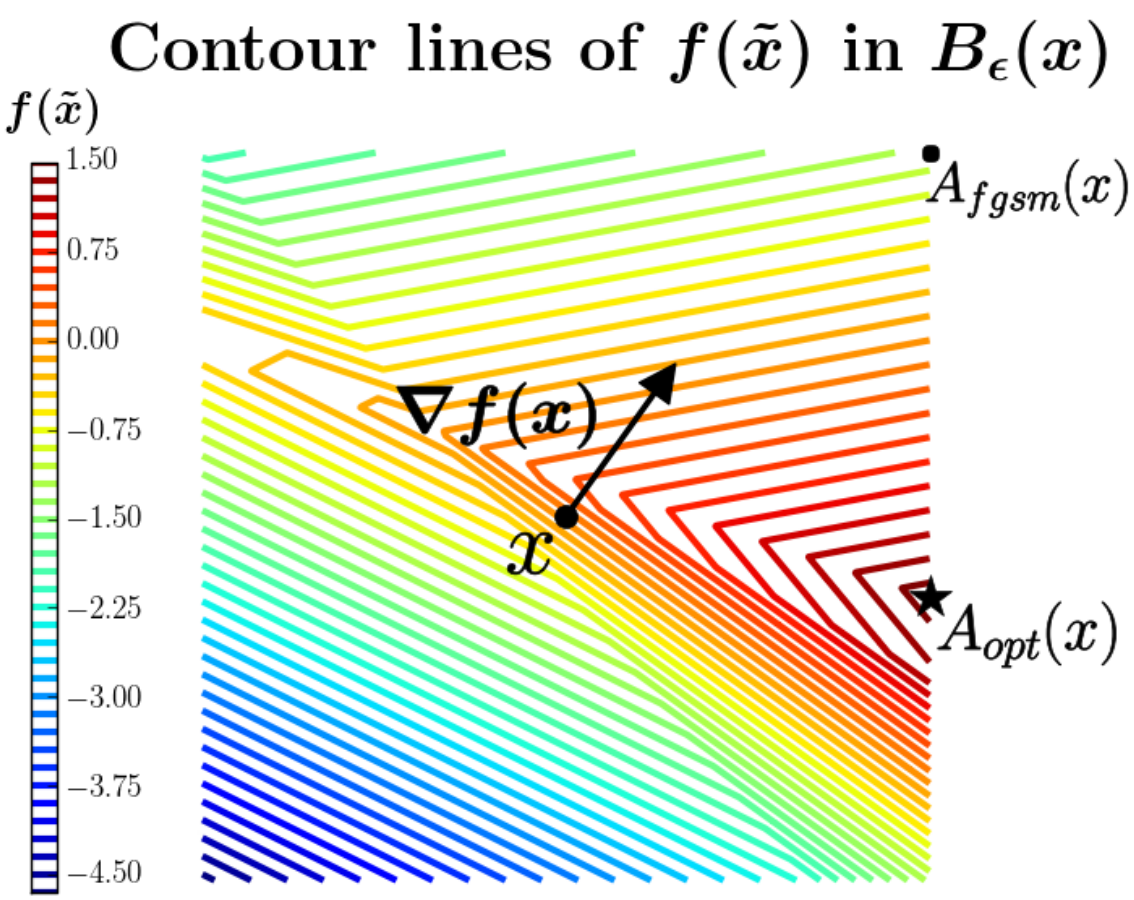

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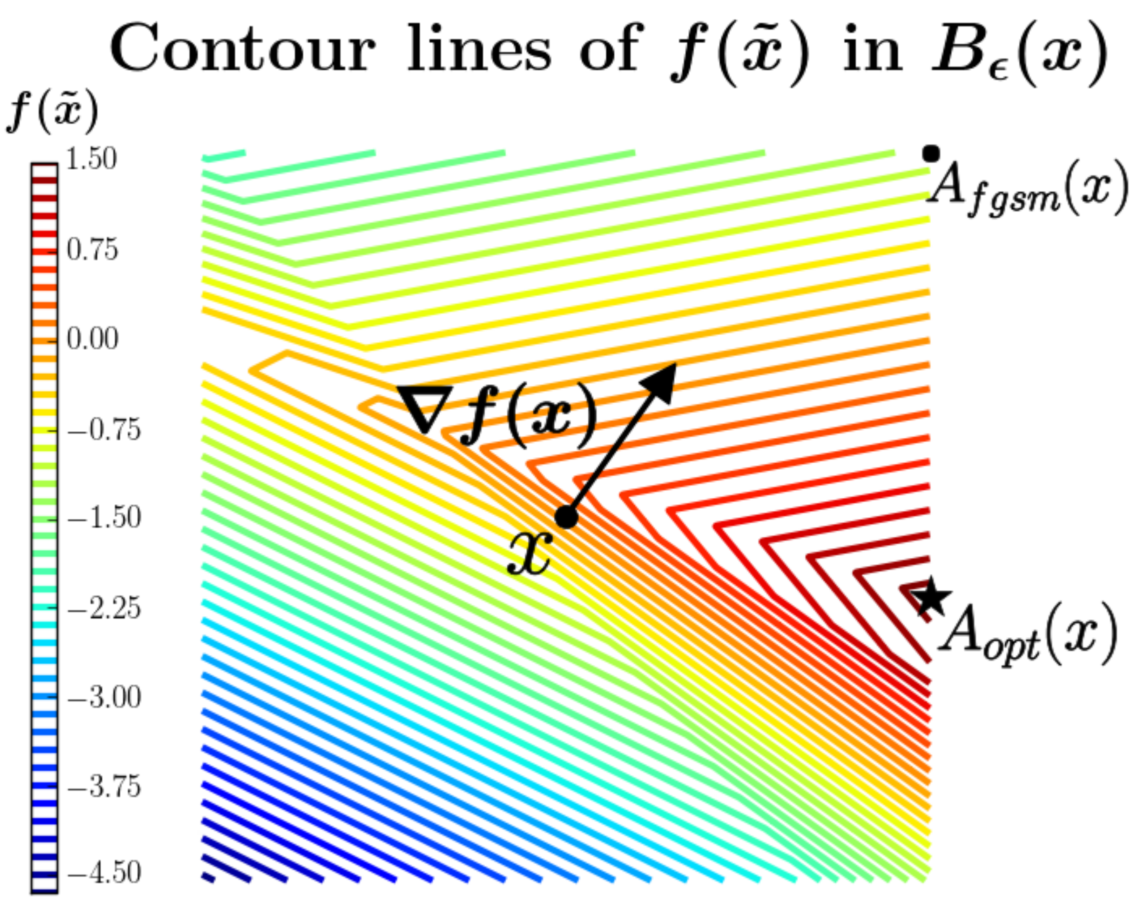

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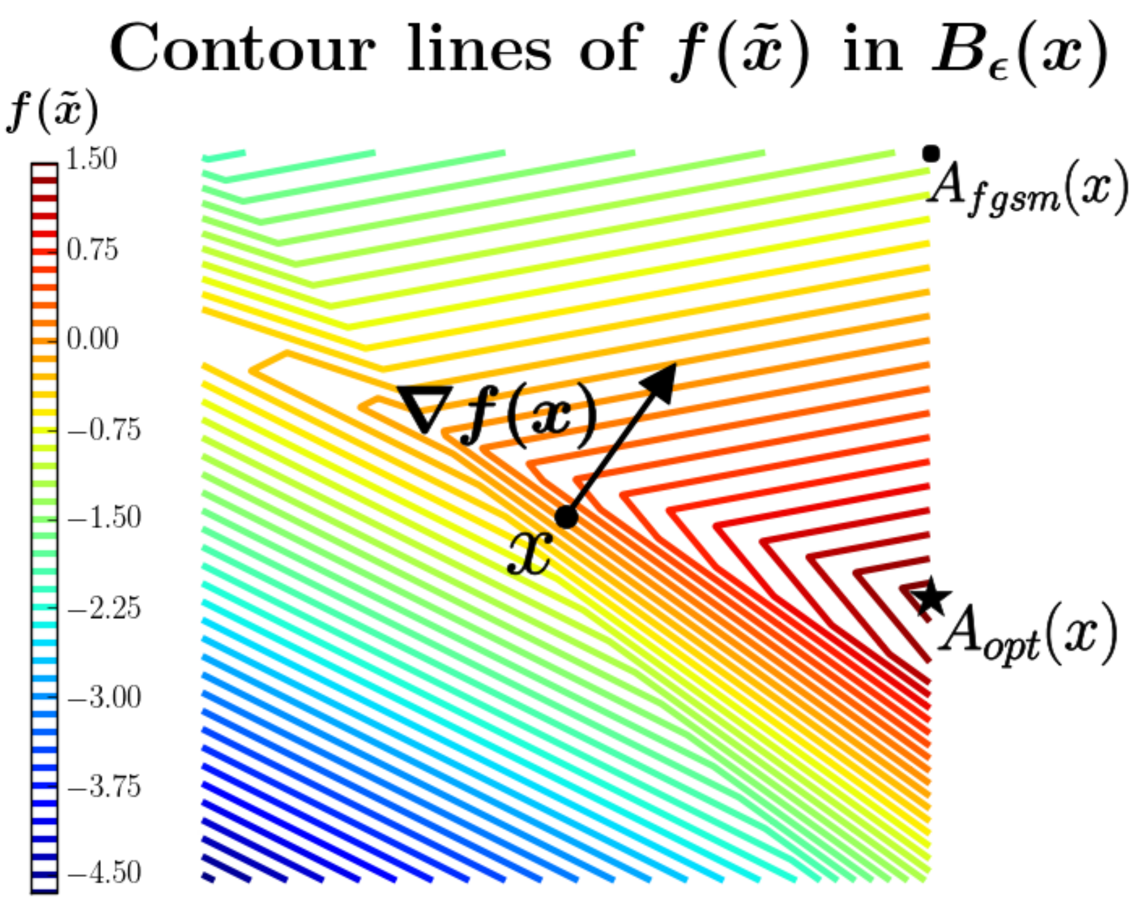

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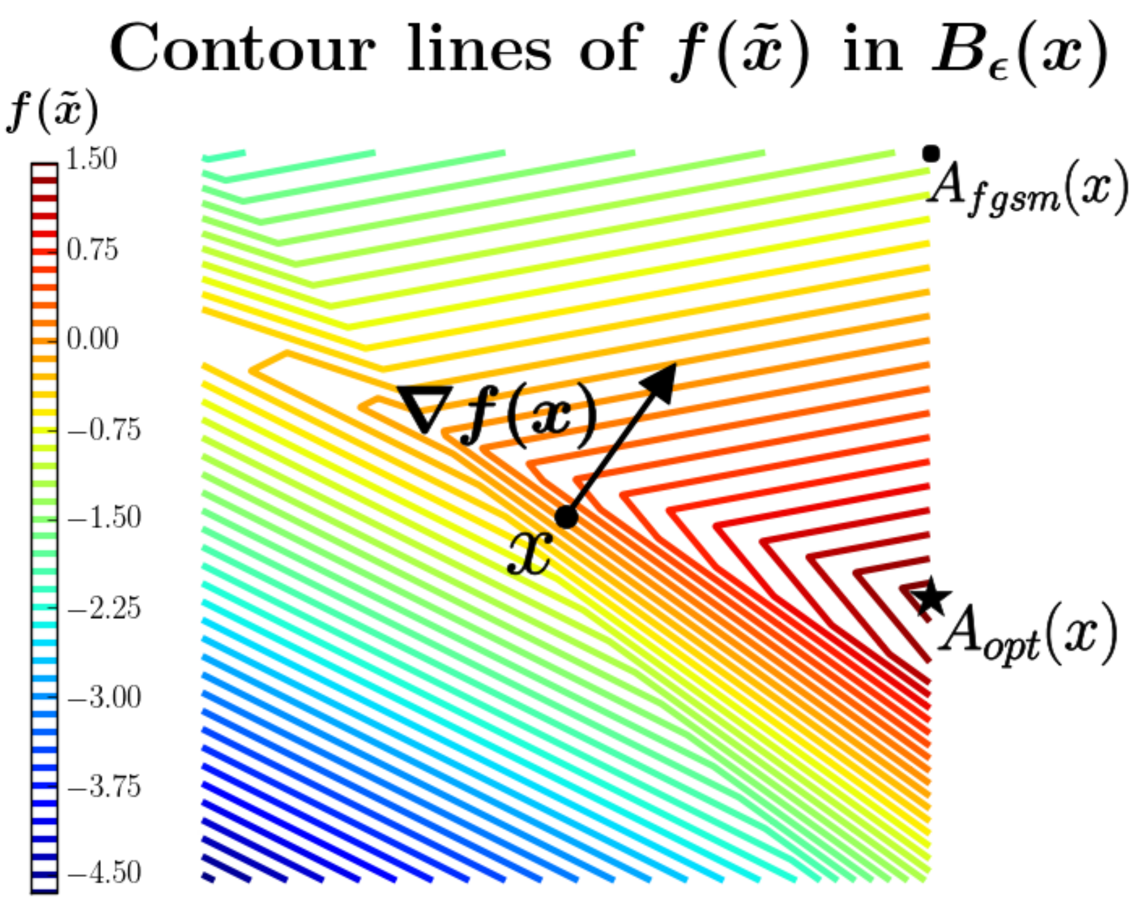

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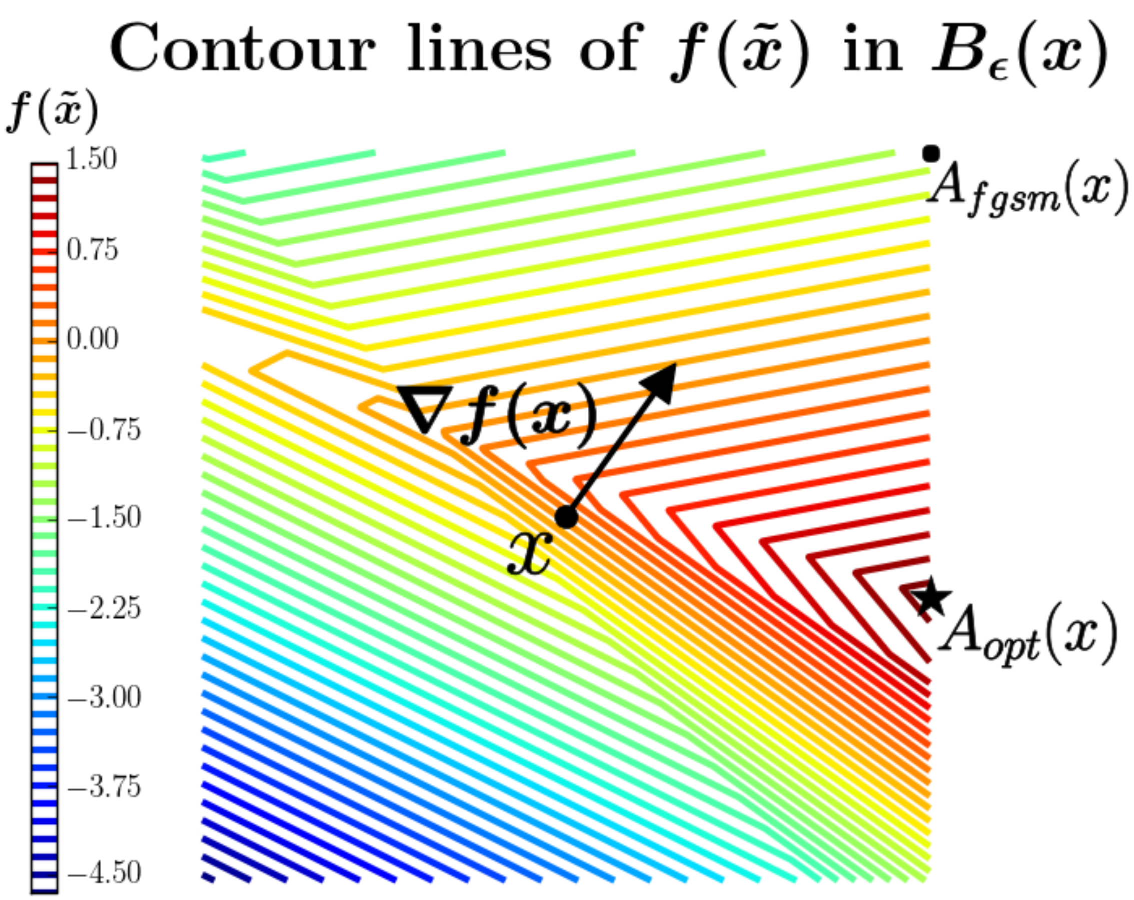

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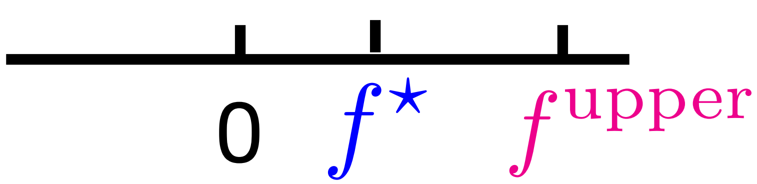

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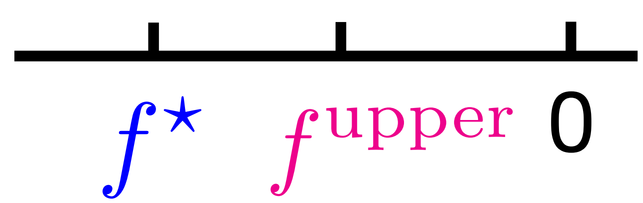

Robust and certified

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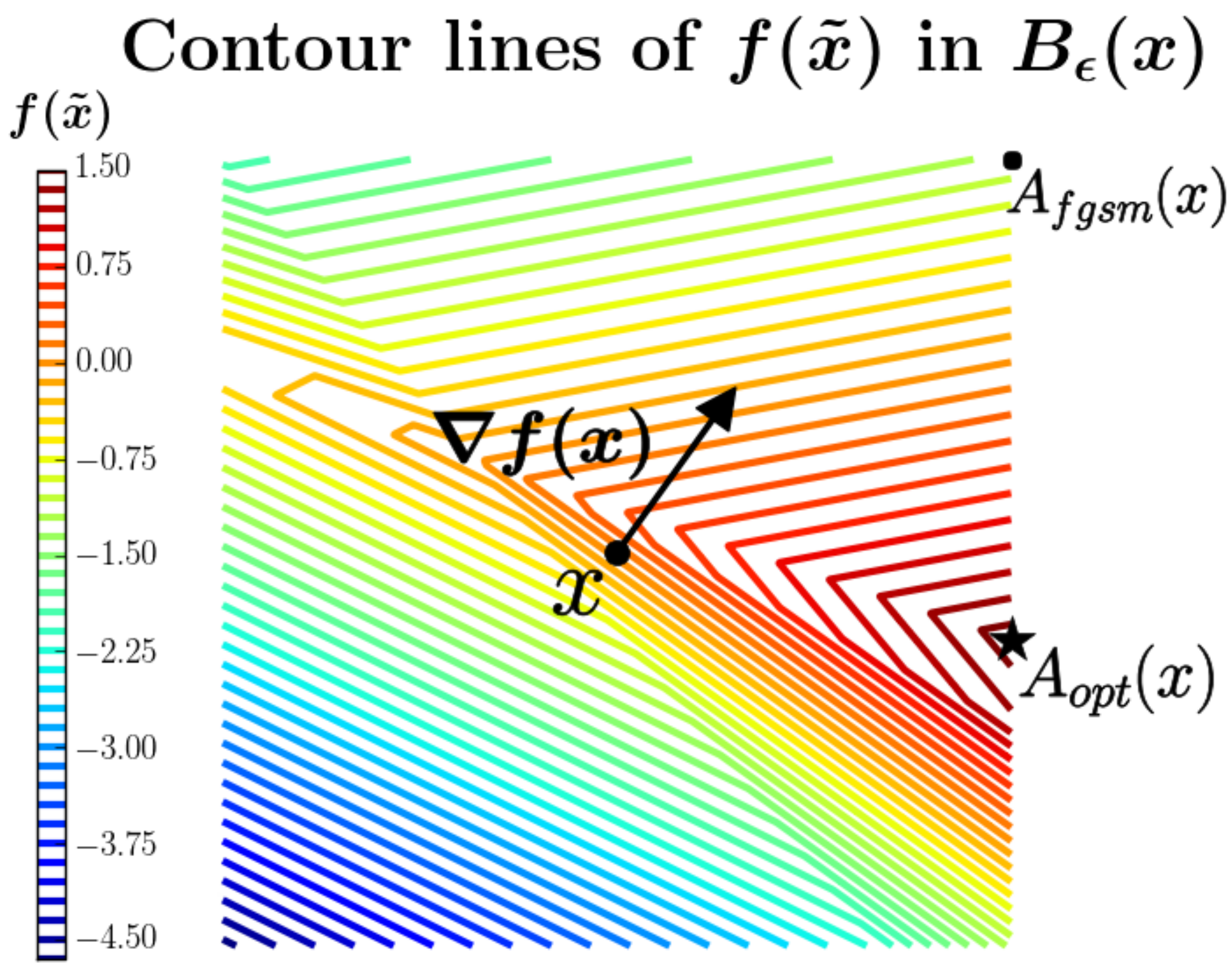

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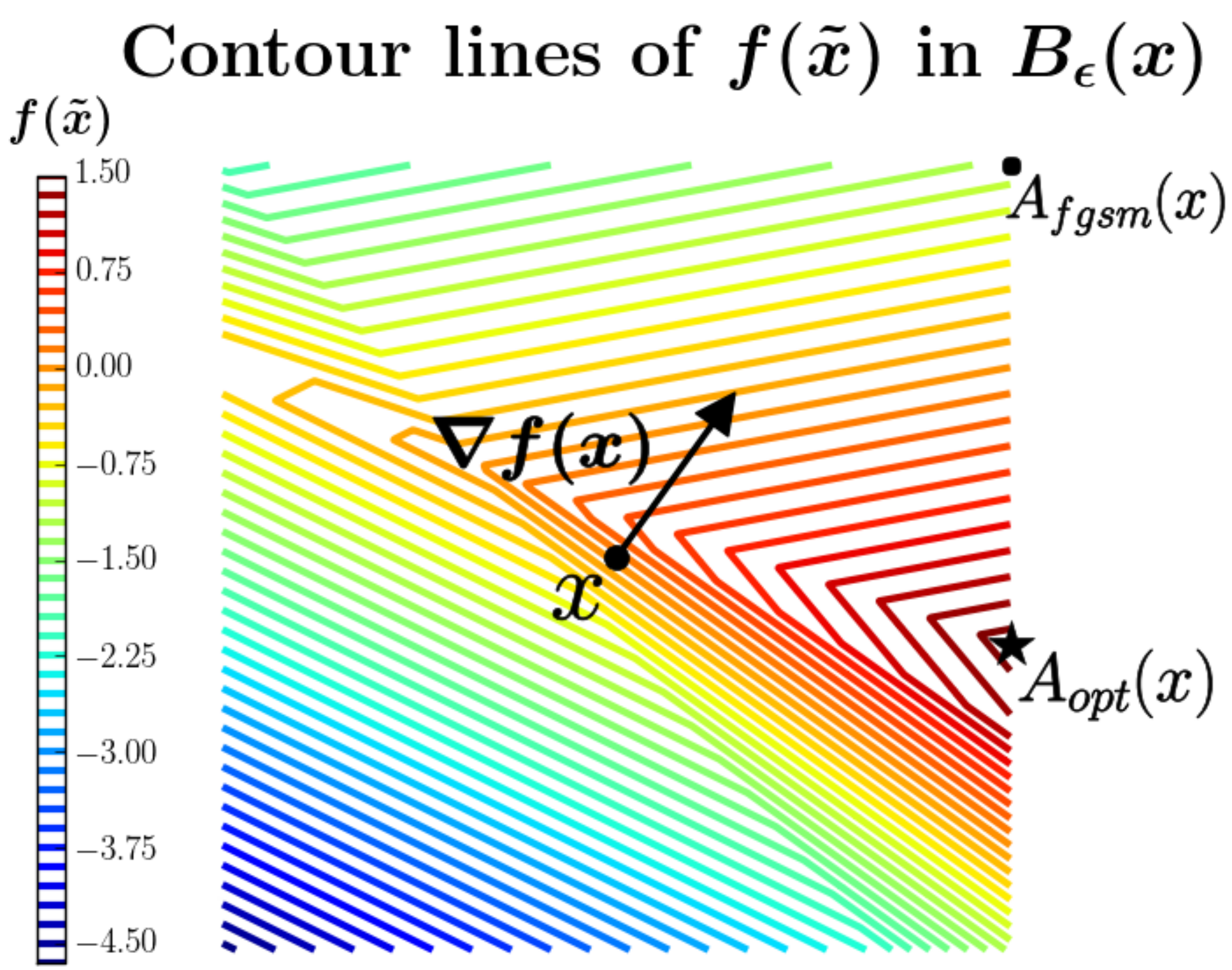

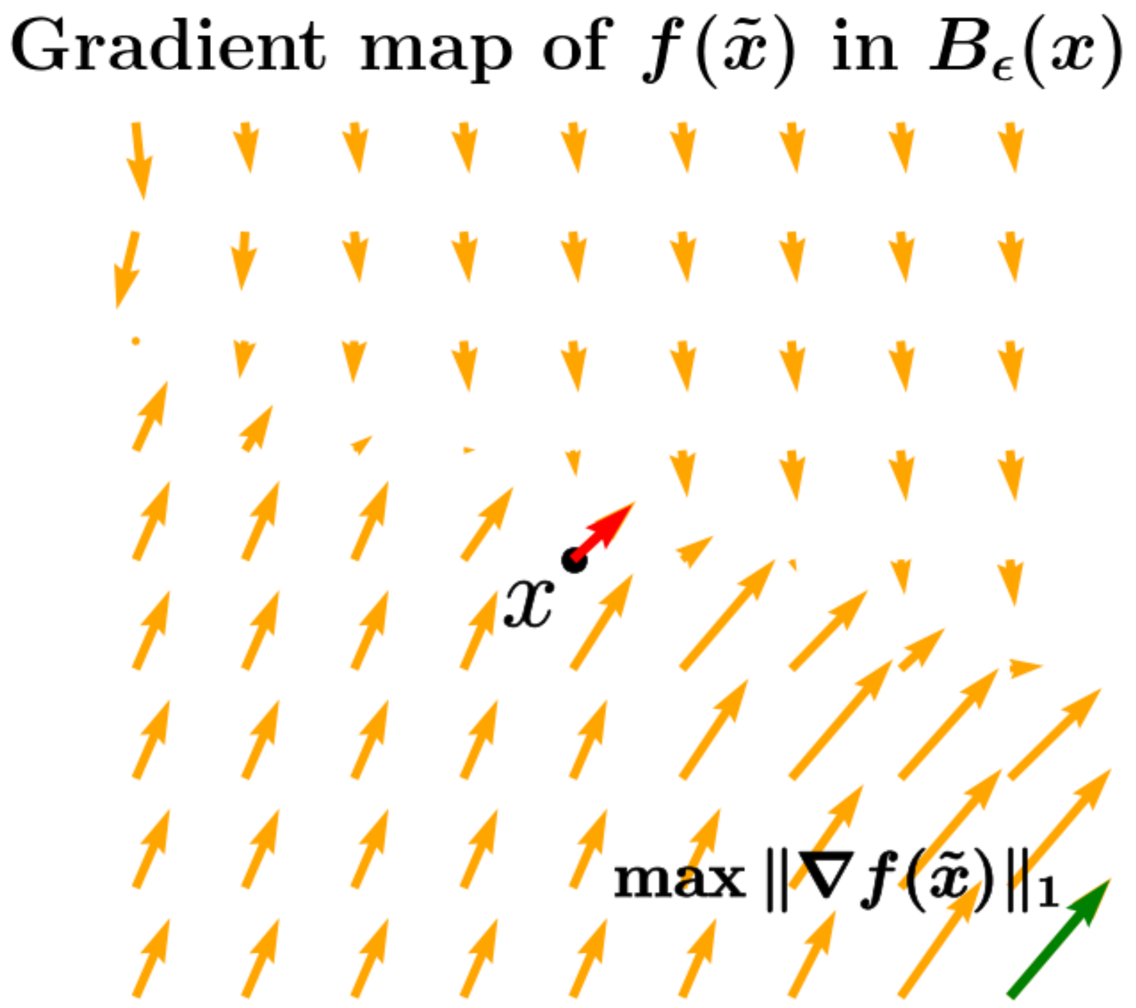

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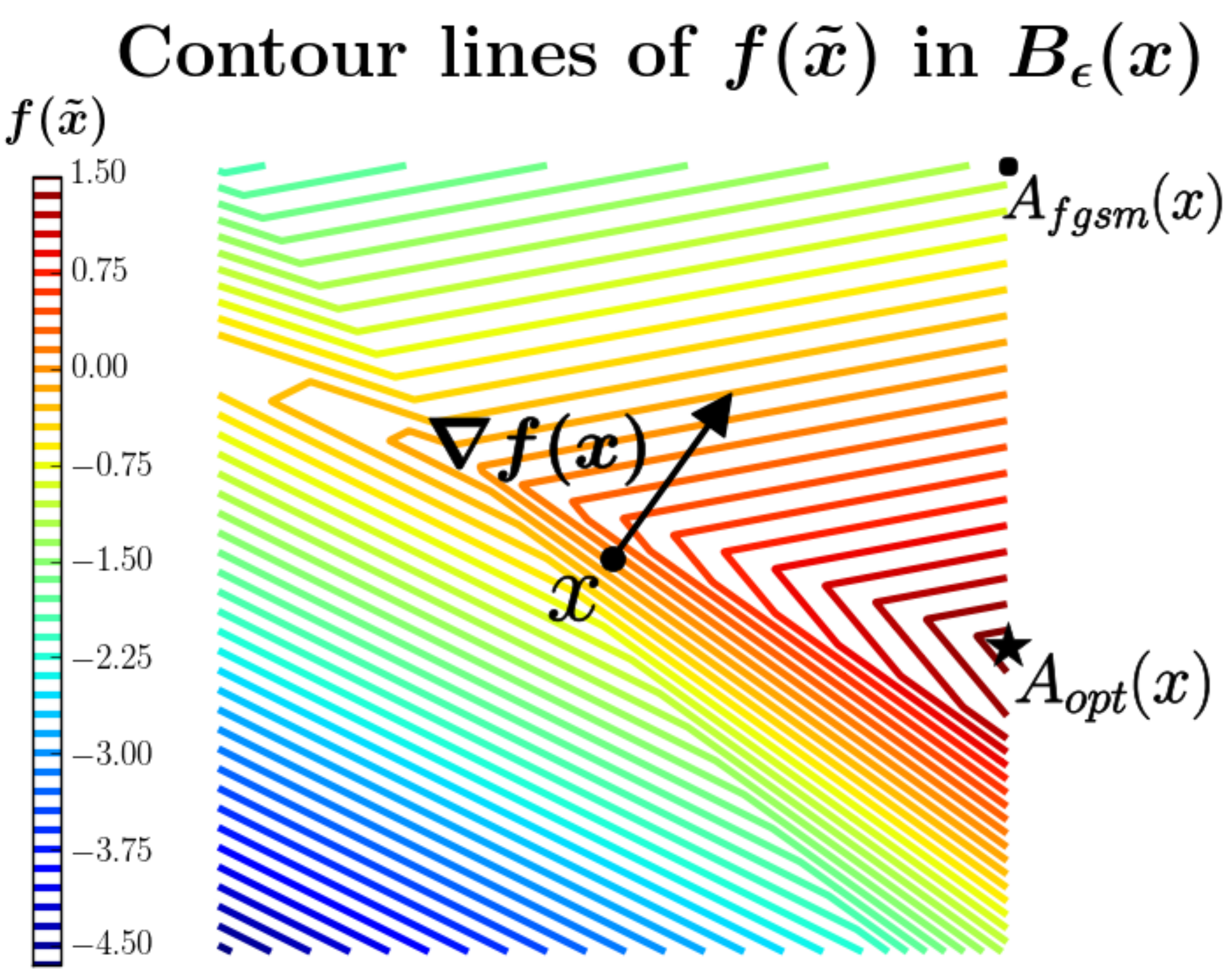

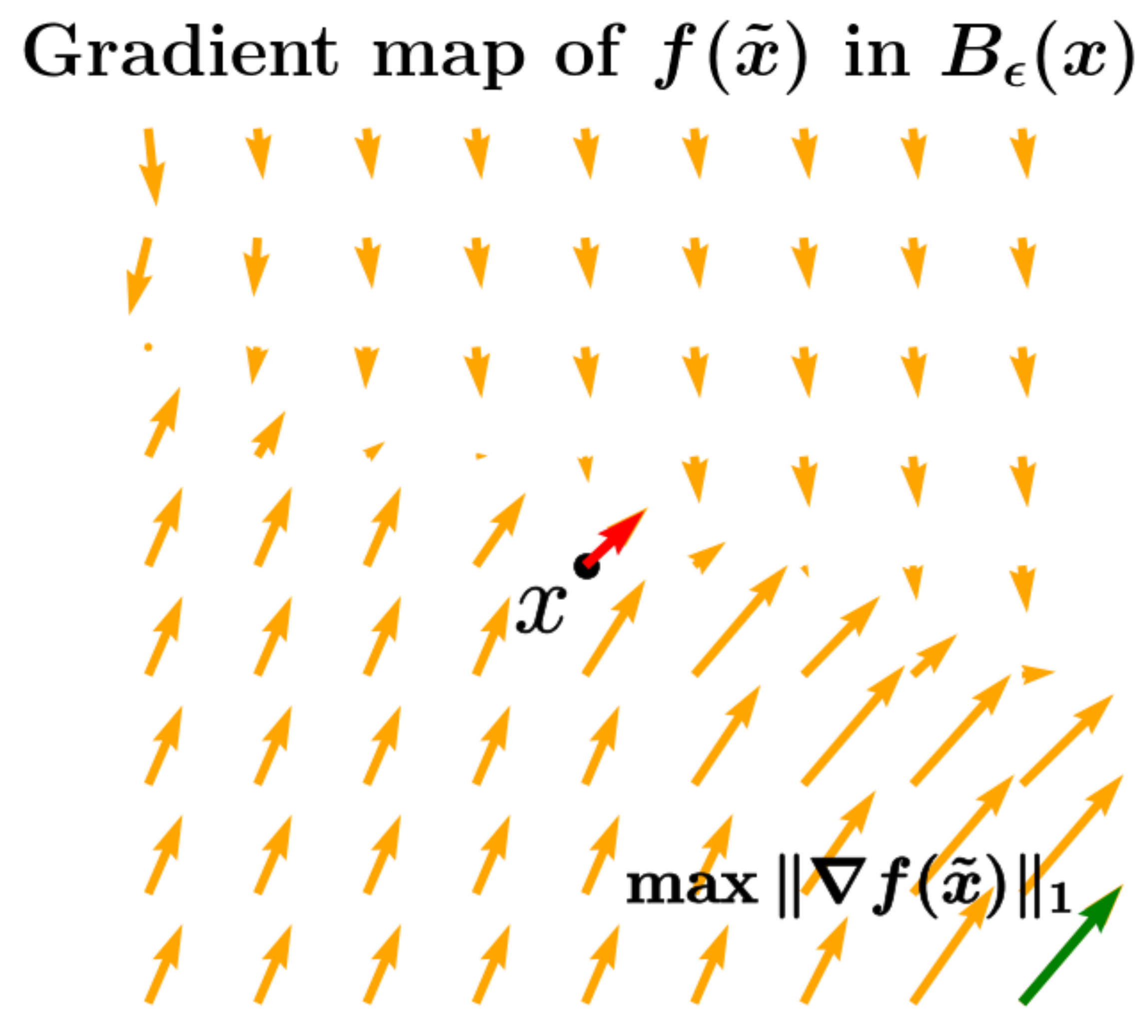

Key idea: Uniformly bound gradients

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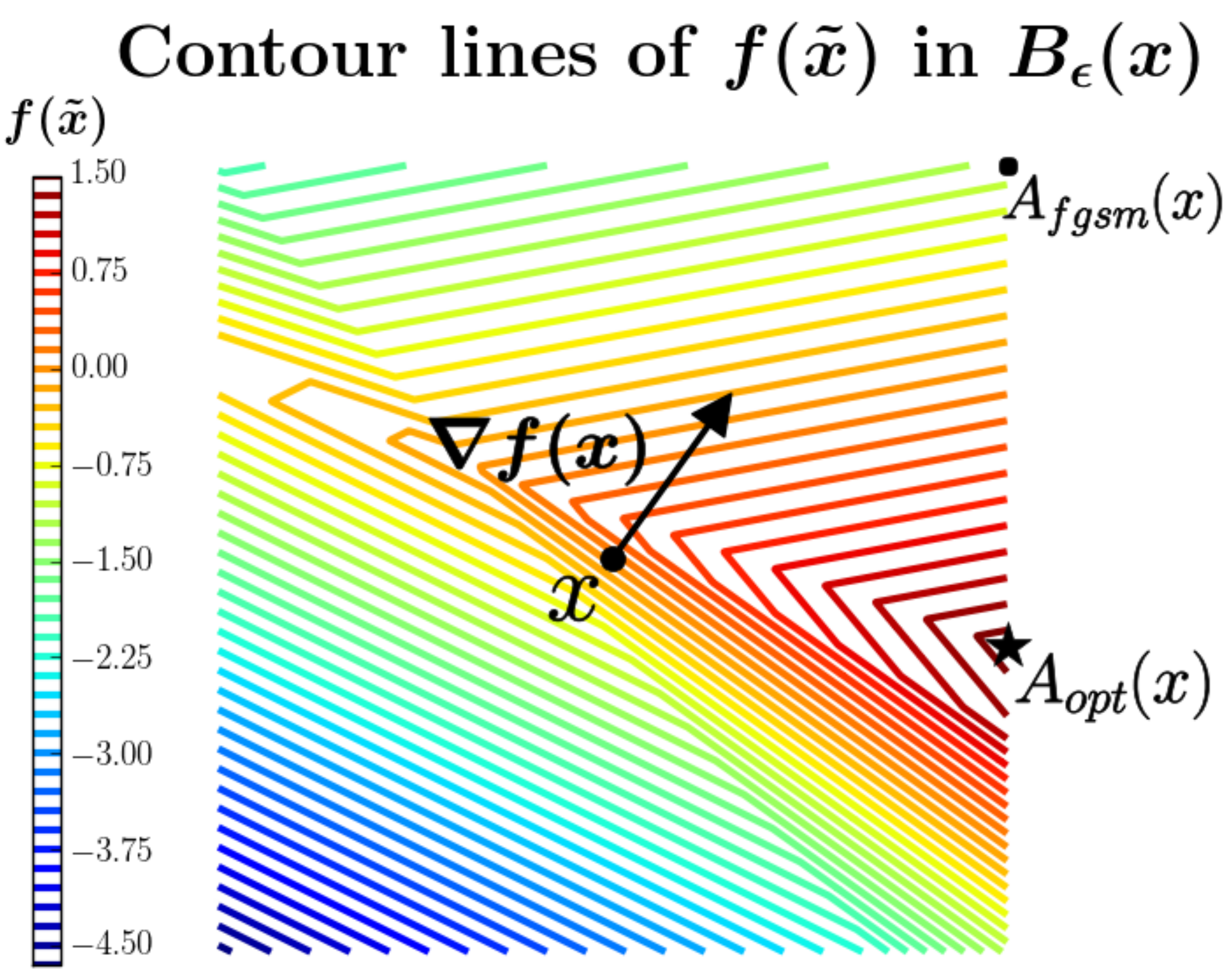

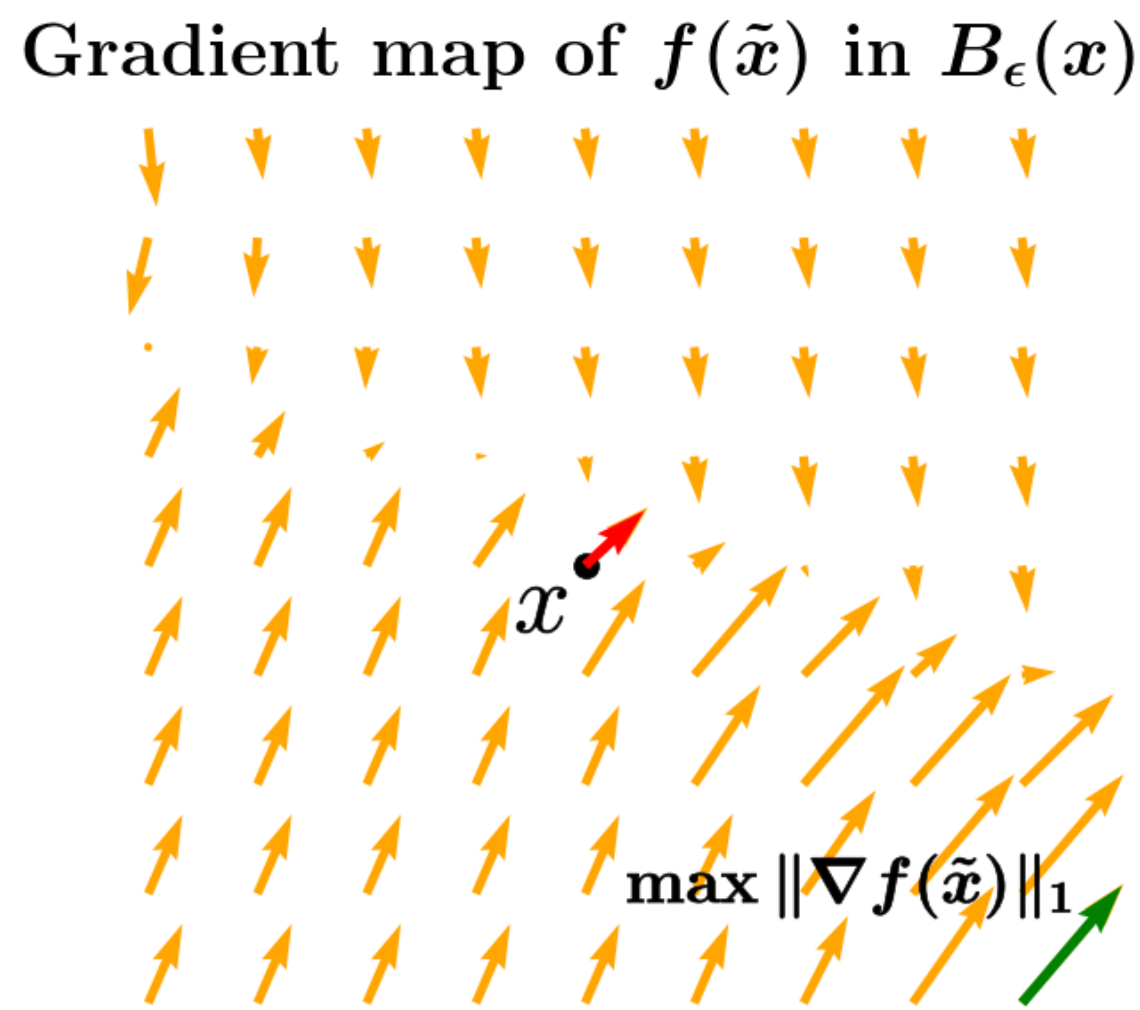

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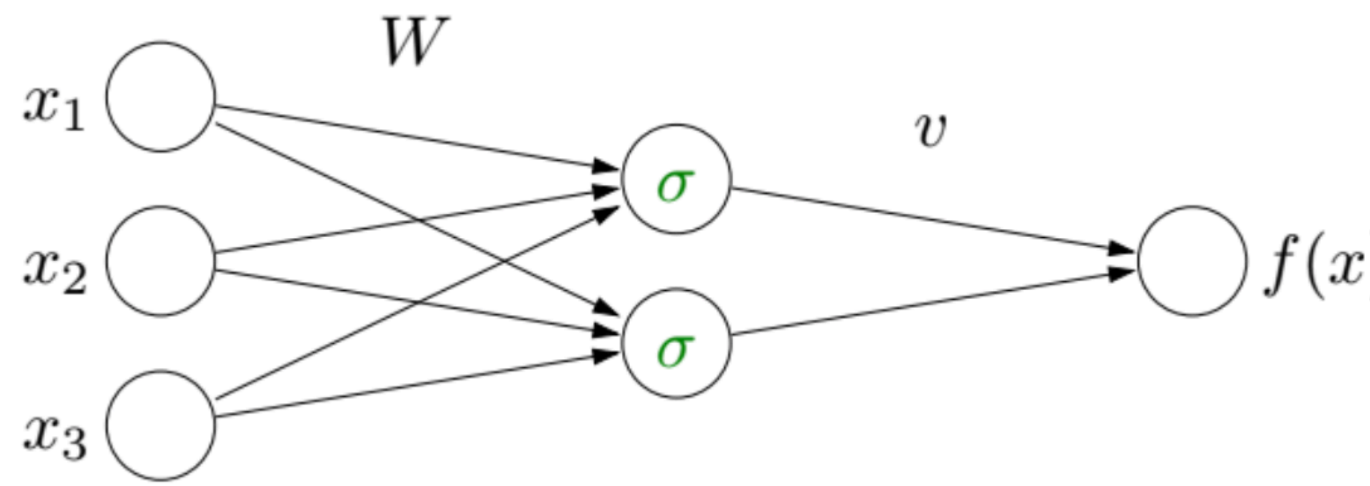

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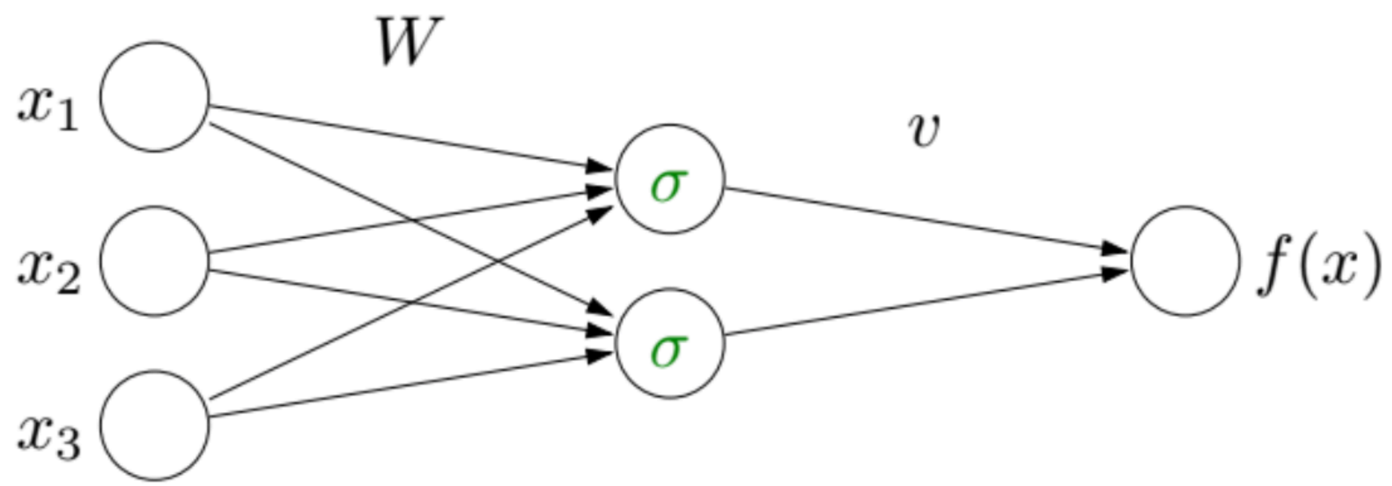

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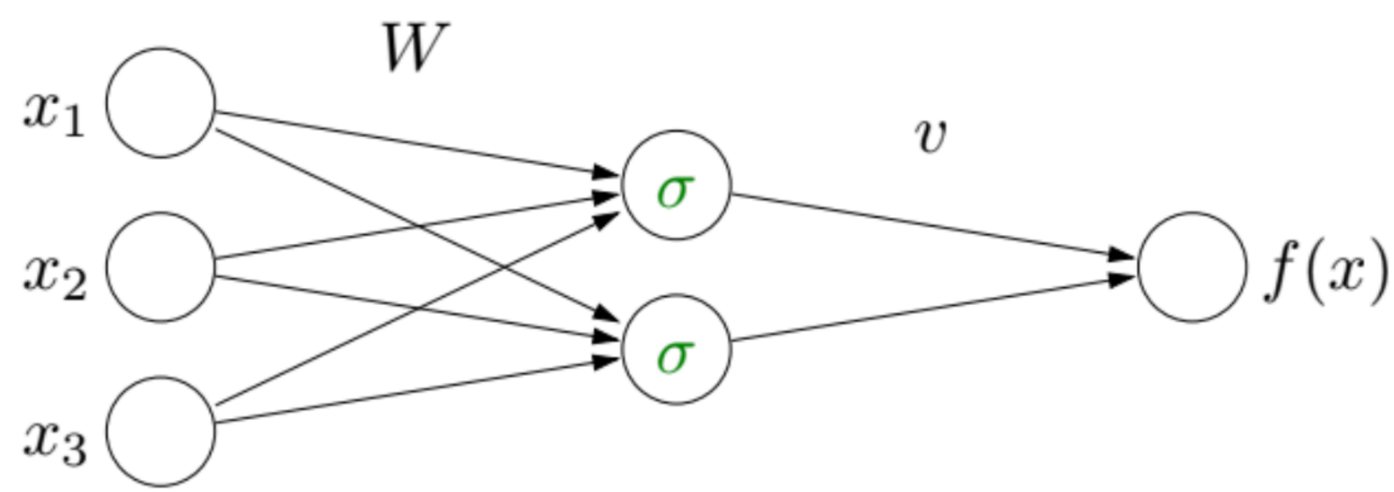

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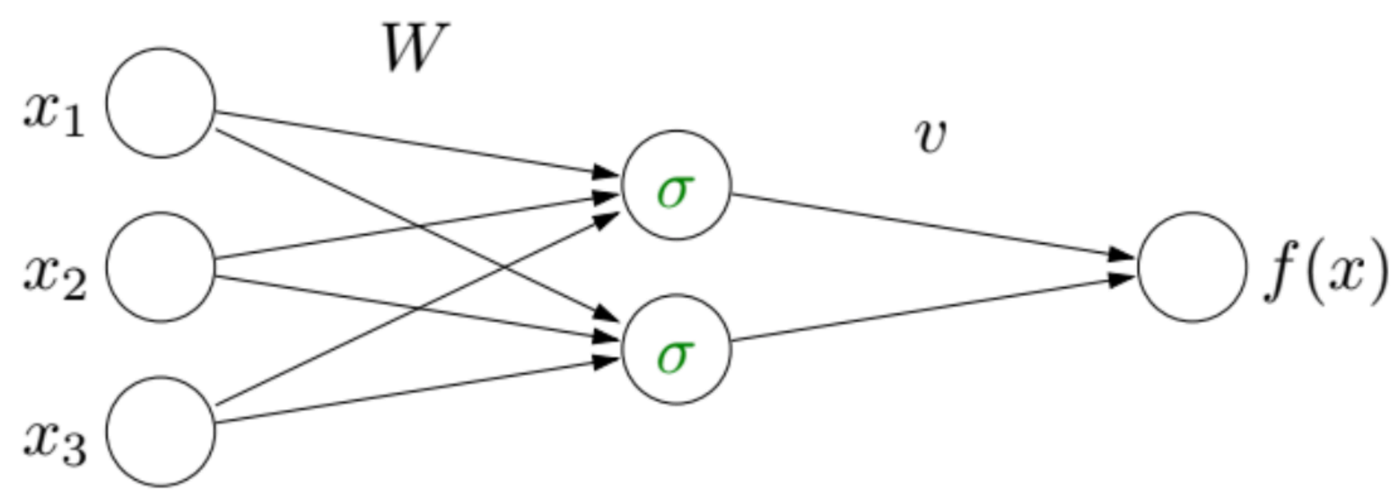

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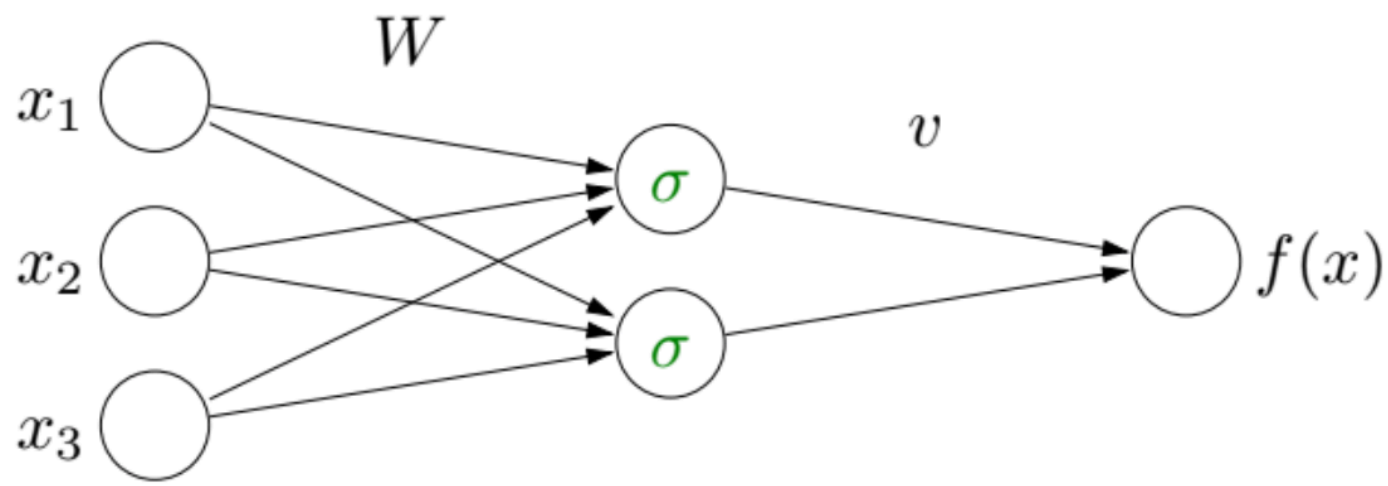

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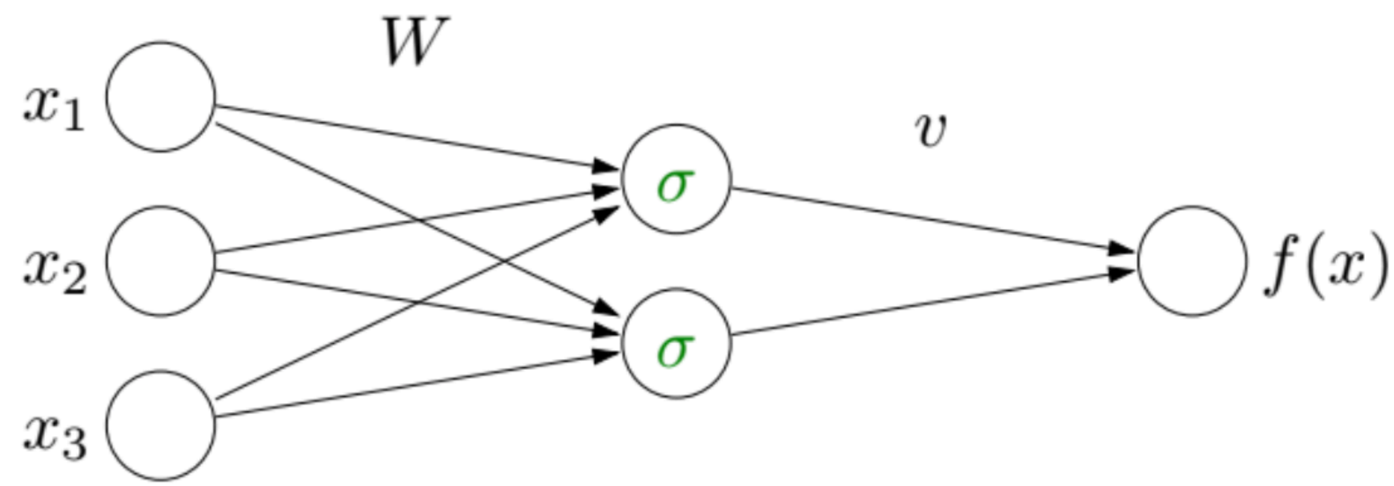

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optimize over activations

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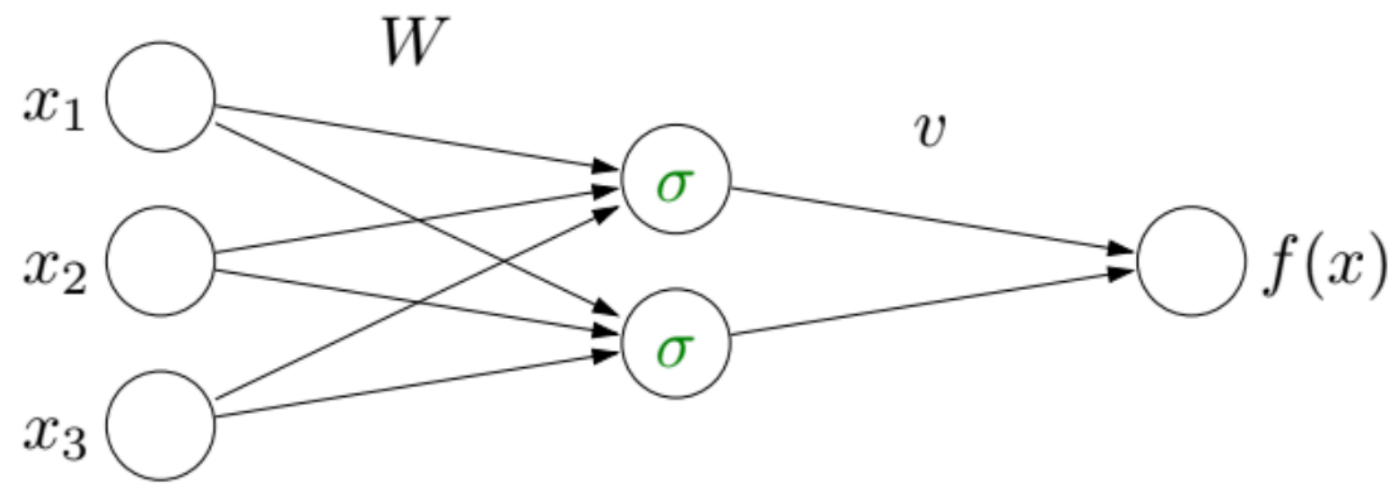

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Bound on gradient: $\|\nabla f(\tilde{x})\|_1 \quad = \|W^\top \mathrm{diag}(v) \sigma'(W\tilde{x})\|_1$

$$\leq \max_{s \in [0,1]^m, t \in [-1,1]^d} t^\top W^\top \mathrm{diag}(v) s$$

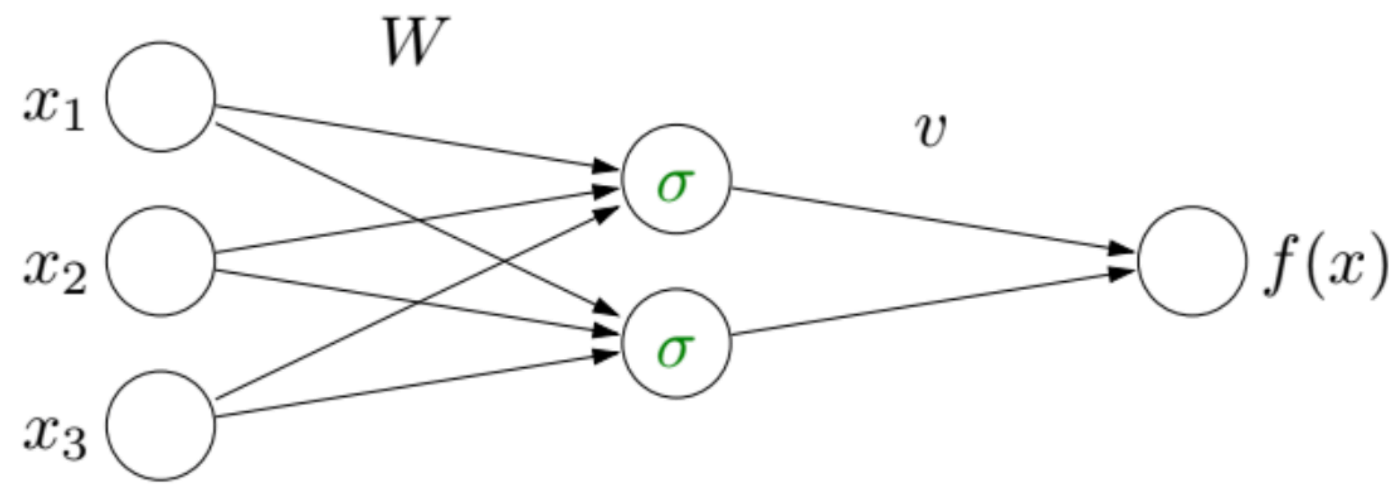
optimize over activations

optimize over signs of perturbation

Two layer networks

Key idea: Uniformly bound gradients

$$f(\tilde{x}) \leq f(\bar{x}) + \epsilon \max_{\tilde{x}} \|\nabla f(\tilde{x})\|_1$$



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Final step: SDP relaxation (similar to MAXCUT) leads to **Grad-cert**

Relaxation → Training

Relaxation $\longrightarrow$ Training

Training a neural network

Relaxation ⟶ Training

$\boxed{\text{Training a neural network}}$

Objective:

Relaxation ⟶ Training

Training a neural network

Objective: $$\min_{W,v} \underbrace{\sum_{i=1}^{n} L(z_i, W, v)}_{\text{training loss}} + \underbrace{\max_{P \succeq 0, P_{ii} \le 1} \mathrm{tr}(M(W,v)P)}_{\text{regularizer}}$$

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Regularizer: $D \cdot \lambda^{+}_{\max}\big((M(v,W) - \mathrm{diag}(c)\big) + \mathbf{1}^\top \max(c, 0)$

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Just one max eigenvalue computation for gradients

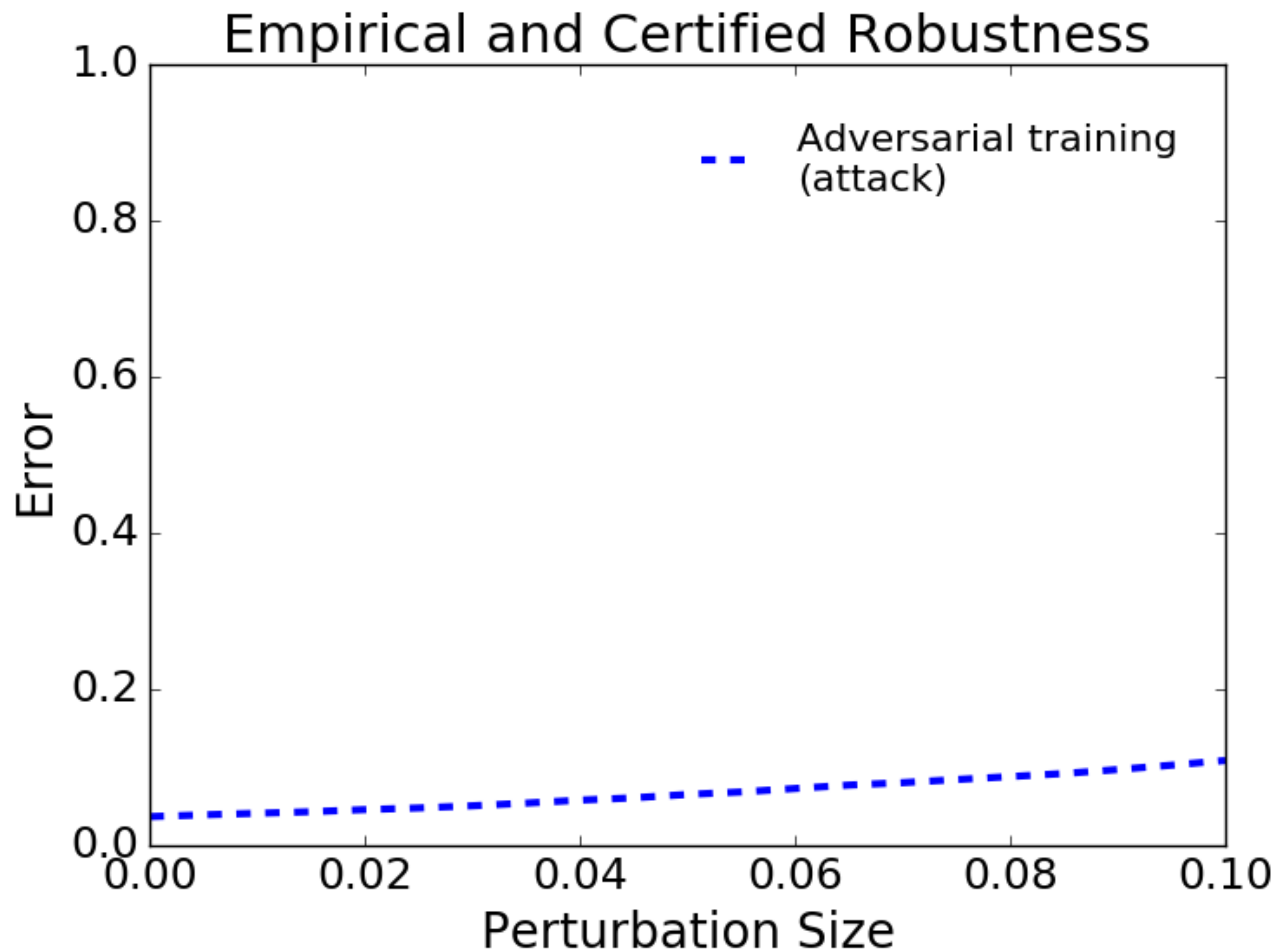
Results on MNIST

Results on MNIST

Attack: Projected Gradient Descent attack of Madry et al. 2018

Adversarial training: Minimizes this lower bound on training set

Results on MNIST



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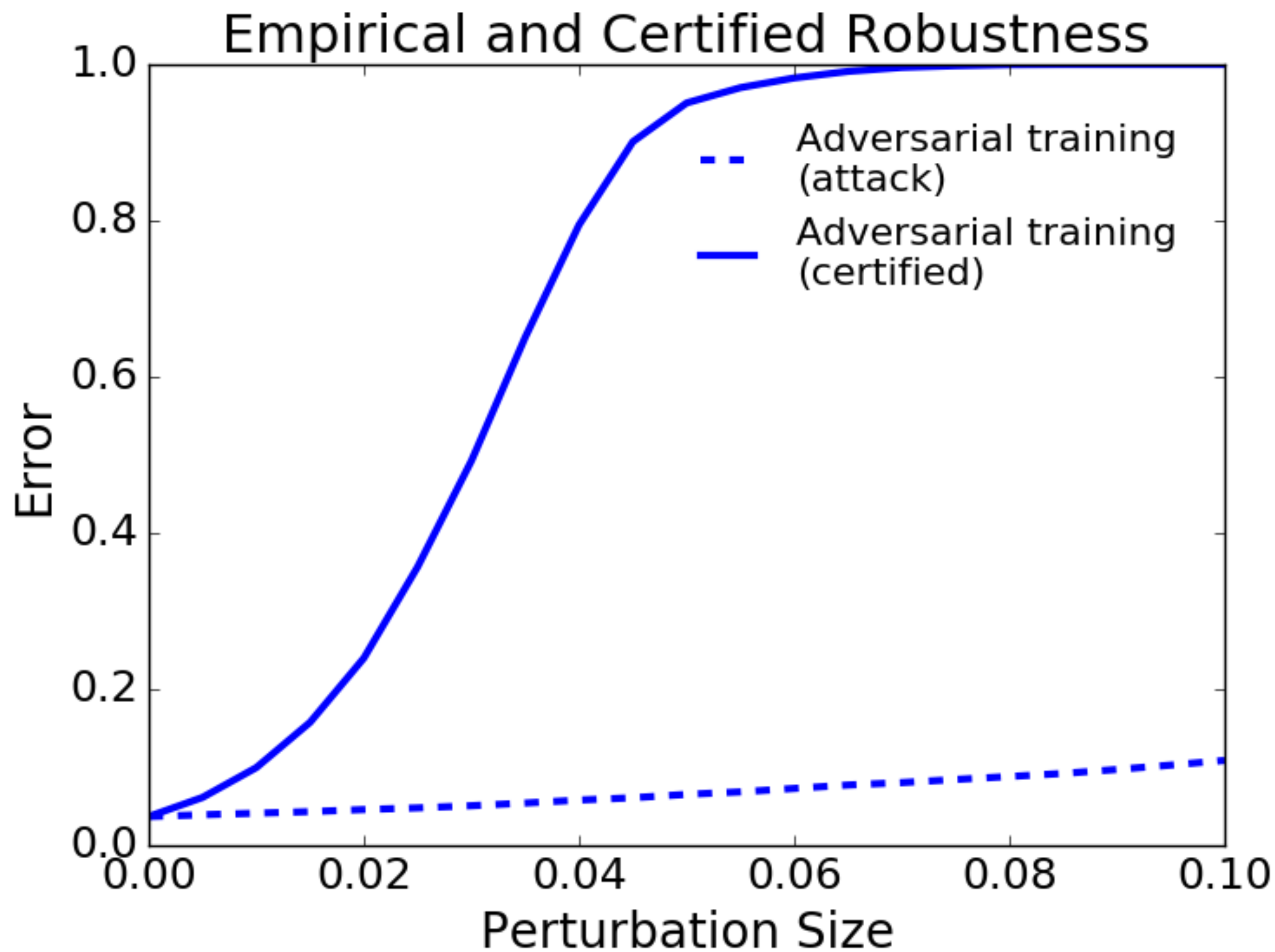

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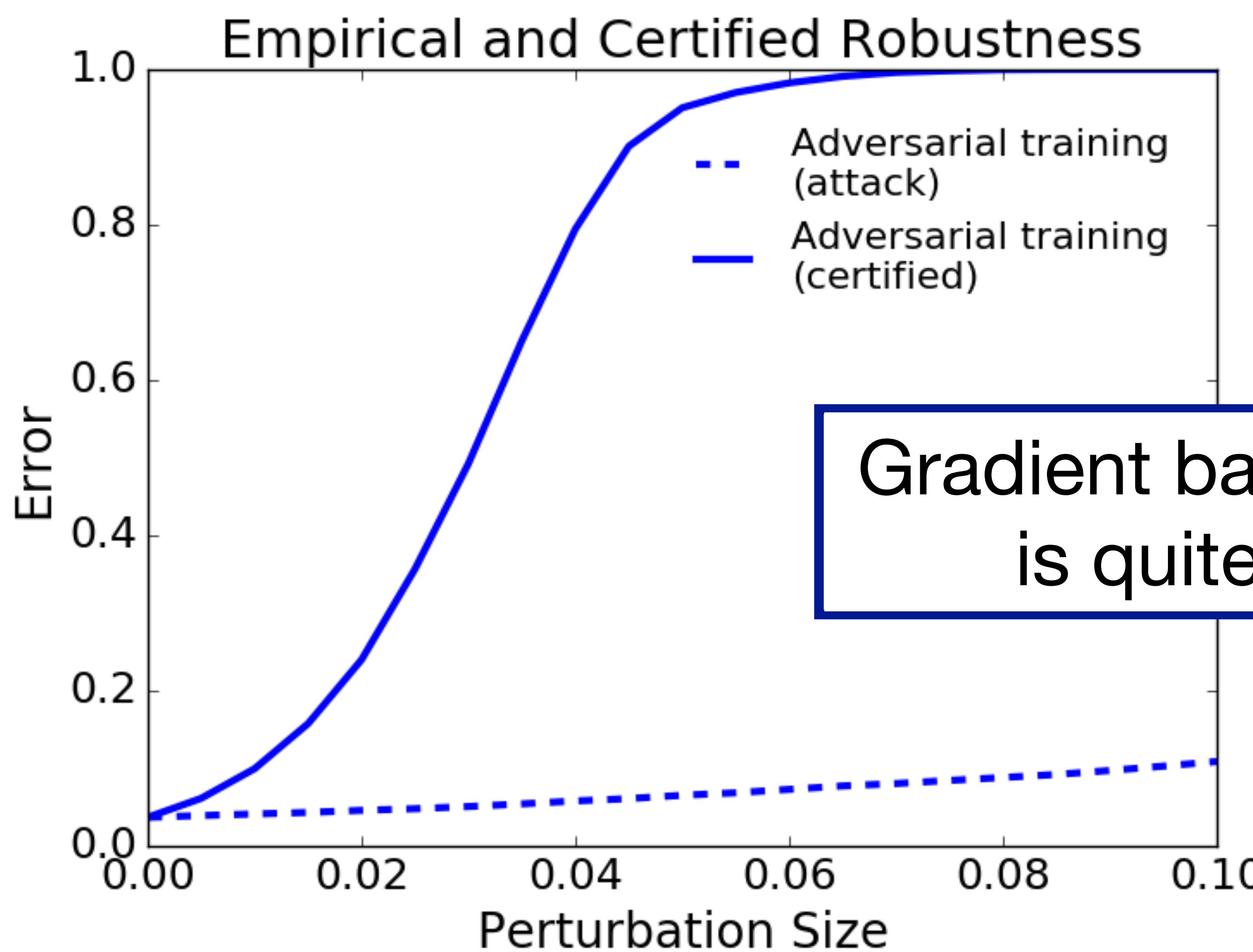

Gradient based bound is quite loose

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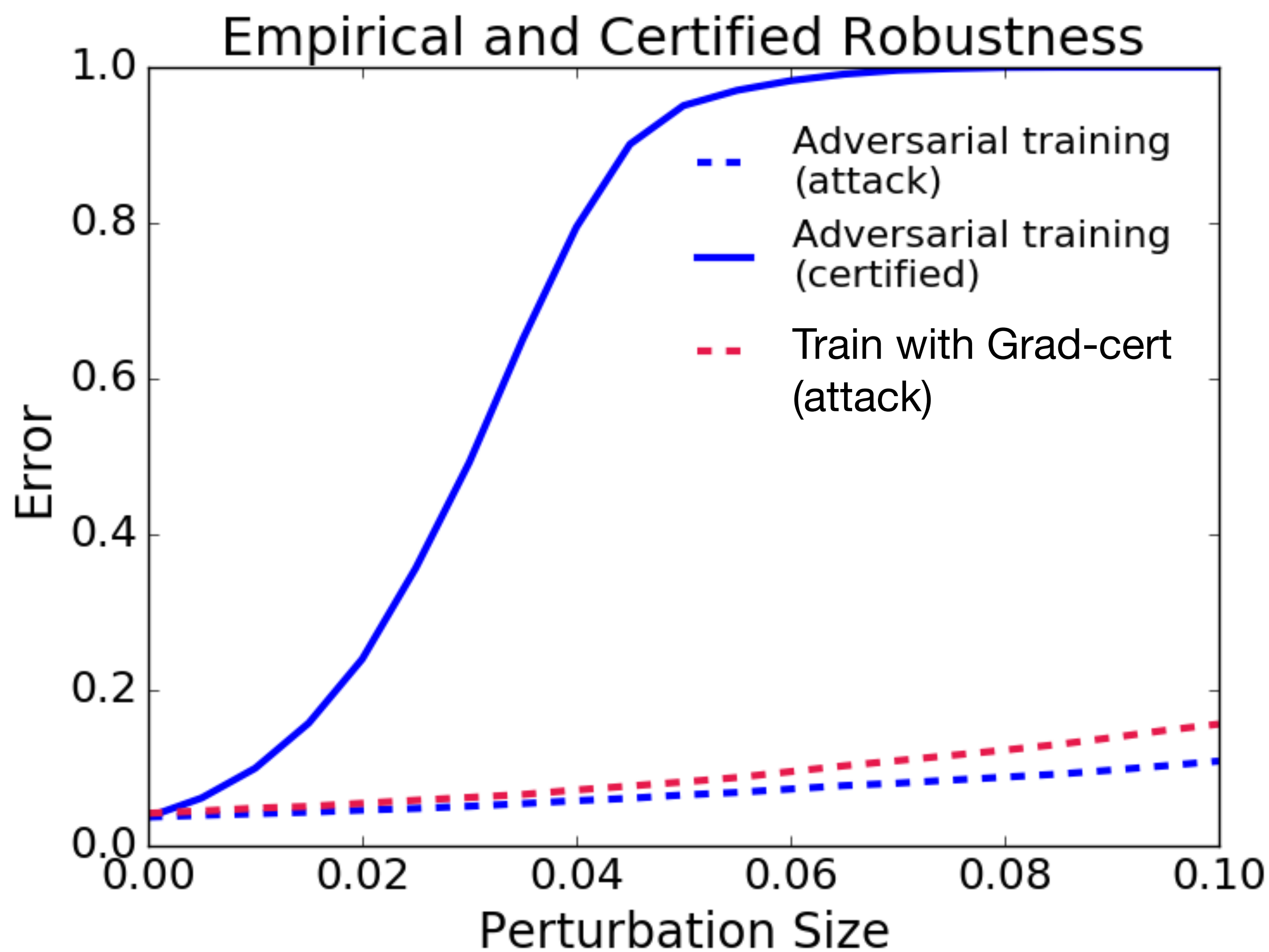
Results on MNIST

Train with Grad-cert
(attack)

Results on MNIST



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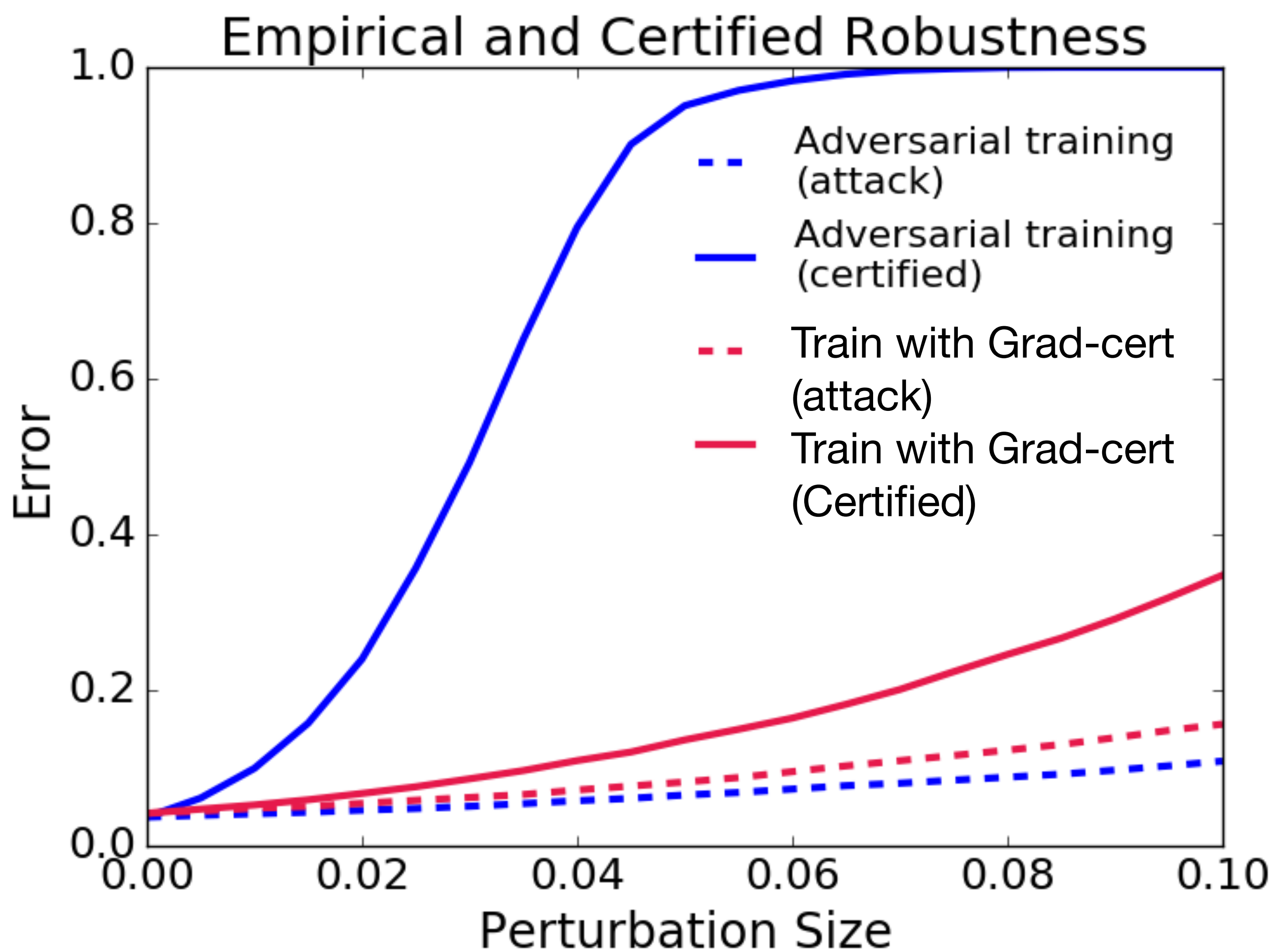
Our method: Minimize gradient based upper bound

Results on MNIST



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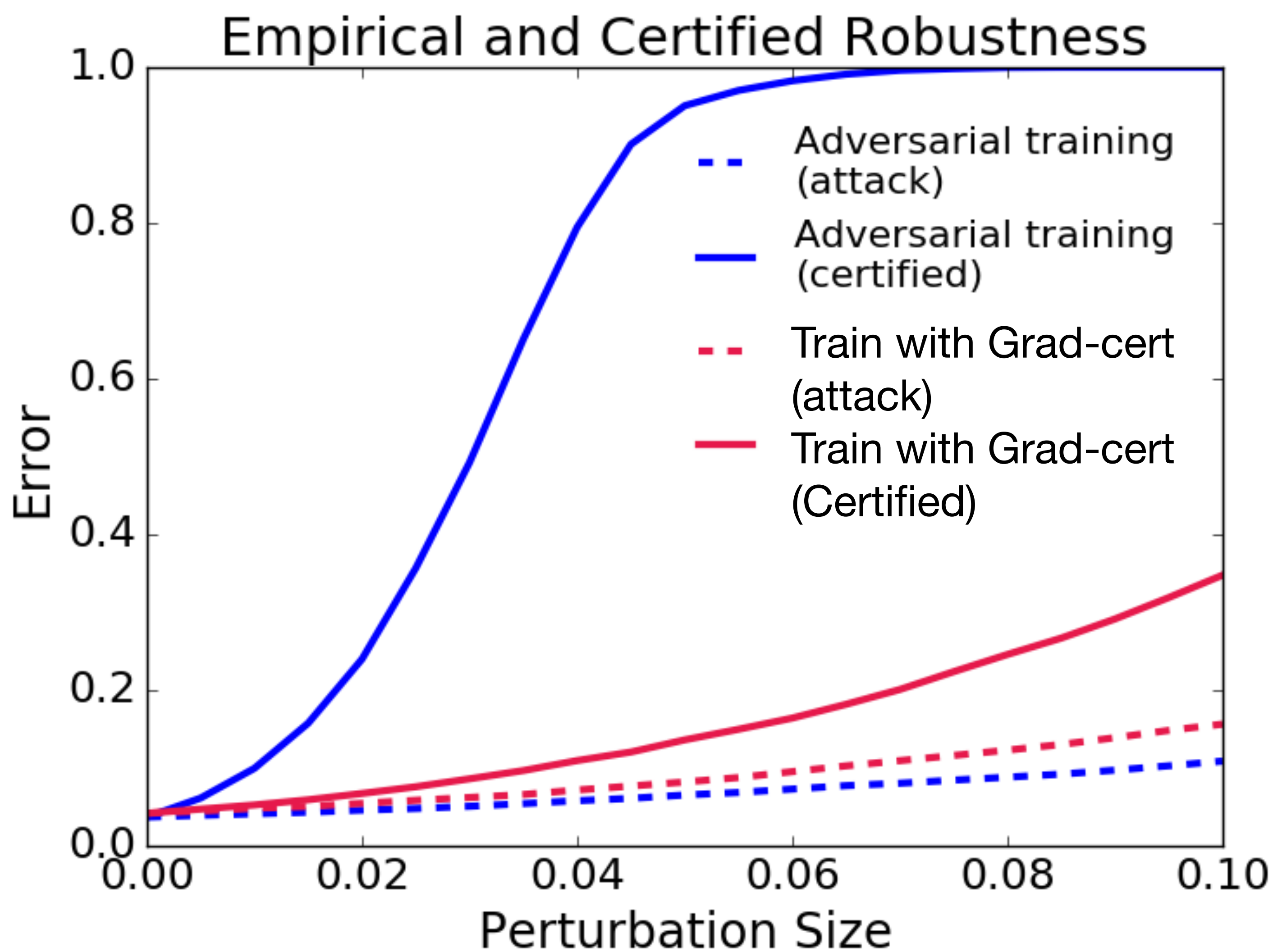

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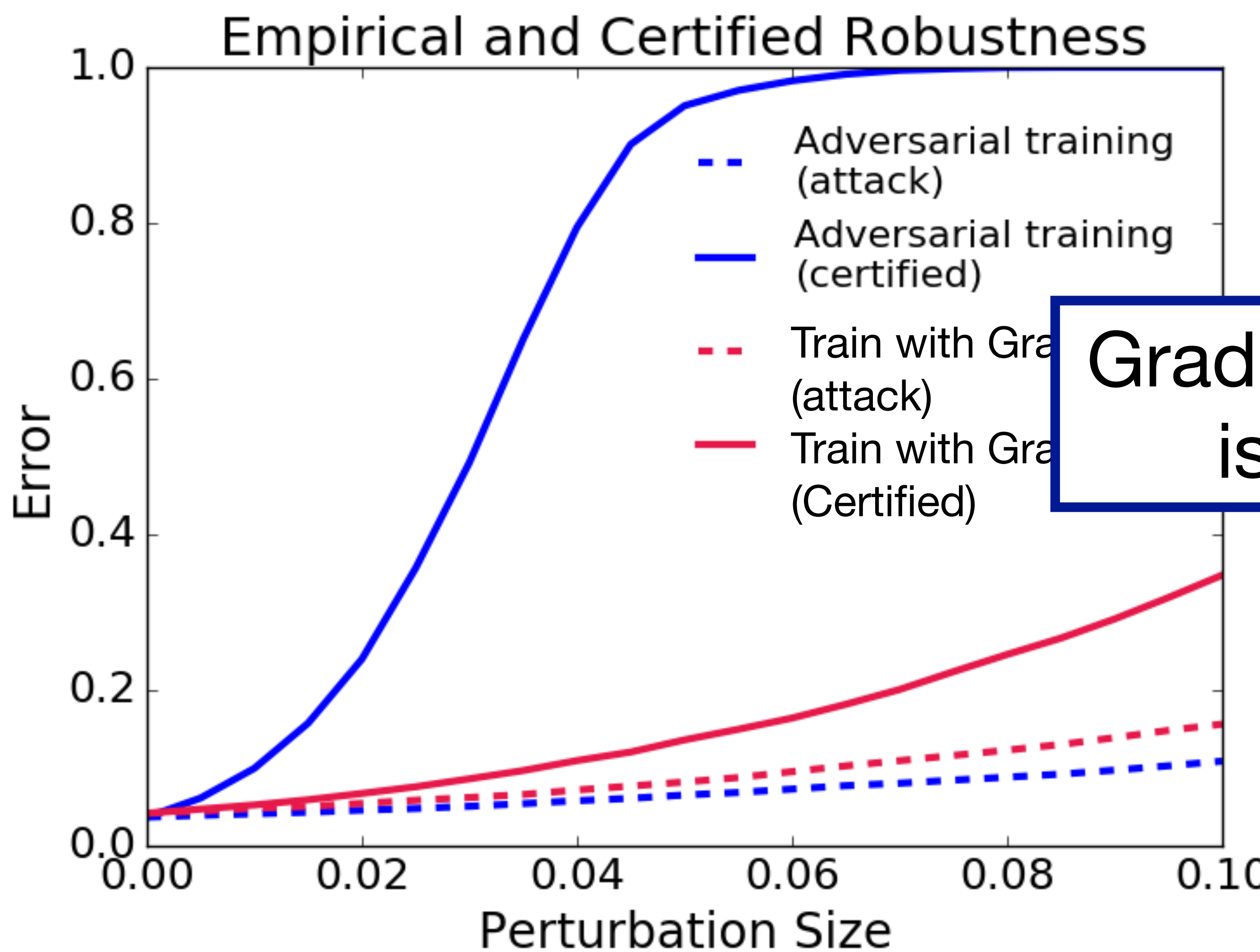

Gradient based bound is much better!

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Training a network to minimize gradient upper bound
finds networks where the bound is tight

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Comparison with Wong and Kolter 2018 (LP-cert)

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Network	PGD-attack	LP-cert	Grad-cert
LP-NN	22%	26%	93%
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Can we certify such “foreign” networks?

Summary so far…

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- Training against the bound makes it tight
- LP-cert and Grad-cert are tight only on training
- Goal: Efficiently certify foreign multi-layer networks

New SDP-cert relaxation

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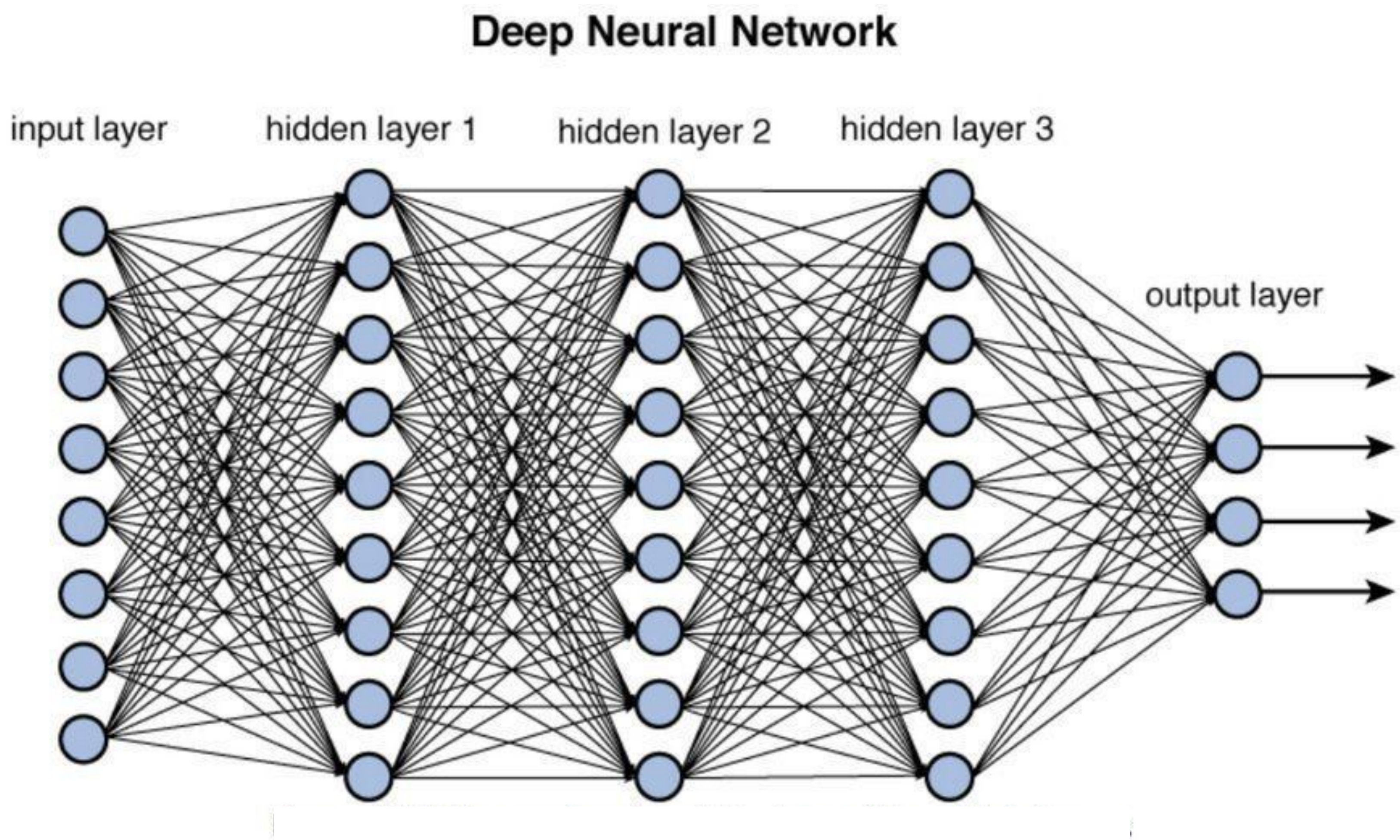

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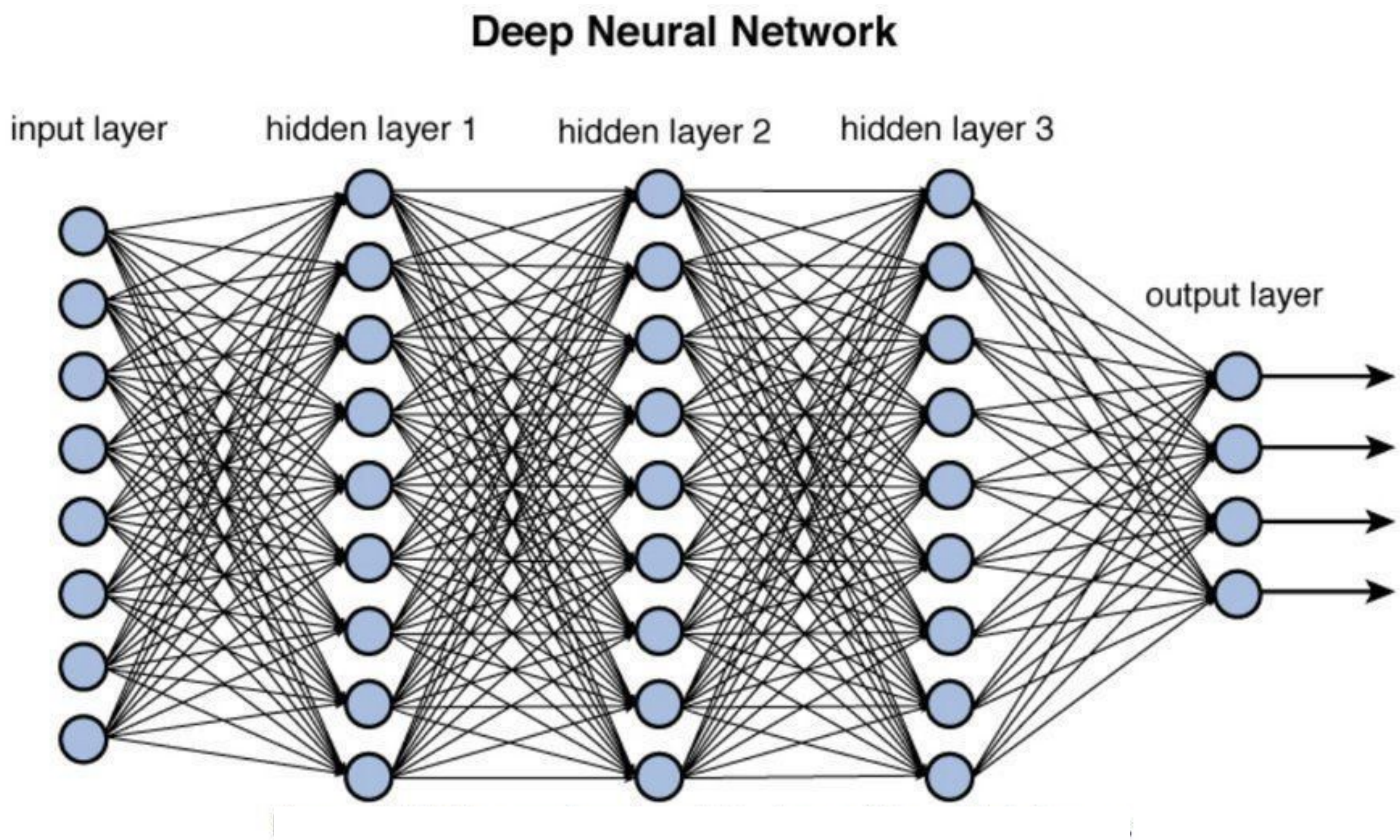

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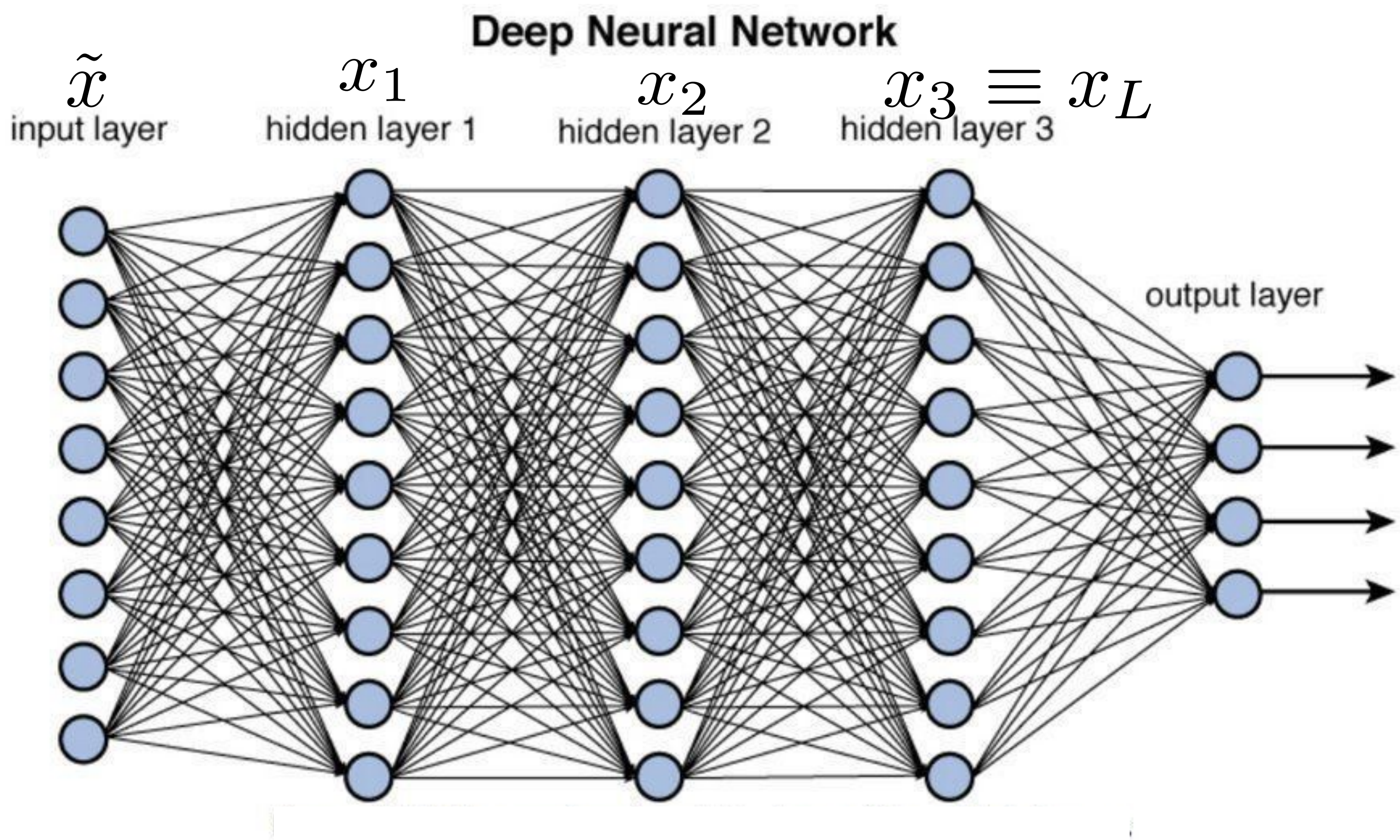
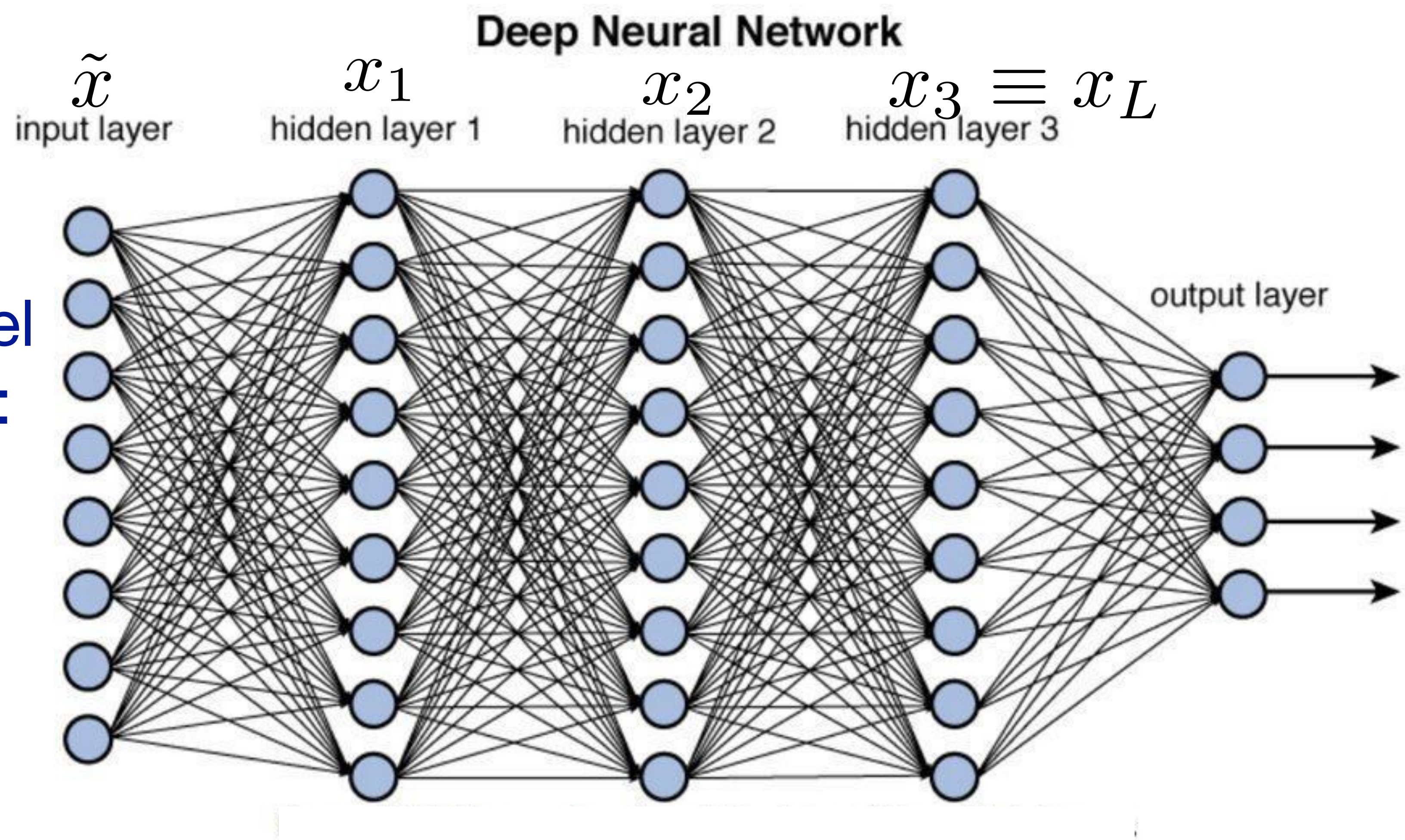


New SDP-cert relaxation

Attack model constraints:



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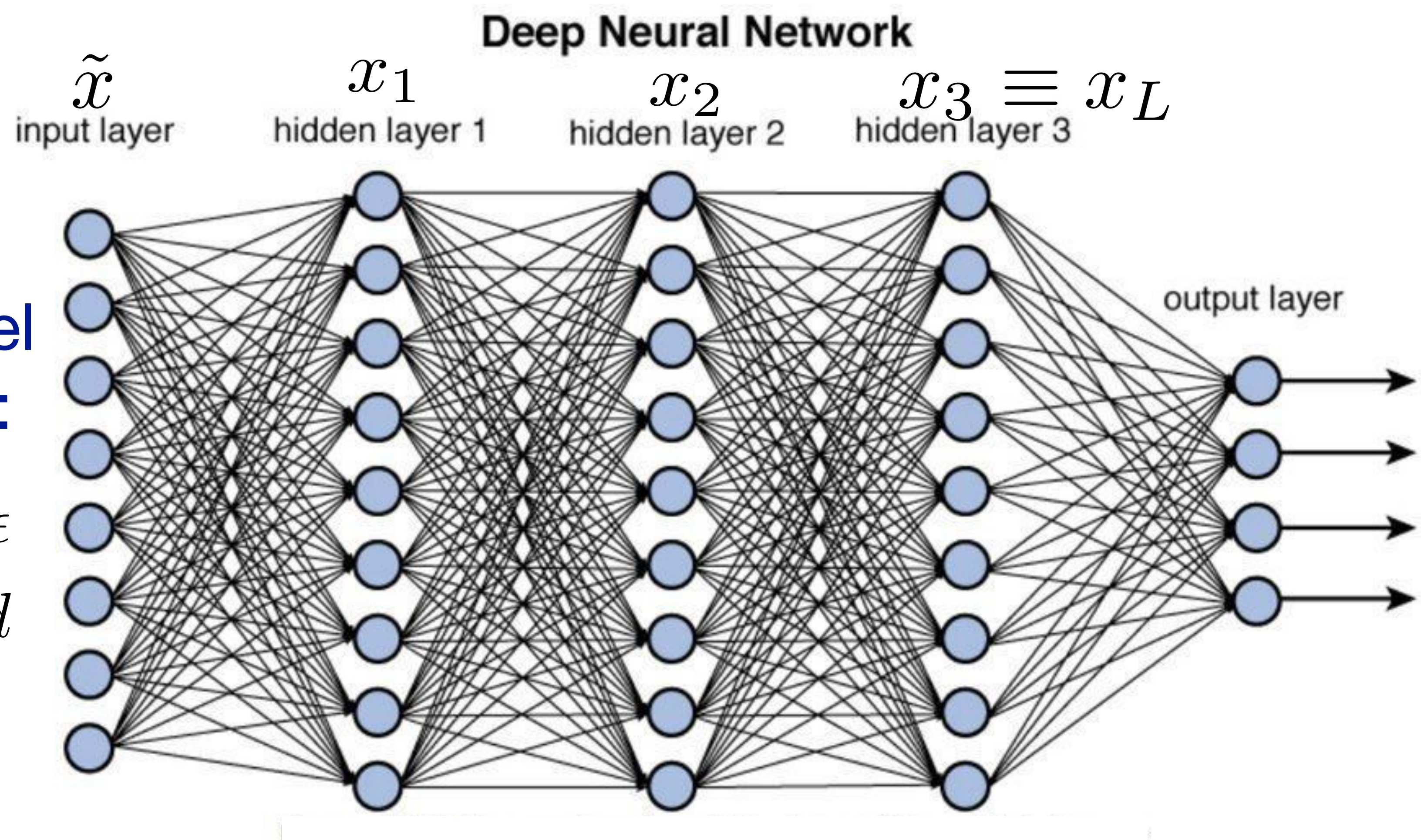

Attack model constraints:

$|\bar{x} - \tilde{x}|_i \leq \epsilon$

for $i = 1, 2, \ldots d$

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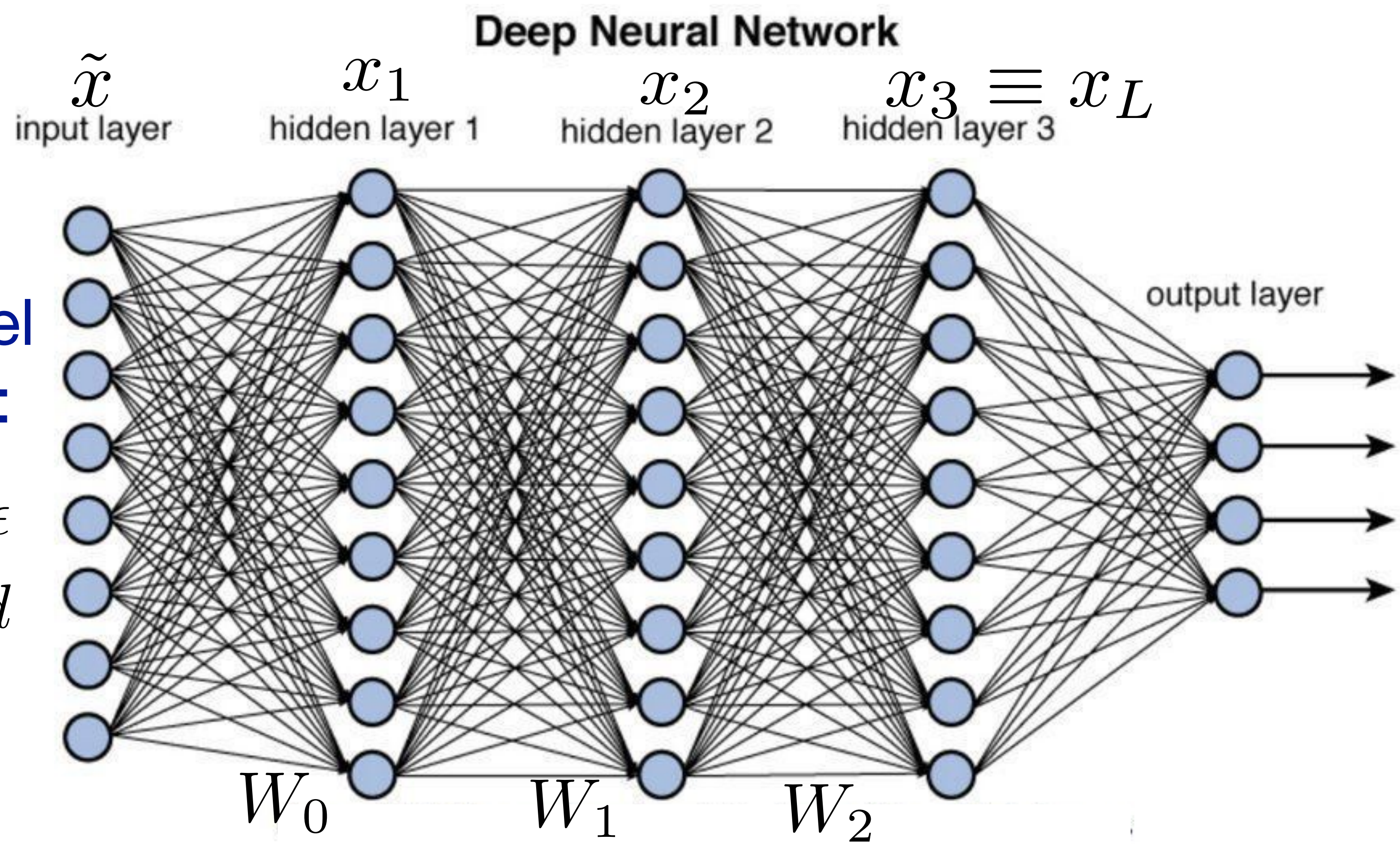

Neural net constraints

$$x_i = \mathrm{ReLU}(W_{i-1}x_{i-1})$$

for $i = 1, 2, \dots L$

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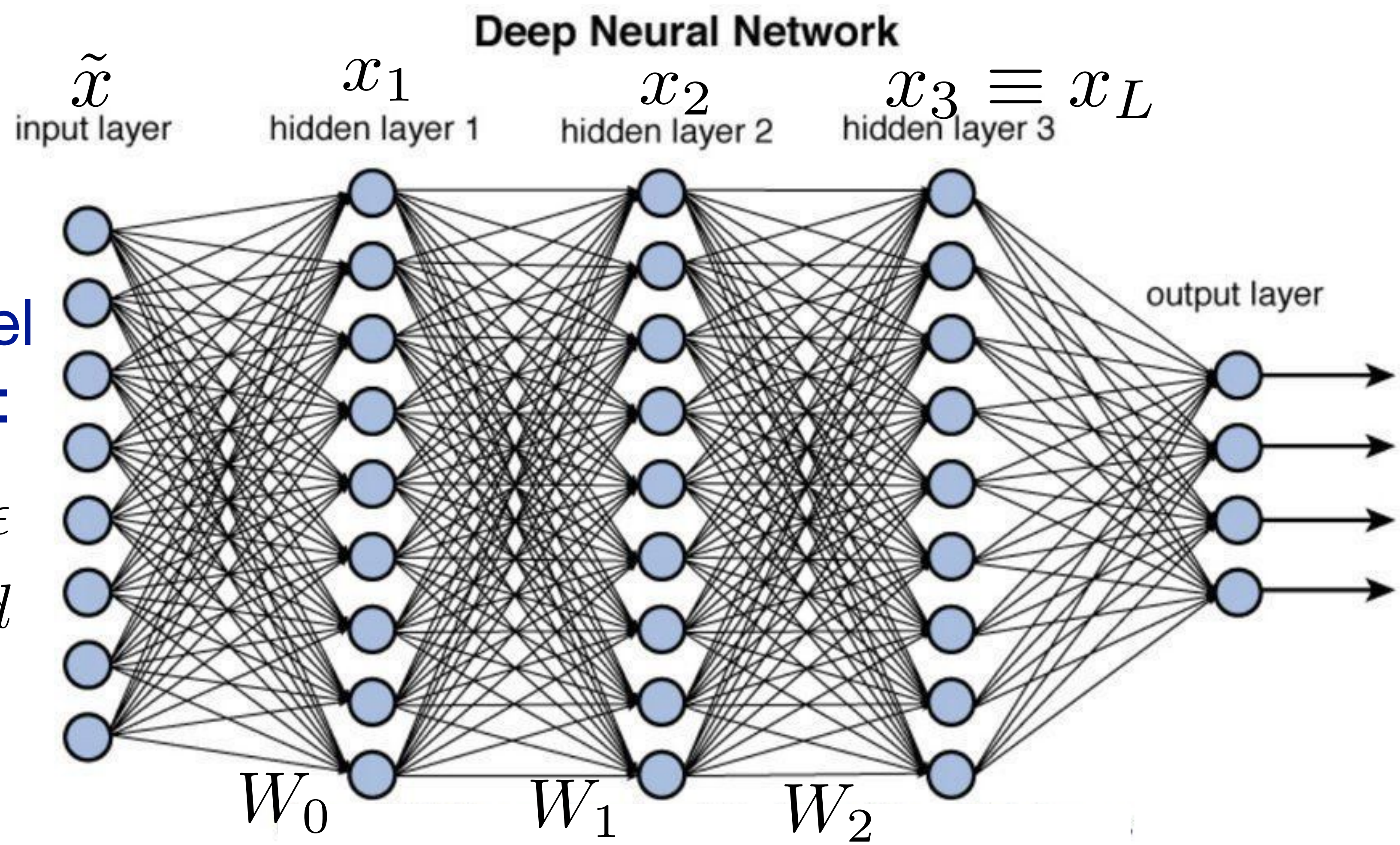

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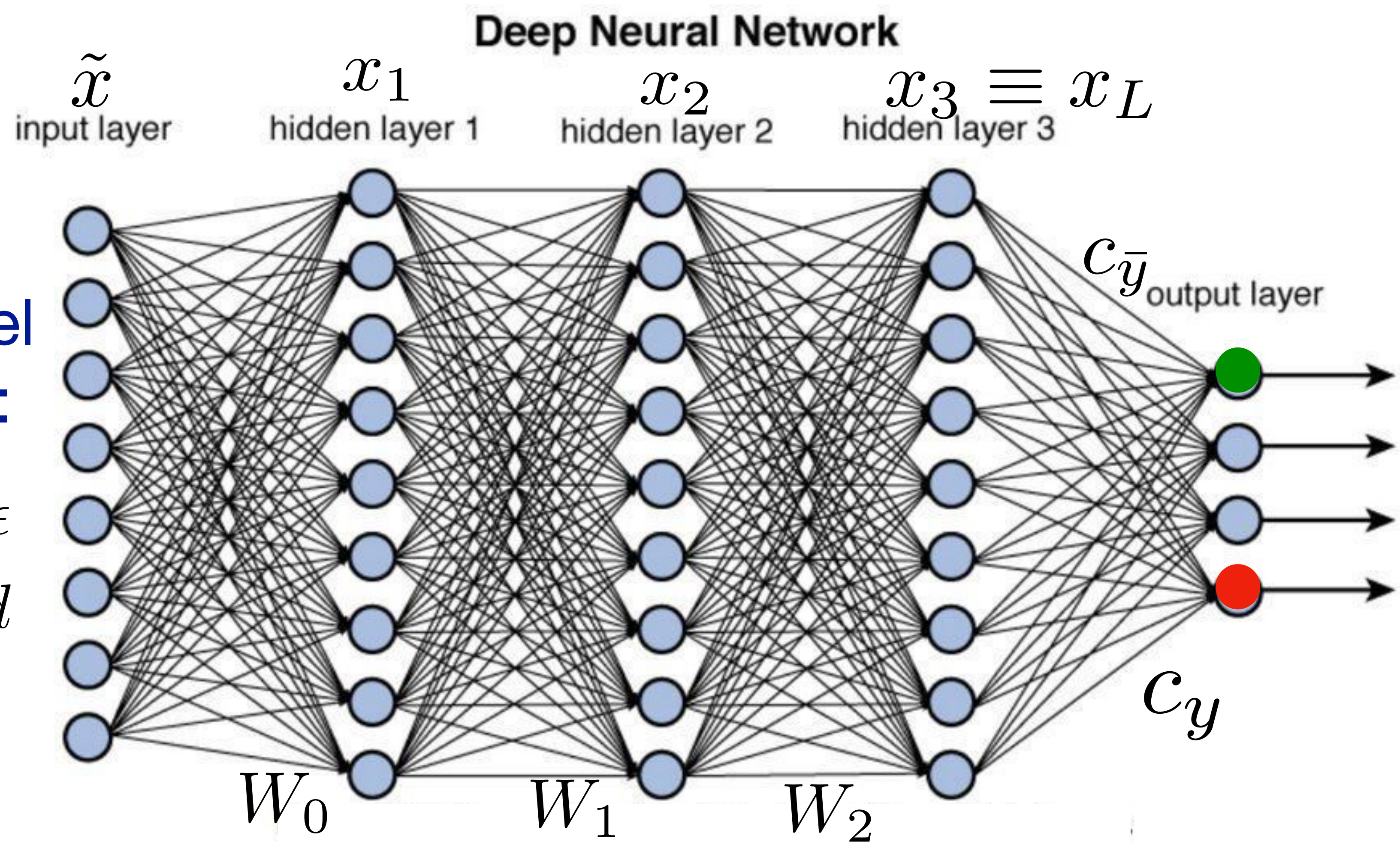

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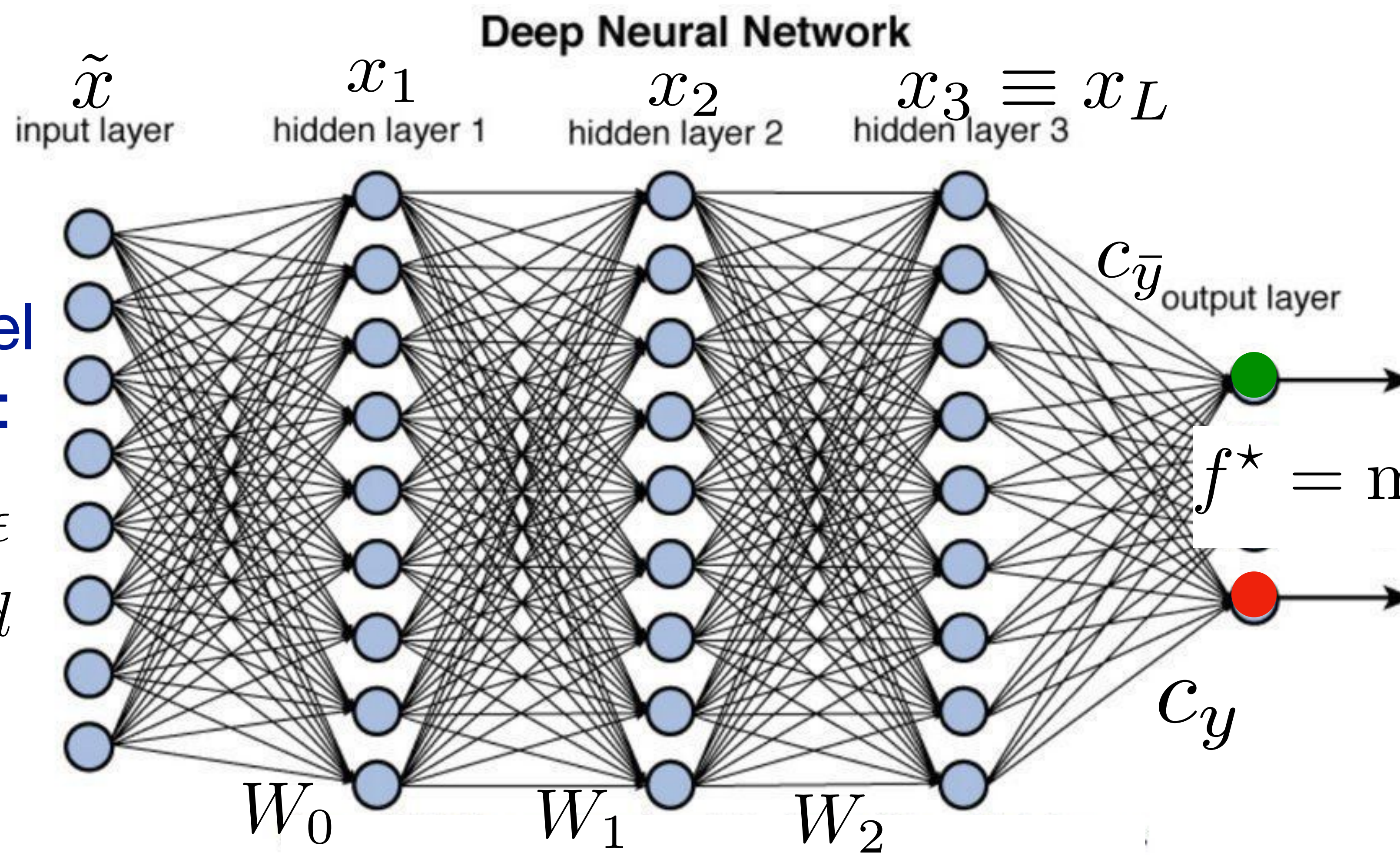

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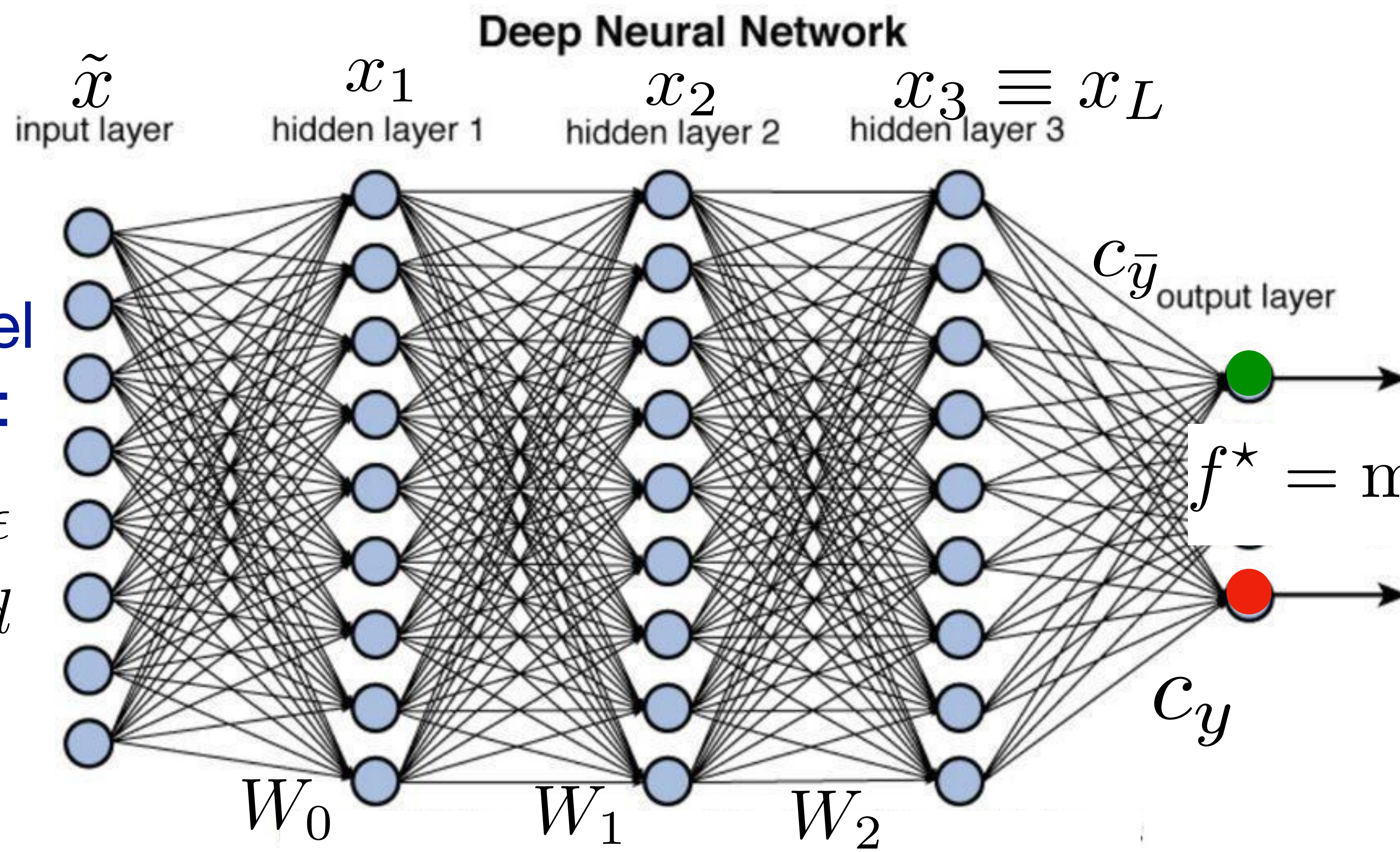

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Source of non-convexity is the ReLU constraints

Handling ReLU constraints

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Consider single ReLU constraint $z = \max(0, x)$

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z is greater than $x, 0$ $\left\{ \begin{array}{ll} z \geq x & \text{Linear} \\ z \geq 0 & \text{Linear} \end{array} \right.$

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Can relax quadratic constraints to get a semidefinite program

SDP relaxation

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Single ReLU constraint $z = \max(0, x)$ $\equiv$ Linear + Quadratic constraints

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$$M = \begin{bmatrix} 1 & x & z \\ x & x^2 & xz \\ z & xz & z^2 \end{bmatrix}$$

SDP relaxation

Single ReLU constraint $z = \max(0, x)$ $\equiv$ Linear + Quadratic constraints

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ReLU constraints as linear constraints on matrix entries

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Generalizes to multiple layers: large matrix M with all activations

SDP relaxation

SDP relaxation

Interaction between different hidden units

SDP relaxation

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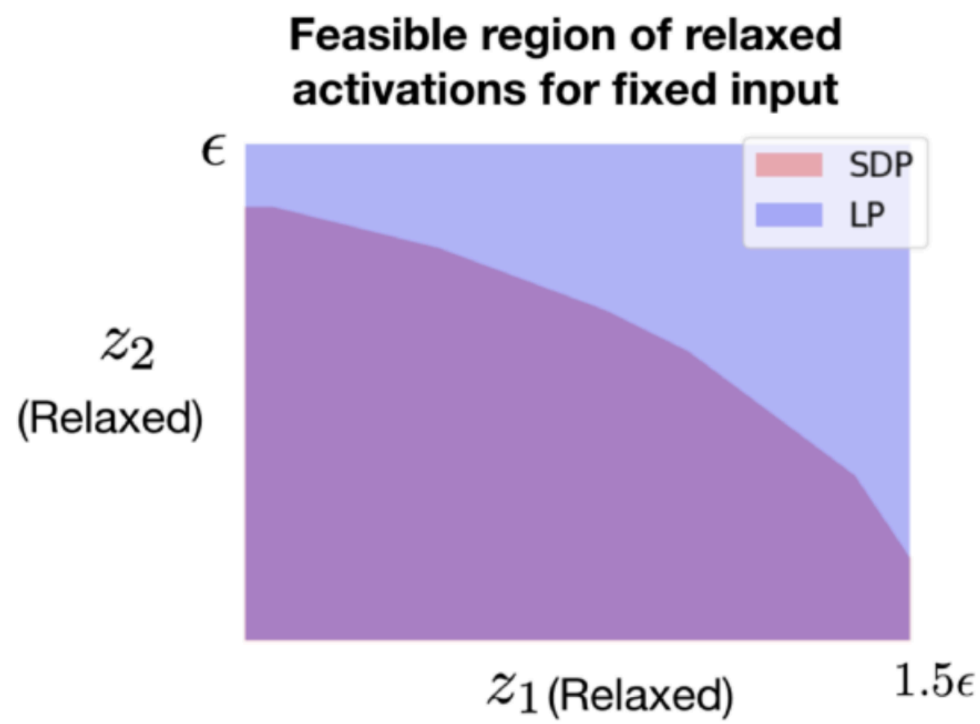

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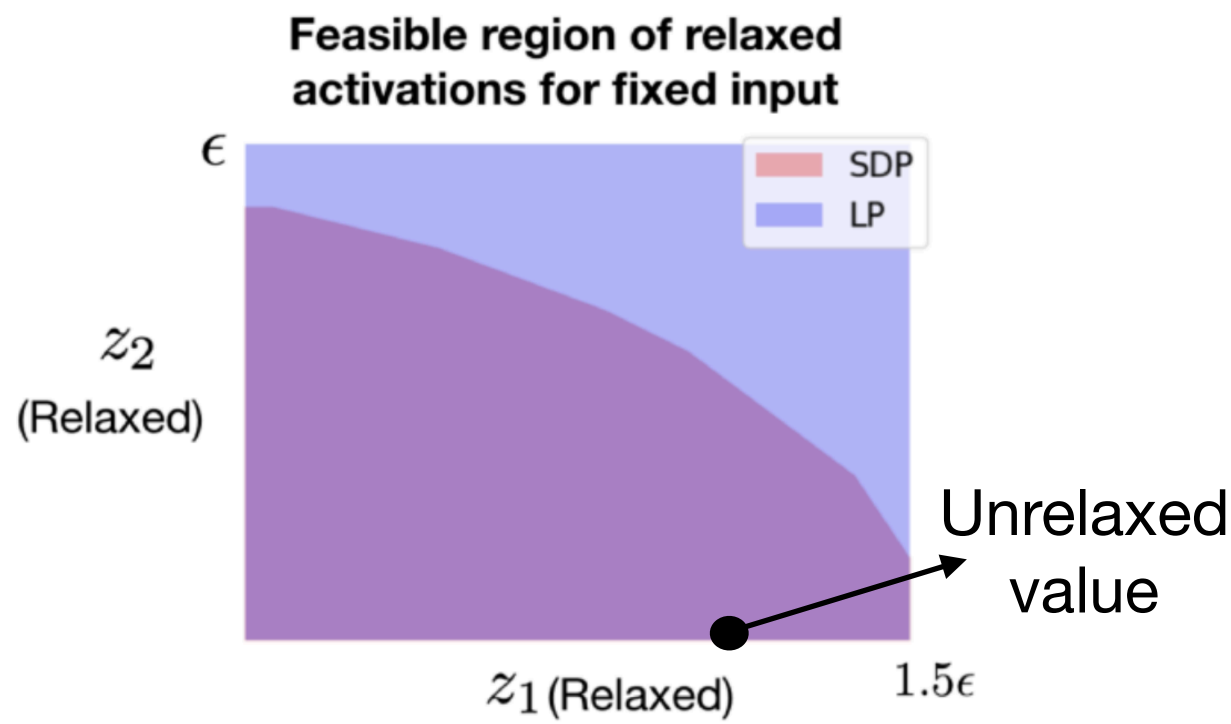

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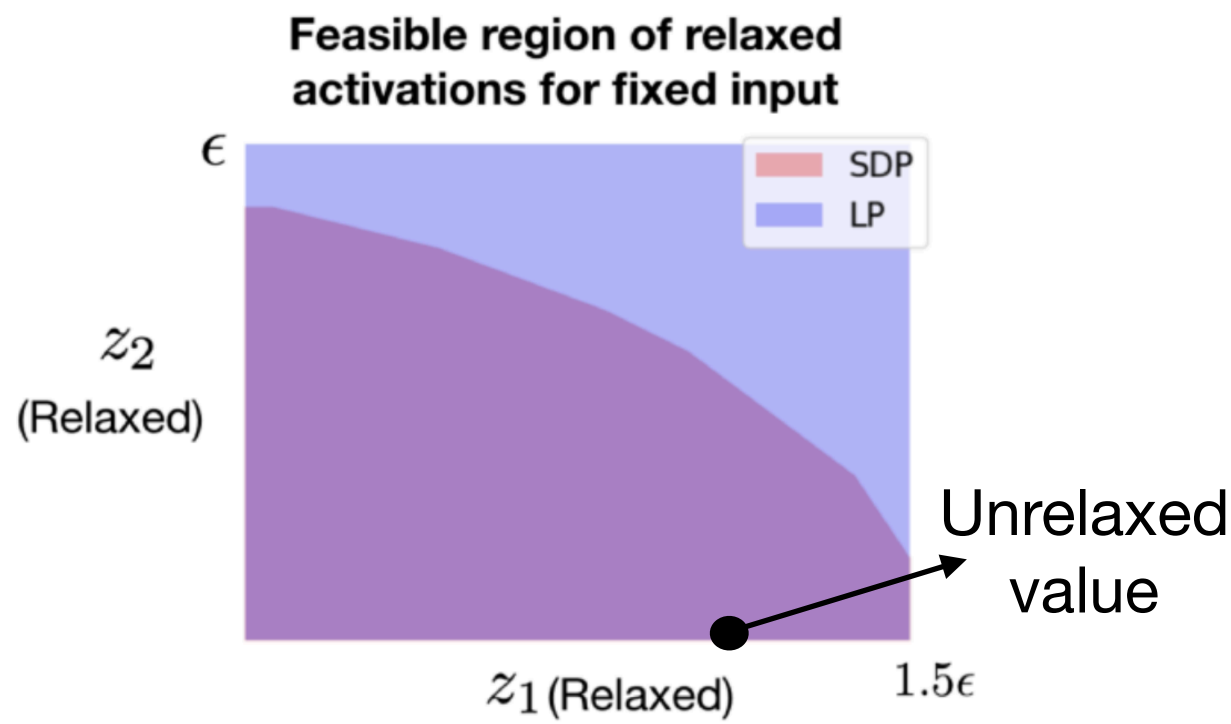

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SDP reasons jointly

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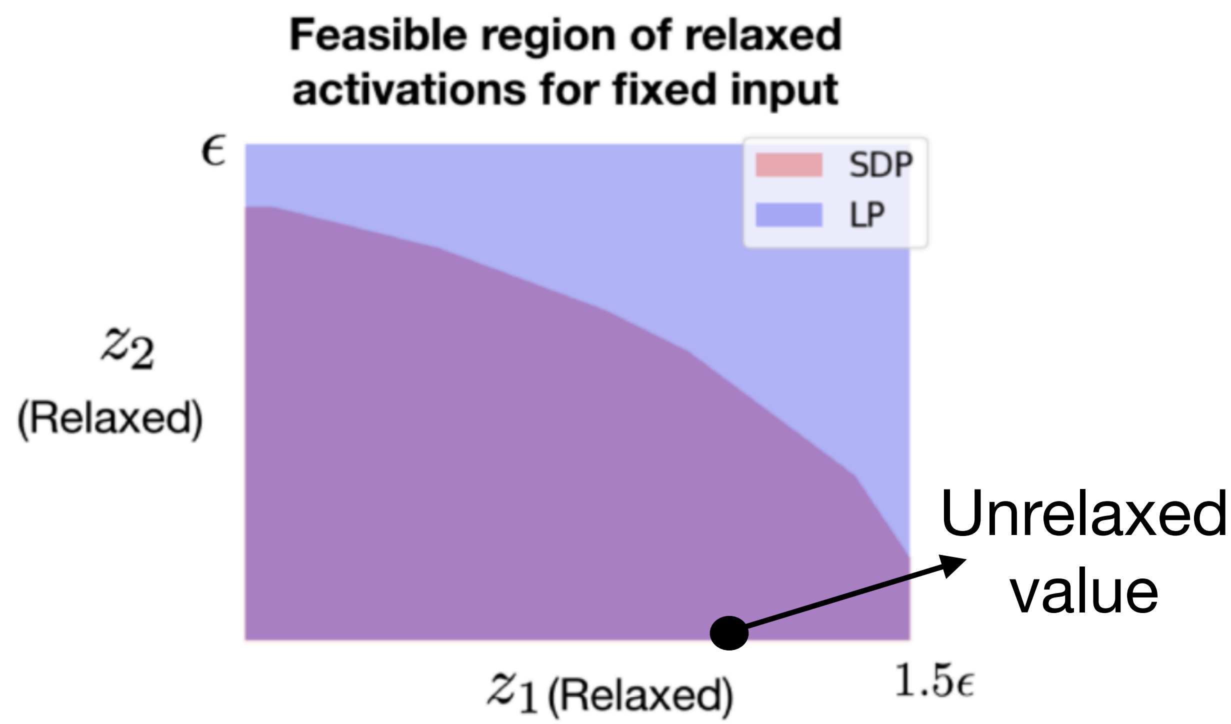

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Theorem: For a random two layer network with m hidden nodes and input dimension d, opt(LP) = $\Theta(md)$ and opt(SDP) = $\Theta(m\sqrt{d} + d\sqrt{m})$

Results on MNIST

Results on MNIST

Three different robust networks

Results on MNIST

Three different robust networks

Grad-NN
[Raghunathan et al. 2018]

Results on MNIST

Three different robust networks

Grad-NN
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Results on MNIST

Three different robust networks

Grad-NN	LP-NN	PGD-NN
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Grad-NN	LP-NN	PGD-NN
[Raghunathan et al. 2018]	[Wong and Kolter 2018]	[Madry et al. 2018]

	Grad-NN	LP-NN	PGD-NN
Grad-cert	35%	93%	N/A
LP-cert	97%	22%	100%
SDP-cert	**20%**	**20%**	**18%**
PGD-attack	15%	18%	9%

Results on MNIST

Three different robust networks

Grad-NN
[Raghunathan et al. 2018]

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LP-cert	97%	22%	100%
SDP-cert	**20%**	**20%**	**18%**
PGD-attack	15%	18%	9%

SDP provides good certificates on all three different networks

Results on MNIST

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[Madry et al. 2018]

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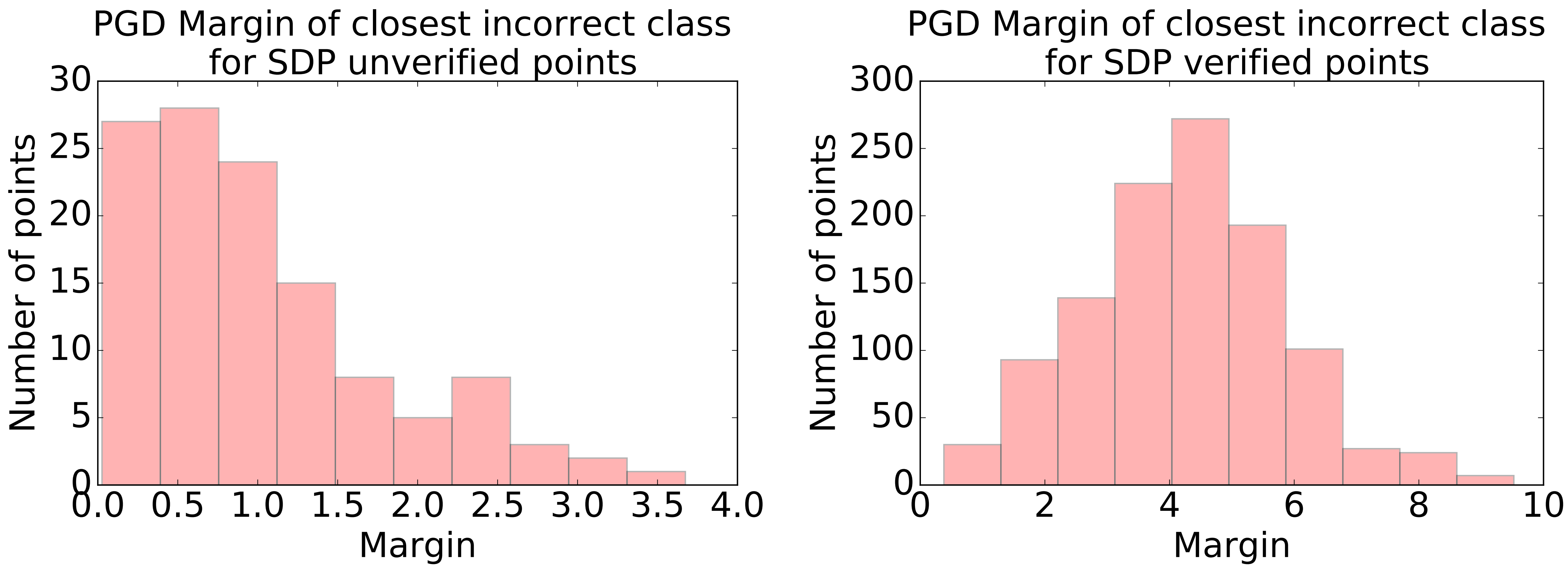

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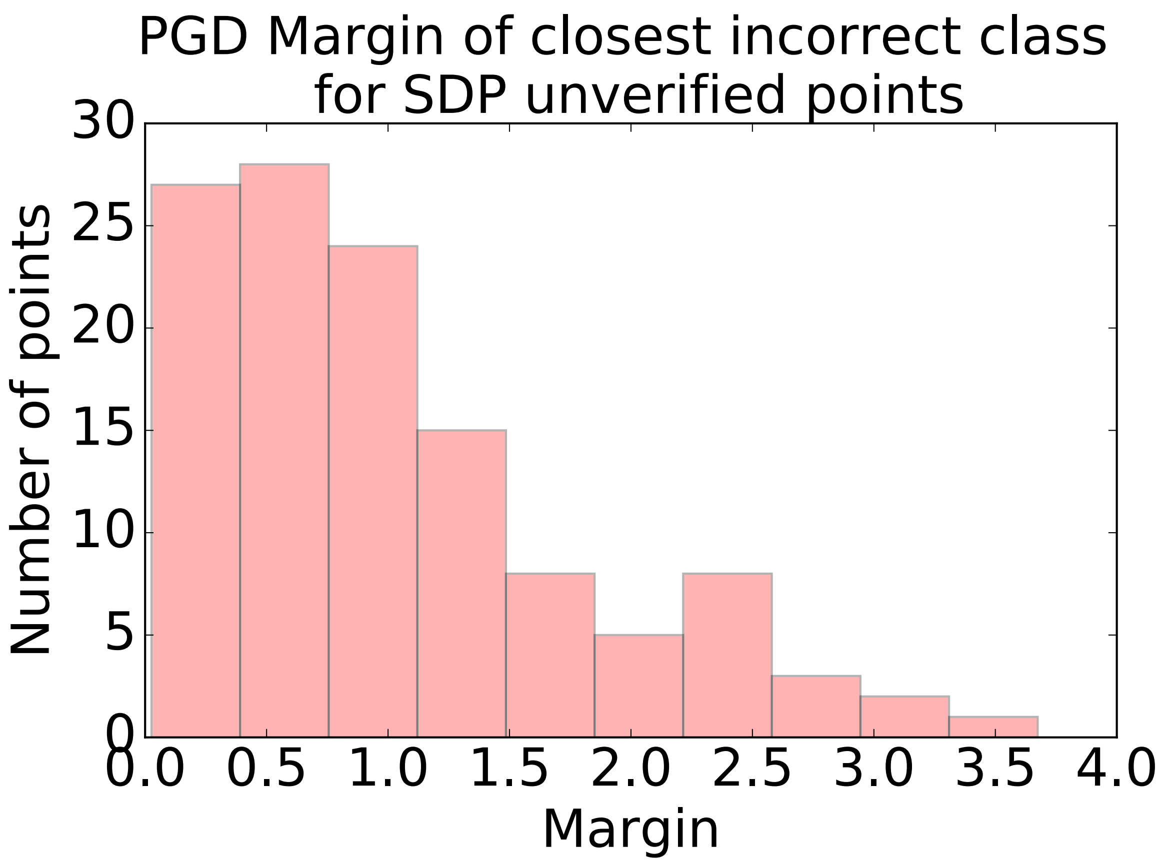

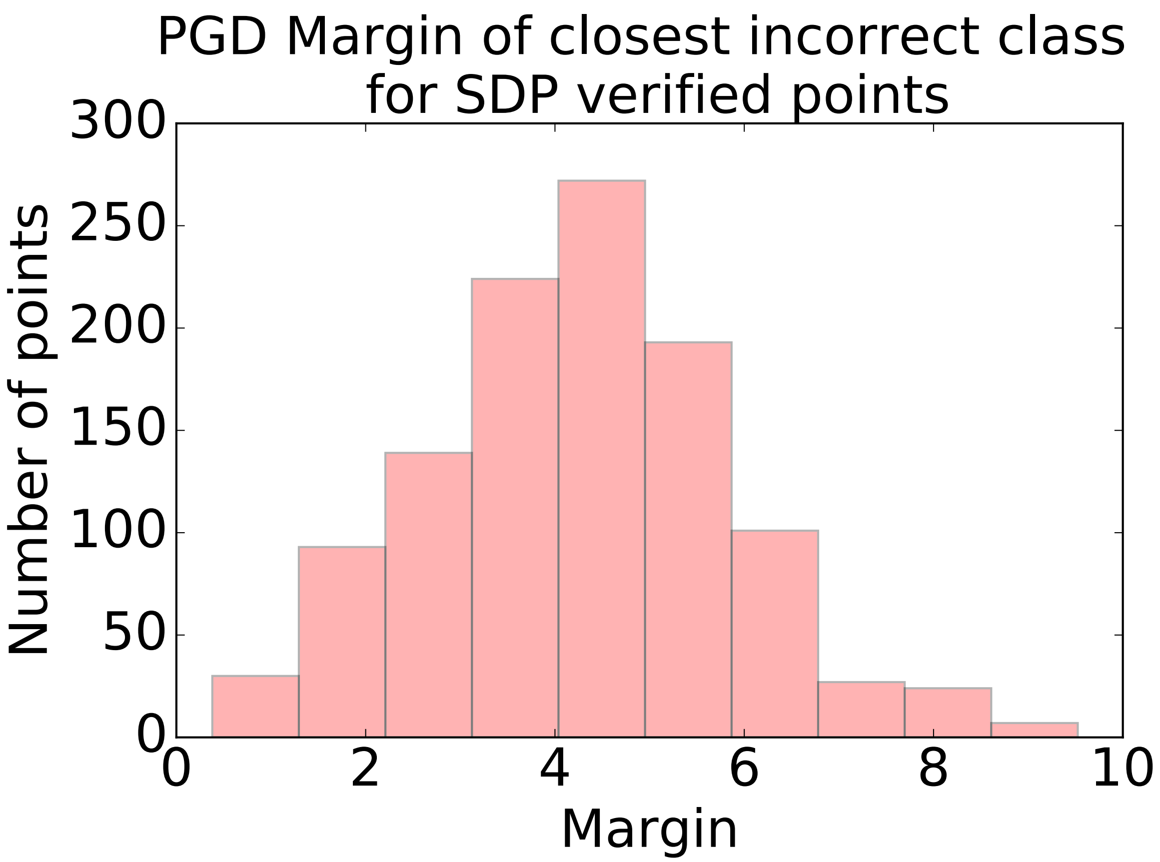

Uncertified points are more vulnerable to attack

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  - Adversarial examples expose limitations of current systems
  - How do we get models to learn **"the right thing"**?

Thank you!

Jacob Steinhardt

Percy Liang

Google

Open Philanthropy Project

"Certified Defenses against Adversarial Examples"
https://arxiv.org/abs/1801.09344 [ICLR 2018]

"Semidefinite Relaxations for Certifying Robustness to Adversarial Examples"
https://arxiv.org/abs/1811.01057 [NeurIPS 2018]