IN THIS ISSUE / DANS CE NUMÉRO

Canadian Mathematical Society
Société mathématique du Canada

CMS NOTES de la SMC

February / février 2017

President's Notes / Notes du président

Michael Bennett *(UBC)*
CMS President / Président de la SMC

Allow me to ring in the New Year by thanking CMS members, volunteers and partners across Canada for their contributions and support through 2016 and by summarizing a few recent Society accomplishments, events and initiatives.

I am pleased to report that 6,550 students wrote the 2016 Canadian Open Mathematics Challenge (COMC) competition this past November. We have seen a steady increase in COMC participation over the past five years and anticipate the trend to continue in future with the support of our generous program partners that include provincial governments, universities, organizations and individual volunteers.

The 2016 CMS Winter Meeting welcomed over 430 participants to Niagara Falls. The program, developed by scientific directors Hans Boden and Bartosz Protas, both from McMaster University, and the scientific committee comprising of Stephen Anco (Brock), Huaxiong Huang (York & Fields Institute), Kumar Murty (Toronto), Jean-Christophe Nave (McGill) and Yael Karshon (Toronto) was meticulously crafted with over 25 sessions and 10 plenary lectures. Highlights from the meeting included an enlightening public lecture by Dror Bar-Natan (Toronto); engaging plenary lectures from Karol Zyczkowski (Jagiellonian), Diane Maclagan (Warwick), Glen Van Brummelen (Quest), Natasa Sesum (Rutgers), Tom Salisbury (York) and Mark Ablowitz (Colorado at Boulder); an industry panel luncheon with NSERC and MITACS; and a celebratory

Continued on page 6

Permettez-moi d'entamer la nouvelle année en remerciant les membres et les bénévoles de la SMC de tout le pays de leur contribution et de leur soutien tout au long de l'année 2016, et en résumant quelques réalisations et évènements récentes.

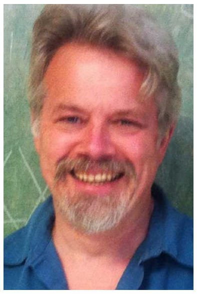
Je suis heureux d'annoncer que 6550 étudiants ont participé au Défi ouvert canadien de mathématiques 2016 (DOCM) en novembre dernier. Nous constatons une hausse graduelle de la participation au COMC au fils des cinq dernières années et nous prévoyons que la tendance se maintiendra à l'avenir avec l'appui de nos généreux partenaires du programme, notamment les gouvernements provinciaux, les universités, les organisations et les bénévoles.

La Réunion d'hiver 2016 de la SMC a rassemblé plus de 430 participants à Niagara Falls. Le programme, mis au point par les directeurs scientifiques Hans Boden et Bartosz Protas de l'Université McMaster et par le comité scientifique composé de Stephen Anco (Brock), Huaxiong Huang (York et Institut Fields), Kumar Murty (Toronto), Jean-Christophe Nave (McGill) et Yael Karshon (Toronto) a été méticuleusement élaboré et a donné lieu à plus de 25 sessions et 10 conférences plénières. Parmi les temps forts de la Réunion, mentionnons une conférence publique inspirante prononcée par Dror Bar-Natan (Toronto); des conférences plénières offertes par Karol Zyczkowski (Jagiellonian), Diane Maclagan (Warwick), Glen Van Brummelen (Quest), Natasa Sesum (Rutgers), Tom Salisbury (York) et Mark Ablowitz (Université du Colorado à Boulder); un dîner avec des experts de l'industrie, le CRSNG et MITACS, ainsi qu'un banquet de remise des prix avec vue sur les chutes Niagara. Merci à Lia Bronsard, présidente sortante de la SMC, d'avoir assisté à la Réunion d'hiver en mon

Suite à la page 6

How The Light Gets In

Robert Dawson, *Editor-in-Chief*



Last November, Canada lost one of its greatest poets with the death of Leonard Cohen. I suspect that over the following week many of us saw things through the lens of his verse - certainly I saw several Internet postings and newspaper articles applying the cynical lyrics of "Everybody Knows" to the American election. But the words I found going around in my head were from "Anthem":

> Forget your perfect offering.
> There is a crack, a crack in everything,
> That's how the light gets in."

It's easy to think of mathematics as a perfect offering, a Platonic ideal. But we know that in reality mistakes get made - sometimes actual errors of fact that somehow escape the eyes of our colleagues and editors, sometimes merely errors of judgement in stressing the wrong things. To pick one example out of many, some early work in algebraic topology looks rather clumsy by modern standards. So much effort is spent computing things that would have been almost trivial with more sophisticated (but still accessible) tools! But of course, those modern tools were developed because of the pioneering work - and if the pioneers had kept their work in their desks, the development of the more modern techniques would have been greatly delayed.

Another familiar example is Euclid's "Elements". It's well-known that Euclid's axioms are not wholly rigorous: for example, while the rational plane obeys any reasonable interpretation of those axioms, Proposition I.1 is false there! But to make Euclid rigorous (except in hindsight) would have required waiting for nineteenth century notions of continuity and completeness. Mathematics was clearly better served by timely publication!

Those flaws did not go unnoticed; but for most of the intervening time, the main flaw in Euclid's work was often thought to be that he (essentially) left the uniqueness of parallels as a postulate, rather than proving it from more basic axioms. Menelaus of Alexandria showed, around 100 AD, that the existence of parallels could be denied; but for another seventeen centuries or so, most geometers backed the wrong horse. Gauss may have shown that hyperbolic geometry was possible, but if so he did not publish; it was Bolyai and Lobachevski who showed that Euclid's postulate was indeed necessary. This time the crack was not an error on Euclid's part, but a wrong conjecture on the part of the mathematical community. But - as always - the light got in.

C'est ainsi qu'entre la lumière

En novembre dernier, le Canada a perdu un de ses plus grands poètes avec le décès de Leonard Cohen. J'imagine que la semaine suivante, nous étions nombreux à voir les choses à travers le prisme de ses vers; j'ai d'ailleurs lu plusieurs publications sur l'internet et des articles dans les journaux qui appliquaient les paroles cyniques de sa chanson « Everybody Knows » aux élections américaines. De mon côté, les mots qui m'ont le plus impressionné étaient plutôt ceux de « Anthem » :

> Forget your perfect offering.
> There is a crack, a crack in everything,
> That's how the light gets in.
>
> (Oubliez vos offrandes parfaites
> Il y a une fissure en toute chose
> C'est ainsi qu'entre la lumière.)

Il est facile de voir les mathématiques comme une offrande parfaite, un idéal platonicien. Mais nous savons qu'en réalité, des erreurs se commettent - parfois des erreurs factuelles qui échappent aux yeux de nos collègues et des rédacteurs, parfois simplement des erreurs de jugement comme nous insistons à poursuivre une mauvaise notion. Un exemple parmi tant d'autres : certains des premiers résultats en topologie algébrique semblent plutôt maladroits selon les normes actuelles. On consacre tant d'effort à calculer des éléments qui auraient presque été insignifiants avec des outils plus sophistiqués (mais tout de même accessibles)! Mais bien sûr, ces outils modernes ont été développés grâce aux travaux de pionniers, et si ces pionniers avaient gardé leurs travaux pour eux, le développement des techniques modernes aurait été grandement retardé.

Un autre exemple connu est celui des *Éléments* d'Euclide. Il est bien connu que les axiomes d'Euclide ne sont pas totalement rigoureux : par exemple, malgré que le plan rationnel obéisse à toute interprétation raisonnable de ces axiomes, la Proposition I.1 est fausse dans ce cas! Mais pour qu'Euclide devienne rigoureux (sauf en rétrospective), il aurait fallu attendre les notions de continuité et d'exhaustivité du XIXe siècle. Les mathématiques étaient nettement mieux servies par une publication opportune!

Ces défauts ne sont pas passés inaperçus. Toutefois, dans l'intervalle, on croyait généralement que le principal défaut du travail d'Euclide était qu'il laissait (essentiellement) comme postulat l'unicité des parallèles plutôt que de la prouver à partir des axiomes de base. Ménélas d'Alexandrie a montré, autour de 100 ans avant notre ère, qu'on ne pouvait nier l'existence des parallèles; mais pendant environ 17 siècles, la plupart des géomètres ont misé sur le mauvais cheval. Gauss a peut-être montré que la géométrie hyperbolique était possible, mais s'il l'a fait, il n'a rien publié à ce sujet; ce sont plutôt Bolyai et Lobachevski qui ont montré que le postulat d'Euclide était en effet nécessaire. Cette fois, la fissure n'était pas une erreur d'Euclide, mais une mauvaise conjecture de la part de la communauté mathématique. Mais, comme toujours, la lumière est rentrée par la fissure.

Letters to the Editors

The Editors of the NOTES welcome letters in English or French on any subject of mathematical interest but reserve the right to condense them. Those accepted for publication will appear in the language of submission. Readers may reach us at the Executive Office or at **notes-letters@cms.math.ca**

Lettres aux Rédacteurs

Les rédacteurs des NOTES acceptent les lettres en français ou en anglais portant sur n'importe quel sujet d'intérêt mathématique, mais ils se réservent le droit de les comprimer. Les lettres acceptées paraîtront dans la langue soumise. Les lecteurs peuvent nous joindre au bureau administratif de la SMC ou à l'adresse suivante : **notes-lettres@smc.math.ca**.

NOTES DE LA SMC

Les Notes de la SMC sont publiés par la Société mathématique du Canada (SMC) six fois par année (février, mars/avril, juin, septembre, octobre/novembre et décembre).

CMS NOTES

The CMS Notes is published by the Canadian Mathematical Society (CMS) six times a year (February, March/April, June, September, October/November and December).

Canadian Mathematical Society - Société mathématique du Canada
209-1725 St. Laurent Blvd., Ottawa, ON, Canada K1G 3V4 tel 613-733-2662

notes-articles@cms.math.ca | smc.math.ca | cms.math.ca
ISSN :1193-9273 (imprimé/print) | 1496-4295 (électronique/electronic)

CMS Notes
Volume 49 No. 1, February 2017

The Calendar brings current and upcoming domestic and select international mathematical sciences and education events to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Denise Charron, *Canadian Mathematical Society, (managing-editor@cms.math.ca)*

Le calendrier annonce aux lecteurs de la SMC les activités en cours et à venir, sur la scène pancanadienne et internationale, dans les domaines des mathématiques et de l'enseignement des mathématiques. Vos commentaires, suggestions et propositions sont le bienvenue.

Denise Charron, *Société mathématique du Canada (redacteur-gerant@smc.math.ca)*

FEBRUARY 2017 FÉVRIER

2	CRM: Lecture by Daniel Wise (2016 CRM-Fields-PIMS Prize Recipient), CRM, Montreal, Que.
5-10	BIRS Workshop: Newton-Okounkov Bodies, Test Configurations, and Diophantine Geometry, BIRS, Banff, Alta.
12-17	BIRS Workshop: Mathematical Approaches to Evolutionary Trees and Networks, BIRS, Banff, Alta.
13-17	Fields Workshop: Heights and Applications to Unlikely Intersections, Fields Institute, Toronto, Ont.
19-24	BIRS Workshop: Validating and Expanding Approximate Bayesian Computation Methods, BIRS, Banff, Alta.
21-23	Fields Distinguished Lecture Series: Umberto Zannier, Fields Institute, Toronto, Ont.
26-Mar 3	BIRS Workshop: Brain Dynamics and Statistics: Simulation versus Data, BIRS, Banff, Alta.

MARCH 2017 MARS

5-10	BIRS Workshop: Optimization and Inference for Physical Flows on Networks, BIRS, Banff, Alta.
7-9	Fields Special Lecture Series: Littlewood Lecture Series: Robert Vaughan, Fields Institute, Toronto, Ont.
9-12	Fields Workshop: Algebraic Varieties With a Special Emphasis on Calabi-Yau Varieties and Mirror Symmetry, Fields Institute, Toronto, Ont.
10	PIMS/UBC Distinguished Colloquium: Michel Brion, University of British Columbia, Vancouver, B.C.
12-17	BIRS Workshop: New Trends in Arithmetic and Geometry of Algebraic Surfaces, BIRS, Banff, Alta.
13-17	Fields Workshop: Efficient Congruencing and Translation-invariant Systems, Fields Institute, Toronto, Ont.
19-24	BIRS Workshop: Communication Complexity and Applications, II, BIRS, Banff, Alta.
23	PIMS/UBC Distinguished Colloquium: Barry Simon, University of British Columbia, Vancouver, B.C.
24-28	Nirenberg Lectures in Geometric Analysis at the CRM: Camillo De Lellis (Universität Zürich), CRM, Montreal, Que.
27-Apr 7	CRM School and Workshop: Combinatorics on Words and Tilings, CRM, Montreal, Que.

APRIL 2017 AVRIL

2-7	BIRS Workshop: Mostly Maximum Principle, BIRS, Banff, Alta.
9-14	BIRS Workshop: Generated Jacobian Equations: from Geometric Optics to Economics, BIRS, Banff, Alta.
16-21	BIRS Workshop: Geometric Structures on Lie Groupoids, BIRS, Banff, Alta.
23-28	BIRS Workshop: Quantum Field Framework for Structured Light Interactions, BIRS, Banff, Alta.
24-May 5	CRM School and Workshop: Bridges Between Automatic Sequences, Algebra and Number Theory, CRM, Montreal, Que.
30-May 5	BIRS Workshop: Phase Transitions Models, BIRS, Banff, Alta.

2017 CMS Winter Meeting

December 8-11, 2017
University of Waterloo, Waterloo, Ontario

CALL FOR SESSIONS

The Canadian Mathematical Society (CMS) welcomes and invites proposals for sessions for the 2017 CMS Winter Meeting in Waterloo. Proposals should include a brief description of the focus and purpose of the session, the expected number of speakers, as well as the organizer’s name, complete address, telephone number, e-mail address, etc. All sessions will be advertised in the CMS Notes, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the scientific directors.

Scientific Directors:

Ken Davidson, University of Waterloo, **krdavids@uwaterloo.ca**
Cam Stewart, University of Waterloo, **cstewart@uwaterloo.ca**

Réunion d’hiver de la SMC 2017

8-11 décembre 2017
Université de Waterloo, Waterloo, Ontario

NOUS VOUS INVITONS

La Société mathématique du Canada (SMC) vous invitent à proposer des séances pour la Réunion d’hiver de la SMC 2017 qui se tiendra à l’Université de Waterloo du 8 au 11 décembre 2017. Ces propositions de séances doivent presenter une brève description de l’orientation et des objectifs de la séance, le nombre de conférenciers prévu, de même que le nom, l’adresse complète, le numéro de téléphone et l’adresse électronique de l’organisateur. Toutes les séances seront annoncées dans les Notes de la SMC, sur le site Web SMC et dans les AMS Notices. Les conférenciers devront présenter un résumé, qui sera publié sur le site Web SMC et dans le programme de la réunion. Toute personne qui souhaiterait organiser une séance est priée de faire parvenir une proposition aux directeurs scientifiques.

Directeurs scientifiques:

Ken Davidson, Université de Waterloo, **krdavids@uwaterloo.ca**
Cam Stewart, Université de Waterloo, **cstewart@uwaterloo.ca**

CMS Notes
Volume 49 No. 1, February 2017

Continued from cover

awards ceremony overlooking Niagara Falls. Thank you to Lia Bronsard, CMS Past President, for attending the Winter Meeting on my behalf. I hope to see many of you at the next CMS meeting which will occur as part of the 2017 Mathematical Congress of the Americas (MCA 2017) from July 24-28.

As I had reported in my first President's Note in the September issue of the CMS Notes (CMS Notes Vol. 48:4 [2016] pp.1, 6-7.), many of the challenges that we face, at least on a certain level, are financial. In order for the Society to continue to finance projects and programs that our members value, we must expand our fundraising efforts to cover the increasing costs. Throughout 2016, the Society ran several fundraising campaigns under the direction of Gerri Jensen and David Rodgers including the CMS – Air Canada Miles campaign that garnered over 500,000 Aeroplan miles for student travel to CMS training camps, competitions and meetings. We plan to increase our focus on fundraising throughout 2017 in order to support ongoing programs and projects including math camps, competitions, educational journals like *Crux Mathematicorum*, and CMS meetings.

Continued involvement from our community is needed in order for the Society to reach its full potential. I encourage you to become a member of the CMS, renew your membership and to begin volunteering on a committee in an area of personal interest. If you have specific questions about what volunteering entails, I encourage you to contact the current members or chair of the committee. A list of the current Executive and Board appointments and vacancies can be found at **https://cms.math.ca/Docs/commlist.html**.

The success of our Society is reflective of and dependent on guidance from our membership. I continue to seek input from members of the Society to help us in shaping the future of the CMS. What, as a Society, should we be doing? Your guidance is welcomed and appreciated as we review our values in the coming years. Please submit your ideas, feedback or comments to me at **president@cms.math.ca**.

Suite de la couverture

nom. J'espère vous voir en grand nombre à la prochaine Réunion de la SMC, qui aura lieu dans le cadre du Congrès mathématique des Amériques 2017 (CMA 2017) du 24 au 28 juillet.

Comme je l'avais mentionné à l'occasion de mon premier Mot du président dans le numéro de septembre des *Notes de la SMC* (vol. 48:4 [2016] p. 1, 6-7), une grande partie des défis à relever sont, au moins à un certain niveau, surtout d'ordre financier. Afin que la Société continue de financer des projets et des programmes que nos membres apprécient, nous devons élargir nos efforts de collecte de fonds pour couvrir les coûts croissants. Tout au long de l'année 2016, la Société a organisé plusieurs campagnes de financement sous la direction de Gerri Jensen et David Rodgers, y compris la campagne SMC – miles Air Canada, qui a recueilli plus de 500 000 miles Aéroplan afin de permettre à des élèves et des étudiants de se rendre à des camps d'entraînement, à des concours ou aux Réunions de la SMC. Nous prévoyons augmenter nos efforts de collecte de fonds tout au long de l'année 2017 afin de soutenir les programmes et les projets en cours, y compris les camps de mathématiques, les revues pédagogiques comme *Crux Mathematicorum*, ainsi que les Réunions la de SMC.

La participation continue de notre communauté est nécessaire pour que la Société atteigne son potentiel. Je vous encourage à devenir membre de la SMC, à renouveler votre adhésion et à commencer à faire du bénévolat au sein d'un comité dans un domaine qui vous intéresse. Si vous avez des questions précises sur ce que le bénévolat suppose, je vous encourage à communiquer avec les membres actuels ou avec le président du comité concerné. La liste des membres actuels et des postes vacants au comité exécutif et au conseil d'administration se trouve à **https://smc.math.ca/Docs/comliste.html**

Le succès de notre Société est le reflet des orientations communiquées par nos membres et repose sur eux. Je continue à consulter les membres pour qu'ils nous aident à forger l'avenir de la Société. Que devrions-nous faire à titre de Société? Nous sommes sensibles à vos conseils, qui seront les bienvenus à mesure que nous réviserons nos valeurs dans les années à venir. Merci de m'envoyer vos idées, vos réactions ou vos commentaires à **president@smc.math.ca**.

Supporting Mathematicians in Canada in 2017

In 2017, the CMS is doing many things to support mathematicians:

- Hosting the 2017 Mathematical Congress of the Americas (MCA) in Montreal (July 24-28);
- Working to ensure interesting and productive meeting sessions; and
- Encouraging collaboration and scientific research through membership activities.

The CMS will also be involved in several educational and outreach activities for students, including sponsorship of:

- The CMS Student Committee;
- Poster contests for Graduate students;
- Math Team Canada at the International Mathematical Olympiad (IMO);
- Math Competitions; and
- Math Camps.

While your membership dues cover part of the costs to operate the CMS, due to declining publication revenue we need to supplement income by asking for donations from you, your colleagues, corporations and foundations. Please consider donating now at **Canada Helps**.

More ways you can help:

Donate Air Canada Miles at: **Aeroplan** (last year the almost 500,000 donated provided travel for students to meetings and the IMO);

Identify mathematicians who work in private industry who we can contact to support the work of the CMS;

Volunteer for a CMS Committee;

Consider the CMS as part of your Estate Planning; and

Encourage your colleagues to join the CMS.

Appuyez les mathématiciens du Canada en 2017

En 2017, la SMC a de nombreux projets pour soutenir les mathématiciens :

- Coparrainer le Congrès mathématique des Amériques (CMA) qui se tiendra en juillet prochain à Montréal;
- Organiser des sessions intéressantes et productives à ses Réunions;
- Encourager la collaboration et la recherche scientifique par des activités pour ses membres.

La SMC organisera également plusieurs activités éducatives et de sensibilisation pour les élèves et les étudiants, y compris le parrainage :

- du Comité des étudiants de la SMC;
- de concours de présentation par affiche pour les étudiants diplômés;
- de l'Équipe Math Canada qui participe à l'Olympiade internationale de mathématiques (OIM);
- de concours mathématiques; et
- de camps mathématiques.

Bien que vos cotisations couvrent une partie des coûts de fonctionnement de la SMC, nous devons trouver d'autres sources de revenus en sollicitant des dons auprès de vous, de vos collègues, d'entreprises et de fondations, en raison des revenus de publication en déclin. Songez à faire un don maintenant à **CanaDON**.

Vous pouvez aussi nous aider :

En donnant des milles d'Air Canada à : **Aéroplan** (l'année dernière, les quelque 500 000 milles donnés ont couvert les déplacements d'élèves qui ont assisté à nos Réunions et à l'OIM);

En identifiant des mathématiciens du secteur privé que nous pouvons contacter pour les inviter à adhérer ou à faire un don à la SMC;

En devenant bénévole à l'un des comités de la SMC;

En nommant la SMC comme bénéficiaire dans le cadre de votre planification successorale; et

En encourageant vos collègues à adhérer à la SMC.

Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, *Dalhousie University (notes-reviews@cms.math.ca)*

Les comptes-rendus de livres présentent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l'enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont le bienvenue.

Karl Dilcher, *Dalhousie University (notes-critiques@smc.math.ca)*

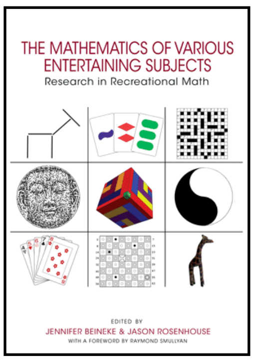
The Mathematics of Various Entertaining Subjects: Research in Recreational Mathematics

Edited by Jennifer Beineke and Jason Rosenhouse

Princeton University Press, 2015

ISBN 978-1400881338

Reviewed by *Richard Nowakowski*, Dalhousie University



Ever "been in the situation of being able to solve a puzzle one day yet be unable to make headway at all … on a puzzle that is ostensibly of the same level"? Then the answer to the question "Should You Be Happy?" to receive a copy of this book is Yes! Just to be sure, if you think your answer is 'probably' then read Peter Winkler's essay first. This book contains an amusing and bemusing collection of essays. Some, such as "Parallel Weighings of Coins" by Tanya Khovanova, are excellent introductions to the topics. Others explore a single topic explaining in detail the solution and the mathematics involved. All include at least a brief history, a bibliography and open questions. After reading through the book, this reviewer is still working through details. It will be the one mathematics book I will be allowed to take on the Xmas break.

It is difficult to do justice to each chapter. Here is a little taste.

The book is divided in 6 parts. In the first part, the authors present a collection of problems with a common theme. Peter Winkler looks at probabilistic puzzles starting with a baseball problem: your team, the underdogs, looses the first game of the World Series. You miss the next two and then hear that these games were split. In this situation and others, Peter answers the question: "Should you be happy?" Anany Levitin looks at puzzles that require exactly one move, including the classic re-arrangement puzzles and dissection-with-one-cut problems. Robert Bosch, Tim Chartier & Michael Rowan look at creating mazes based on an image using, amongst others, the Travelling Salesman problem, Minimum-cost spanning subgraph, and Phyllotaxis. Jennifer & Lowell Beineke explore 7 puzzles where graphs (can) play an integral part in the solution; from Mortal Combat (a game colouring battle) to Schwenk's version of non-transitive dice where even the order relations depend on how many throws are to be made.

The essays in Part II are extensions of some well known puzzles. Some of these extensions are in novel directions. Max Alekseyev & Toby Berger introduce randomness into the Tower of Hanoi and by using statistics and electric networks, they note that if the monks are not perfect then the end of the world is still far off. Julie Beier & Carolyn Yackel consider groups that can be associated with tetraflexagons and end with some intriguing questions. Tanya Khovanova re-examines Knop's problem of finding a counterfeit coin where there are multiple scales that can be used in parallel but each weighing takes 1 minute and there is a time constraint. John McSweeney analyzes the difficulty of Crossword puzzles using graphs. An answer is a vertex and there is an edge between two answers that cross each other. The analysis is then based on a random graph model related to epidemic modeling! Derek Smith gives a purely combinatorial solution to how to make a $2k+1x2k+1x2k+1$ cube out of $2k+11x1x1$ cubes and, for $1 \leq I \leq k$, six rectangular blocks of size $ix2k+1-ix2$.

Part III has Card 'Tricks'. Neil Calkin & Colm Mulcahy consider generalizations of a special shuffle found by Mulcahy. Using decks with different numbers of suits, Dominic Lanphier & Laura Taalman consider the problem of: Is it easier to get a Straight, A Flush or a Full House? They invoke Diophantine equations to show that there is no deck in which the probabilities are equal. Robert Vallin looks at the continued fractions generated by a certain card trick.

Part IV looks at Games. Maureen Carroll & Steven Dougherty and David Molnar look at Maker-Maker games, respectively, Tic-Tac-Toe on Affine planes, and variants of the connection games Athol and Begrid, developed by Mark Steere, which have Hex and Y as special cases. Gary Gordon & Elizabeth McMahon show how the card game SET leads to error correcting and detecting codes.

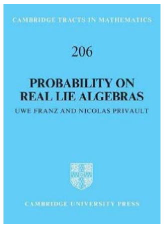
The last section deals with Fibonacci numbers. Leigh Braswell & Tanya Khovanova expand on Vaserlind, Guy & Larson's Cookie Monster problem—the Monster eats cookies but each has an expiration date, how should the Monster maximize the number of cookies eaten? They show that n-nacci numbers (add the last n terms) are involved. Finally, Stephen Lucas looks at expressing numbers in terms of Fibonacci numbers, extending past the Zeckendorf representation.

Short Review : Probability on Real Lie Algebras

by Uwe Franz and Nicolas Privault

Cambridge University Press, 2016

ISBN: 978-1-107-12865-1



This contents of this book, published in the well-known *Cambridge Tracts* series, are best summarized by quoting the book description: “This monograph is a progressive introduction to non-commutativity in probability theory, summarizing and synthesizing recent results about classical and quantum stochastic processes on Lie algebras. In the early chapters, focus is placed on concrete examples of the links between algebraic relations and the moments of probability distributions. The subsequent chapters are more advanced and deal with Wigner densities for non-commutative couples of random variables, non-commutative stochastic processes with independent increments (quantum Lévy processes), and the quantum Malliavin calculus. This book will appeal to advanced undergraduate and graduate students interested in the relations between algebra, probability, and quantum theory. It also addresses a more advanced audience by covering other topics related to non-commutativity in stochastic calculus, Lévy processes, and the Malliavin calculus.”

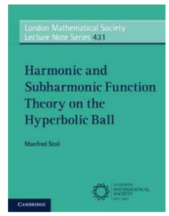
Each of the twelve chapters ends with a set of exercises, with solutions provided at the end of the book. There is also an 18-page appendix which gathers some background and complements on orthogonal polynomials, moments and cumulants, the Fourier transform, adjoint action on Lie algebras, nets, closability of linear operators, and tensor products.

Short Review : Harmonic and Subharmonic Function Theory on the Hyperbolic Ball

by Manfred Stoll

Cambridge University Press, 2016

ISBN: 978-1-107-54148-1



According to the book’s description, “This comprehensive monograph is ideal for established researchers in the field and also graduate students who wish to learn more about the subject. The text is made accessible to a broad audience as it does not require any knowledge of Lie groups and only a limited knowledge of differential geometry. The author’s primary emphasis is on potential theory on the hyperbolic ball, but many other relevant results for the hyperbolic upper half-space are included both in the text and in the end-of-chapter exercises.”

In the preface the author further mentions that his development of the theory is analogous to the approach taken by W. Rudin (*Function Theory in the Unit Ball of* $\mathbf{C}^n$, Springer, 1980) and in his own earlier book (*Invariant Potential Theory in the Unit Ball of* $\mathbf{C}^n$”, Cambridge, 1994). He also states that with only one or two exceptions, this book is self-contained and the only prerequisites are a standard beginning graduate course in real analysis.

After a brief review of Möbius transformations and a chapter on Möbius self-maps on the unit ball, the book is divided into eight further chapters, each ending with a set of exercises (102 in total). Some of these exercises are quite substantial and introduce additional material related to the relevant chapter.

Finally, it is worth mentioning that part of the first draft of the book was written while the author was on sabbatical leave at the CRM; the author also acknowledges the help and motivation provided by researchers in Montréal.

Book Reviews in Crux Mathematicorum

Crux Mathematicorum (https://cms.math.ca/crux/), another CMS publication, also has a Book Reviews column, featuring reviews that might interest readers of the CMS Notes as well. On campuses with institutional membership (as is the case with almost all Canadian universities) there will be free electronic access to *Crux Mathematicorum*. During the past year, the following books were reviewed:

A Mathematical Space Odyssey : Solid Geometry in the 21st Century, by Claudi Alsina and Roger B. Nelsen. MAA Press, 2015. (Vol. 41, #4).

The Ellipse : A Historical and Mathematical Journey, by Arthur Mazer. Wiley, 2010. (Vol. 41, #8).

Trigonometry: A Clever Study Guide, by James Tanton, MAA Press, 2015. (Vol. 41, #9).

Patterns of the Universe : A Coloring Adventure in Math and Beauty, by Alex Bellos and Edmund Harriss. The Experiment Publishing, 2015. (Vol. 41, #9).

Statistics Done Wrong: The Woefully Complete Guide, by Alex Reinhart. No Starch Press, 2015. (Vol. 42, #1).

Problems for Metagrobologists: A Collection of Puzzles With Real Mathematical, Logical or Scientific Content, by David Singmaster. World Scientific, 2016. (Vol. 42, #4).

Education Notes brings mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Jennifer Hyndman, *University of Northern British Columbia (hyndman@unbc.ca)*
John McLoughlin, *University of New Brunswick (johngm@unb.ca)*

Notes pédagogiques présentent des sujets mathématiques et des articles sur l'éducation aux lecteurs de la SMC dans un format qui favorise les discussions sur différents thèmes, dont la recherche, les activités et les nouvelles d'intérêt. Vos commentaires, suggestions et propositions sont le bienvenue.

Jennifer Hyndman, *University of Northern British Columbia (hyndman@unbc.ca)*
John McLoughlin, *University of New Brunswick (johngm@unb.ca)*

Group exams: the good, the bad and the students' comments

Kseniya Garaschuk,
University of the Fraser Valley,
kseniya.garaschuk@ufv.ca

Disclaimer. No teaching technique will work for every instructor, every class, every university. Below is an account of my experiences with one particular teaching tool, as well as the summary of an experimental study with 6 instructors and hundreds of students. As the characters and events in this article are all real, their opinions should be taken into account.

Many of us use group work in our classes whether through think-pair-share, "now you try one" or any type of activity that encourages or even requires students to work together. Peer instruction and its benefits are so widely recognized that some universities have clicker bases installed in every large classroom. By using group work in class, you are signalling to your students that it is something that you value as a teaching and learning tool, something that you find important enough to dedicate class time to, something that you want to make sure they get exposed to and hence, practice doing. Yet, our learning outcomes as well as course assessments tend to be based solely on individual performance.

So we are faced with a dilemma: how do we align our active instructional strategies with generally static course assessments? Furthermore, shouldn't learning happen as a result of every component and activity of the course, including exams?

The biggest issue that I struggle with when it comes to traditional evaluations is their immediate expiration, which is particularly damaging with mid-semester tests. The test itself and the material tested become dead to students the moment the test is over – so dead, in fact, that students may not even bother picking up test remains. How many of you have boxes of old tests lying around your office? Many students do not consider such evaluations as learning opportunities; they rarely go back to reflect on their mistakes and correct their understanding of the concept or adjust their study habits. This "from-test-to-test" approach compartmentalizes the material. However, in most mathematics courses, the material builds upon itself and hence misconceptions need to be fixed or they will hinder further understanding of the subject matter.

So how do we design an assessment to fit all of those needs?

I will discuss one type of collaborative testing that I have experimented with in my teaching. We will talk about so-called *two-stage group exams*. (Note that the term exam is being used interchangeably with tests in this discussion.) The two stages alluded to are formatted as follows:

- Stage 1 Individual: standard formal assessment
- Stage 2 Group: students revisit the same or similar problems in small groups.

The total exam grade is calculated as a combination of the grades from the two stages with the individual stage grade normally being about 75-80% of the total grade. Ordinarily, the students are also given the best of individual and combined individual+group work grade. You can vary the type of assessment (background or test review done in this way is particularly effective) and approach to group stage (repeat the whole test, change question format, add harder questions, etc.). It is important to point out that these stages occur back-to-back in a single class period.

This format has several advantages on top of including active group work on exams. First of all, it gives individuals an opportunity to do the work themselves initially, so everyone is able to contribute to the group stage. Secondly, while the final grade largely contains evaluation of their individual performance, the graded group stage motivates students to participate and defend their opinions. In general, the active immediate review of the material is where learning happens during this type of exam. Finally, as opposed to options such as group projects, this method can be reasonably implemented in a large classroom.

I first tried this format as a voluntary (not for a grade) exam review activity in Spring 2015 in my Multivariate Calculus class of 90 students. It was a success as a midterm review held two days before the midterm. It was mostly a failure as a final exam review held a week before the test. Lesson learned – timing matters because students need to be familiar with the material in order to engage in meaningful discussions about it. Another pointer for running this format as a non-graded review is to do it at the beginning of the lecture; if you do it at the end, students will not stick around for the group discussion (the review is dead once it is over too, remember?). But one thing is sure: when it worked, it was the most interactive and effective review class I had ever held.

I wanted more than anecdotal evidence indicating that the two-stage set-up provides a positive learning experience; I wanted to know what I could claim about the influence of this format on student material retention and whether it was suitable for exams in large mathematical classes. I also wanted to know the opinions of students and instructors regarding this kind of assessment. So in the Fall 2015 semester, I ran a study in two very different courses at the University of British Columbia examining feasibility and effectiveness of two-stage exams. We implemented the study in bi-weekly quizzes in two different courses (first-year calculus and second-year matrix algebra, 309 and 525 students respectively), with different question formats (multiple choice and short answer), of different length (30 minutes and 50 minute quizzes) and with different grading schemes (80% individual and 20% group grade versus 50% individual and 50% group grade). We then compared student performance on the quiz and the corresponding topics on the final exam. We also collected student comments and instructor feedback. Quantitatively, the two-stage quiz format did not have a strong influence on material retention. Qualitatively, students preferred this format to traditional quizzes and had many positive things to say about it. You can read the study specifics and analysis of the data collected in our upcoming paper on the topic (contact me for more details). Here, I will simply highlight some outcomes.

In terms of group performance, we see an immediate influence of the group stage on exam averages: obviously, groups tend to perform better on average than individuals. However, a closer examination of the group results yields some interesting observations. In multiple choice questions, groups tend to converge to the right answer no matter how people performed individually. In short answer questions, there is a lot more variety as both group composition in terms of the individual strengths of students and the choice of a scribe matters. Furthermore, knowledge does spread through the group; that is, group answers are actually a group effort as students can collaboratively figure out the correct answer even if most of them failed to do so individually. Moreover, not one single person claims all the answers, so if someone was right about questions 1 and 2, they will not persuade the group of their answer to question 3 simply "by induction". The grade attached to the group stage, no matter how little, motivates students to actively participate and really agree on the answer. (It should be noted that the group must give a single collective answer.)

The fact that we did not see increased material retention is not surprising: there is simply too much studying that goes on between any given quiz (our treatment) and the final exam (the ultimate comparison point). However, it becomes clear that while the final result of studying might be the same, the studying itself was different for the experimental groups (see student comments below). I believe that it was more efficient as students spent little time fixing misconceptions that were addressed in the group stage of the quiz. Here, a peer discussion effect is magnified because of the grade attached to the group work.

As for student comments in general, the majority of students enjoyed the group quizzes and found them useful for their learning. In fact, 79% of 425 responses (54% of all student participants) students said they would pick the two-stage group quiz format over the regular individual quizzes, if they were given the option. Here are some student comments.

"Every time you do a similar question for the rest of your life you will always check to make sure you didn't make that mistake."

Group exams are…

"a chance to see how your peers are approaching the concepts in class and LEARN from them (if you are struggling) or lend them GUIDANCE (if they are struggling)."

"help in seeing the mistakes you made five minutes after you made them. It makes you so angry it permanently burns into your mind."

"a good way to expand on how you solve a question. Working with other people provides an opportunity to gain a different perspective in understanding a concept."

"very helpful! It helps you correct any misconceptions you have about a concept. Furthermore, the more you talk things out, the easier it is to learn and correct yourself."

The few negative comments (15% of all comments received) mainly addressed group dynamics, dealing with unprepared participants and the stress of group work.

Instructor feedback was also generally positive. They liked that the group format allowed them to include more conceptual and/or complicated questions in the group portion of the exam. They trusted the groups to collaboratively figure them out and hence felt that there was genuine learning happening during the exam. Further, being an assessment, this format receives immediate student buy-in and instructors need not supervise the groups to ensure everyone is on the same page, so this format can be used both by instructors who are used to doing group work in class and those who are new to the idea.

So you think you might want to give two-stage group exams a try? Before you do, here are some words of caution.

You need enough TA support. Photocopying, invigilating, marking and entering grades all takes up longer with this format. It is particularly important to ensure you have enough people in the room helping with the exam: the papers need to be handed out to individuals, then collected, then handed out to groups and then collected again, all in a timely and organized fashion. We even devised a system to speed up grade entering just to support this experiment.

Also, running a two-stage quiz requires more class time than running a regular quiz. Therefore, it is hard to implement only in select sections of a multi-sectional course where everyone has to stay on the same pace. Even in a regular course, this activity will take up some non-trivial class time, so make sure to budget for that.

If the course assessment must include a written component, then the short-answer questions should be included on the individual part only. There are too many factors that can affect the group performance on short answer questions, which can cause student frustration and will not adequately assess their knowledge.

I encourage you to give group exams a try. They provide structured assessment that corresponds to the group work used in lectures. They are loud and engaging. Students like them and find them useful for their learning. I particularly enjoyed using this format for reviews and would especially recommend it for the first-day review of background material. (Just don't grade it. You don't want to be *that* instructor.)

If you have any questions, please email me at **kseniya.garaschuk@ufv.ca**. I would love to chat about group exams and hear about your experiences.

Math Education News from Ontario

Peter Taylor, *Queen's University, peter.taylor@queensu.ca*

The Ontario Ministry of Education has embarked on a new, and unique, strategy for renewing its vision of K-12 education. To review the history of this initiative, in 2010 it established the *Knowledge Network for Applied Education Research* (**KNAER**), a partnership with Western University and the Ontario Institute for Studies in Education (OISE), whose objective was "to build, advance, and apply robust evidence of effective practices through promoting research use, synthesizing state-of-the-art knowledge from existing bodies of evidence, and facilitating networks of policy makers, educators, and researchers, working collaboratively, to apply research to practice."

In Phase I (2010-2014) the activities of this partnership were focused on supporting a large number of projects. For Phase II (2015-2020) it was decided to shift the emphasis to the support of what it called "thematic knowledge networks that employ a systems approach to enhance knowledge mobilization on clear and specific priority themes." This past year the Ministry announced the first two of these themes: mathematics education and student and teacher well-being. In spring 2016 it called for proposals to host the mathematics knowledge network and the winning proposal, announced this fall, was that from the Fields Institute Centre for Math Education (**CME**). It has established a *Mathematics Knowledge Network* (**MKN**) with co-directors George Gadanidis (Western) and Donna Kotsopoulos (WLU) along with Dragana Martinovic (Windsor) as co-director of CME. The guiding principles of MKN are:

1) **Addressing educator-identified needs for improving student learning.** Mathematics teaching and learning needs identified by educators will be at the core of our work by engaging educators in reform that is personally and professionally meaningful and rewarding.

2) **Changing attitudes towards mathematics**. We will engage educators in co-designing mathematical learning experiences for students that offer surprise and conceptual insight, and opportunities to share their learning with family, their peers, and the wider community.

3) **Fostering inclusion**. Some groups are more marginalized than others when it comes to mathematics education. We will aim to enhance learning and participation opportunities for all learners.

In short––what are students' real learning needs, how can we communicate more broadly how wonderful mathematics is, and how can we reach marginalized communities?

Planning is well underway with the establishment of four "Communities of Practice"––these will be groups of individuals who collaborate on shared interests to create knowledge that informs and supports educational practice. The "Communities of Practice" are:

- *Mathematics Leadership*
- *Critical Transitions in Student Mathematical Development*
- *Indigenous Knowledge and Math Education*
- *Computational Thinking in Math Education*

The Network has a number of partners, professional organizations and universities. I offer this update here as the MKN representative of CMS, one of these partners. Anyone who is interested in the work of the MKN and might like to get involved is welcome to contact me.

Some resources for extending awareness of mathematics and education

John McLoughlin, *University of New Brunswick*
johngm@unb.ca

The fields of mathematics can readily be isolated in the public domain of discussion. Whether it is the intention to neatly fit mathematics into a box or the limited experience of people with mathematics is open for argument, a resolution of which is not to be obtained here. As mathematicians who care about the discipline it falls upon us to open more avenues for engagement, thus, inviting a broader circle of participation. Beginning with reference to two journals this short piece represents an invitation to those in the mathematical community to engage with resources that may be lesser known than the core mainstream periodicals. It is hoped that one's own interests and curiousities will enable an extension to others in terms of sharing ideas within classes, community conversations, or collegial collaborations.

Journal of Humanistic Mathematics

Stephen Brown introduced me to the *Humanistic Mathematics Network Journal* (*HMNJ*) during my graduate studies in Buffalo. He continues to serve on the editorial board of the *Journal of Humanistic Mathematics*, an online freely accessible journal that has spun out of the original efforts edited by Alvin White of Harvey Mudd College. The homepage for the journal is: **http://scholarship.claremont.edu/jhm/**. Amongst the content of the website is a Resources link that offers various other links including one that captures the archives of all 27 issues of *HMNJ* from 1987 through 2004.

Quoting the website: … *humanistic mathematics* could include a broad range of topics; for our purposes it means "the human face of mathematics." Thus our emphasis is on the aesthetic, cultural, historical, literary, pedagogical, philosophical, psychological, and sociological aspects as we look at mathematics as a human endeavor. More broadly, we aim to provide a forum for both academic and informal discussions about matters mathematical.

Published articles "focus mainly on the doing of mathematics, the teaching of mathematics, and the living of mathematics. We also welcome contributions about the state of the mathematical profession (both in research and in education), underrepresentation issues within the world of mathematics, mathematics across national and cultural boundaries, mathematical fiction and poetry, personal reflections that provide insight to the inner workings of the mathematical mind, and other types of writing which may stimulate discussion among our readers. Overall we are a free platform where many different conversations about mathematics are welcome and encouraged."

Those unfamiliar with the journal are encouraged to browse and see what they find. You may be surprised to find familiar names amongst the authors including Robert Dawson, the editor of ***CMS Notes***, and Chantal Buteau, a contributing author on more than one occasion to ***Education Notes***. Gila Hanna is on an Editorial Board that includes an eclectic and international mix with Philip J. Davis, Ed Dubinsky, Paul Ernest, Reuben Hersh, and Sandra Keith amongst them. A broad range of mathematical experiences including philosophical or aesthetic takes on the discipline will find a place here.

The aesthetic aspects of mathematics emerge in many ways. One can search for poetry, beauty, art, philosophy, creative writing or any such term to find numerous potential contributions of interest. Some articles that caught my attention were: *The Importance of Surprise in Mathematical Beauty* (by V. Rani Satyam), *Raphael's School of Athens: A Theorem in a Painting* (by Robert Haas), *Kaleidoscopes, Chess and Symmetry* (by Tricia M. Brown), and *Is (Some) Mathematics Poetry* (by James Henle).

Philosophy of Mathematics Education Journal

The current issue of the *Philosophy of Mathematics Education Journal* is a special issue dedicated to The Philosophy of Mathematics Education ICME-13. The issue features the papers presented in the topic group at Hamburg. All of the papers are accessible at the link below:

http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome31/index.html

The journal has also recently published a call for a special issue on Mathematics Education and the Environment. Proposals are sought "on the theme of ethics, uncertainty and complexity in relation to the link between mathematics education and the global environmental crisis (e.g., climate change, food security, water, future cities, etc.)."

Some other ideas

The *Canadian Mathematics Education Study Group* (CMESG) held its 40th meeting this past spring at Queen's University where it began. The 2017 meeting will take place at McGill University from June 2-6, 2017. Proceedings of a CMESG Meeting reflect its different flair with an unusual compilation of working group reports, plenary lectures, recent dissertation summaries, and more. The 2016 special issue highlighting excerpts from 40 years of CMESG would make a good starting place for getting a flavour of the offerings to be found in the collection: **http://www.cmesg.org/past-proceedings/**

The *Bridges Organization* oversees an annual conference bringing together art, architecture, culture, education, music and mathematics. The forthcoming Bridges 2017 Conference will be hosted at University of Waterloo from July 27th to 31st. More information can be obtained at **bridgesmathart.org** including a link to the June 2016 *Mathematics Intelligencer* article about the history of Bridges. The website offers resources including virtual art galleries and museums.

Closing Comments

These ideas scratch the surface of what is out there to be engaged with and uncovered by opening wider the curtains on mathematics. May an element of what has been shared here spark a curiousity to learn about something new or perhaps revisit a mathematical moment from your past or simply invite others to (re)kindle a spark for mathematics with greater awareness of its scope.

Research Notes brings mathematical research ideas forth to the CMS readership in a generally accessible manner that promotes discussion of relevant topics including research (both pure and applied), activities, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Patrick Ingram, *York University (notes-research@cms.math.ca)*

Les articles de recherche présentent des sujets mathématiques aux lecteurs de la SMC dans un format généralement accessible qui favorise les discussions sur divers sujets pertinents, dont la recherche (pure et appliquée), les activités et des nouvelles dignes de mention. Vos commentaires, suggestions et propositions sont le bienvenue.

Patrick Ingram, *York University (notes-recherche@smc.math.ca)*

A Problem With A Regular Outlook

Michael Coons, *School of Math. and Phys. Sciences, University of Newcastle, Callaghan, Australia, michael.coons@newcastle.edu.au*

Recall that a sequence $\{f(n)\}_{n\geqslant 0}$ is linearly recurrent if and only if there is a positive integer d, a $d\times d$ matrix $\mathbf{A}$ and $d\times 1$ vectors $\mathbf{w}$, and $\mathbf{v}$ such that $f(n)=\mathbf{w}^T\mathbf{A}^n\mathbf{v}$. Allouche and Shallit [1] introduced the following generalisation. Let $k\geqslant 2$ be an integer. We say that a sequence $\{f(n)\}_{n\geqslant 0}$ is k*-regular* (or just *regular*) if and only if there is a positive integer d, a set of k $d\times d$ matrices $\mathcal{A}_f=\{\mathbf{A}_0,\mathbf{A}_1,\ldots,\mathbf{A}_{k-1}\}$ and $d\times 1$ vectors $\mathbf{w}$ and $\mathbf{v}$ such that

$$f(n)=\mathbf{w}^T\mathbf{A}_{i_0}\mathbf{A}_{i_1}\cdots\mathbf{A}_{i_s}\mathbf{v},$$

where $(n)_k=i_s\cdots i_1 i_0$ is the base-k expansion of the integer n. While the vectors and matrices associated to a given sequence $\{f(n)\}_{n\geqslant 0}$ are not unique, there are canonical choices[1]; we call such a set $\mathcal{A}_f$ the *canonical set*.

The class of k-regular sequences contains k-automatic sequences—those sequences which are outputs of deterministic finite automata. Arguably the simplest non-automatic example is *Stern's diatomic sequence* $\{s(n)\}_{n\geqslant 0}$, which is 2-regular and determined (in the above terminology) by the vectors and matrices

$$\mathbf{w}=\mathbf{v}^T=[1\ 0]\ \text{and}\ (\mathbf{A}_0,\mathbf{A}_1)=\left(\begin{bmatrix}1&1\\0&1\end{bmatrix},\begin{bmatrix}1&0\\1&1\end{bmatrix}\right).$$

Stern's sequence has some pretty amazing properties, but probably the most interesting is that its ratios $\{s(n)/s(n+1)\}_{n\geqslant 0}$ enumerate the nonnegative rational numbers, in reduced form, without repeats.

In this short note, we present a few properties of regular sequences and relate them to a question of Lagarias and Wang.

We first address size. Just as the growth of a linear recurrent sequence is dictated by the eigenvalues of the associated matrix—in particular, by its spectral radius—the growth of a regular sequence is related to the joint spectral radius of a certain set of associated matrices. The *joint spectral radius* of a finite set of matrices $\mathcal{A}=\{\mathbf{A}_0,\mathbf{A}_1,\ldots,\mathbf{A}_{k-1}\}$ denoted $\rho(\mathcal{A})$, is defined as the real number

$$\rho(\mathcal{A})=\limsup_{n\to\infty}\max_{0\leqslant i_0,i_1,\ldots,i_{n-1}\leqslant k-1}\left\|\mathbf{A}_{i_0}\mathbf{A}_{i_1}\cdots\mathbf{A}_{i_{n-1}}\right\|^{1/n},$$

where $\|\cdot\|$ is any (submultiplicative) matrix norm. This quantity was introduced by Rota and Strang [9] and has a wide range of applications. For an extensive treatment, see Jungers's monograph [7]. We define the *growth exponent of* f, denoted $\mathrm{GrExp}(f)$, by

$$\mathrm{GrExp}(f):=\limsup_{\substack{n\to\infty\\ f(n)\neq 0}}\frac{\log|f(n)|}{\log n}.$$

Allouche and Shallit [1] showed that the growth exponent of a k-regular sequence is finite. Recently, we [4] proved the following relating the growth exponent to the joint spectral radius.

Theorem 1. *Let $k\geqslant 1$ and $d\geqslant 1$ be integers and $f:\mathbb{Z}_{\geqslant 0}\to\mathbb{C}$ be a (not eventually zero) k-regular sequence. If $\mathcal{A}_f$ is a canonical set of matrices, then $\log_k\rho(\mathcal{A}_f)=\mathrm{GrExp}(f)$, where $\log_k$ denotes the base-k logarithm.*

While Theorem 1 characterises the maximal order of growth of a regular sequence, it makes no claim about the algebraic character of the value of the growth exponent, though this question could possibly be answered if we knew more about the following property.

Definition 2. A finite set of matrices $\mathcal{A}$ is said to have the *finiteness property* provided there is a specific finite product $\mathbf{A}_{i_0}\cdots\mathbf{A}_{i_{m-1}}$ of matrices from $\mathcal{A}$ such that $\rho(\mathbf{A}_{i_0}\cdots\mathbf{A}_{i_{m-1}})^{1/m}=\rho(\mathcal{A})$.

Arising from the work of Daubechies and Lagarias [5], Lagarias and Wang [8] conjectured that the finiteness property holds for all finite sets of real matrices, though this was shown to be false. A constructive counterexample (reminiscent of the Stern sequence) was given by Hare, Morris, Sidorov and Theys [6].

Theorem 3. (Hare, Morris, Sidorov and Theys [6]) *There is a real number α_*, which starts $\alpha_*=0.749326546330367557943\ldots$, such that the set*

$$\left\{\begin{bmatrix}1&1\\0&1\end{bmatrix},\alpha_*\begin{bmatrix}1&0\\1&1\end{bmatrix}\right\}$$

does not have the finiteness property.

It is an open and interesting question to determine if all finite sets of integer matrices satisfy the finiteness property. If this were the case, then it would imply that given an integer-valued regular sequence f, the number $\rho(\mathcal{A}_f)$ is algebraic over the rationals. It would also imply that the number α_* is irrational—though this is potentially a much easier problem on its own.

1 See [4] for the details of a canonical set, wherein we call such a set a set of matrices *associated to a basis*.

One of the beautiful aspects of regular sequences, is that even though they present themselves as mere combinatorial and algebraic objects, they have a deep analytic side as well.

Theorem 4. (Becker [2]). *If $\{f(z)\}_{n\geqslant 0}$ is an integer valued k-regular sequence for some integer $k\geqslant 2$ and $F(z):=\sum_{n\geqslant 0} f(n)z^n$, then there is a positive integer d and polynomials $a_0(z),\ldots,a_d(z)\in\mathbb{Z}[z]$ with $a_0(x), a_d(z)\neq 0$ such that*

(1) $a_0(z)F(z)+a_1(z)F(z^k)+\cdots+a_d(z)F(z^{k^d})=0.$

That is, the function $F(z)$ is a Mahler function.

In recent joint work with Jason Bell on transcendence tests for Mahler functions, we proved a result on the asymptotics of Mahler functions.

Theorem 5. (Bell and Coons [3]) *Let $F(z)\in\mathbb{Z}[[z]]$ be a Mahler function satisfying (1) that converges in the unit disc. Form the polynomial*

$$p_F(\lambda)=a_0(1)\lambda^d+\cdots+a_{d-1}(1)\lambda+a_d(1).$$

If $p_F(\lambda)$ has distinct roots, then there is an eigenvalue λ_F with $p_F(\lambda_F)=0$, such that as $z\to 1^-$

$$F(z)=\frac{C(z)}{(1-z)^{\log_k \lambda_F}}(1+o(1)), \tag{2}$$

where $\log_k$ denotes the principal value of the base-k logarithm and $C(z)$ is a real-analytic nonzero oscillatory term, which on the interval $(0,1)$ is bounded away from 0 and ∞, and satisfies $C(z)=C(z^k)$.

The power of asymptotics of the form (2) is that they can be used in conjunction with the Mahler functional equation (1) to give similar asymptotic results as z radially approaches any root of unity of degree k^n for any $n\geqslant 0$. Another point of interest in these asymptotics is that the `Mahler eigenvalue' λ_F of $F(z)$ is related to the sequence $\{f(n)\}_{n\geqslant 0}$. In particular, if $g(n):=\sum_{k\geqslant n} f(k)$ is always nonnegative, then $\lambda_F=\rho(\mathcal{A}_g)$, where $\mathcal{A}_g$ is any canonical set of matrices for the k-regular sequence $\{g(n)\}_{n\geqslant 0}$. In this case λ_F is an algebraic number.

Of course, one now needs to figure out how to translate such a statement back to address $\rho(\mathcal{A}_f)$. Therein lies the difficulty… and the fun in the journey.

References

[1] Jean-Paul Allouche and Jeffrey Shallit, *The ring of k-regular sequences*, Theoret. Comput. Sci. 98 (1992), no. 2, 163–197. MR1166363 (94c:11021)

[2] Paul-Georg Becker, *k-regular power series and Mahler-type functional equations*, J. Number Theory 49 (1994), no. 3, 269–286. MR 1307967

[3] Jason P. Bell and Michael Coons, *Transcendence tests for Mahler functions*, Proc. Amer. Math. Soc., to appear.

[4] M. Coons, *Regular Sequences and the Joint Spectral Radius*, Internat. J. Found. Comput. Sci., to appear.

[5] Ingrid Daubechies and Jeffrey C. Lagarias, *Sets of matrices all infinite products of which converge*, Linear Algebra Appl. 161 (1992), 227–263. MR 1142737 (93f:15006)

[6] Kevin G. Hare, Ian D. Morris, Nikita Sidorov, and Jacques Theys, *An explicit counterexample to the Lagarias-Wang finiteness conjecture*, Adv. Math. 226 (2011), no. 6, 4667–4701. MR 2775881 (2012b:15016)

[7] Raphaël Jungers, *The joint spectral radius*, Lecture Notes in Control and Information Sciences, vol. 385, Springer-Verlag, Berlin, 2009, Theory and applications. MR 2507938 (2011c:15001)

[8] Jeffrey C. Lagarias and Yang Wang, *The finiteness conjecture for the generalized spectral radius of a set of matrices*, Linear Algebra Appl. 214 (1995), 17–42. MR 1311628 (95k:15038)

[9] Gian-Carlo Rota and Gilbert Strang, *A note on the joint spectral radius*, Nederl. Akad. Wetensch. Proc. Ser. A 63 = Indag. Math. 22 (1960), 379–381. MR 0147922 (26 #5434)

CMS Notes
Volume 49 No. 1, February 2017

Uniform Boundedness Of Rational Preperiodic Points

Patrick Ingram, *York University, Toronto, pingram@yorku.ca*

Most are introduced to the iteration of rational functions through Newton's method. Given a rational function $f(z) \in \mathbb{C}(z)$ of degree at least 2, one might naturally ask for which $n \geq 1$ there exists a point of *exact period* n *for* f, that is, a $z \in \widehat{\mathbb{C}}$ (the Riemann sphere) with $f^n(z) = z$, but $f^m(z) \neq z$ for all $0 < m < n$. Given that $\mathbb{C}$ is algebraically closed, it is not surprising that the answer is "For every $n \geq 1$," except for a few very special choices of f [6].

The question makes sense over any field F, however, and for fields which are not algebraically closed it is less clear what to expect. For finite fields, no orbit can be larger than the number of points available, but over a field like $\mathbb{Q}$ there is not much one can easily say about the roots of $f^n(z) = z$, other than the general heuristic that most polynomials are irreducible.

The theory of heights gives us some insight. For two integers a and $b \neq 0$ with no common factor we define the *height* of $a/b \in \mathbb{Q}$ to be

$$h(a/b) = \log \max\{|a|, |b|\}.$$

Working over the projective line, the one added point $\infty \in \mathbb{P}^1(\mathbb{Q})$ has height 0 by convention. Note that there are only finitely many points in $\mathbb{P}^1(\mathbb{Q})$ up to any given height; that is, sets of bounded height are finite. This is just because bounded sets of integers are finite.

Theorem 1 (Northcott [10]). *Given a rational function $f(z) \in \mathbb{Q}(z)$ of degree at least 2, the set of $z \in \mathbb{P}^1(\mathbb{Q})$ with finite forward orbit is a set of bounded height. In particular, there are only finitely many periodic points for f in $\mathbb{P}^1(\mathbb{Q})$.*

The first step in proving Theorem 1 is to establish an inequality of the form

$$h(z) \leq \frac{1}{\deg(f)} h(f(z)) + C, \tag{1}$$

where the constant C depends on f but not on z. When z is an integer, and f is a polynomial with integer coefficients, this simply says that $\log|f(z)|$ is roughly $\deg(f)\log|z|$, for $|z|$ large enough. The general case for rational functions involves Hilbert's Nullstellensatz. From (1), an induction shows that

$$h(z) \leq \frac{1}{\deg(f)^n} h(f^n(z)) + \left(1 + \frac{1}{\deg(f)} + \frac{1}{\deg(f)^2} + \cdots\right) C,$$

which bounds $h(z)$ if there are only finitely many values $f^n(z)$, since the geometric series converges.

Theorem 1 raises some interesting questions, however, namely how effective and how uniform the finiteness of periodic points is. Effectivity is not that difficult. With some algebra, we can make the constants in Northcott's Theorem explicit for a given example. Listing all $z \in \mathbb{P}^1(\mathbb{Q})$ up to a cetain height, and checking whose orbits stay within this finite set, we can find all rational periodic points in a predictable, finite time. We can also give an upper bound on the number of points satisfying the height inequality, but it depends delicately on the coefficients of f.

So is there a uniform bound? We clearly cannot eliminate the dependence on d. It is a problem of elementary linear algebra to find a polynomial $f(z) \in \mathbb{Q}[z]$ of degree at most d permuting any $d+1$ distinct rational numbers cyclically. So as $d \to \infty$, we can certainly have rational functions with arbitrarily long periodic cycles over $\mathbb{Q}$. But is there a uniform bound once d is fixed? Even the case of quadratic polynomials seems to be very hard.

Conjecture (Poonen [11]). *If $c \in \mathbb{Q}$, and $f(z) = z^2 + c$ has a point of exact period n in $\mathbb{Q}$, then n is 1, 2, or 3.*

There are certainly infinitely many $c \in \mathbb{Q}$ such that $z^2 + c = z$ admits a rational solution (namely those with $1 - 4c$ a square), and one can parametrize the infinitely many $c \in \mathbb{Q}$ such that $z^2 + c$ has a rational point of period 2 or 3. On the other hand, there are some periods we know do not occur.

Theorem 2 *(Morton [9], Flynn-Poonen-Schaeffer [3], Stoll [12])*
Let $c \in \mathbb{Q}$, and let $f(z) = z^2 + c$.

- *There is no point of exact period 4 for f in $\mathbb{Q}$* [9].
- *There is no point of exact period 5 for f in $\mathbb{Q}$* [3].
- *Assuming the Generalized Birch–Swinnerton-Dyer Conjecture, there is no point of exact period 6 for f in $\mathbb{Q}$.* [12].

These results are established by fixing n, and trying to describe the set of rational solutions to the equation $f^n(z) = z$ in the variables z and c. Each such equation defines an algebraic curve, one of whose components corresponds to examples of points of exact period n. These curves are of general type for $n \geq 4$, and so have only finitely many rational points Faltings' Theorem [2], but we need to know that they in fact have none. There are, of course, many more values of n to deal with before the conjecture is established.

Rather than try to rule out examples for a given n, one could test the conjecture experimentally for a large number of values c. Although the height argument above allows one to check the conjecture for a single value of c, the computation takes a non-trivial amount of time. To test the statement for a large set of parameters, other methods must be employed.

Theorem 3. (Ingram-Hutz [4]) *The Conjecture is true for all $c = a/b \in \mathbb{Q}$, with a and b integers satisfying $\max\{|a|, |b|\} \leq 10^8$.*

So are there any non-trivial families of rational functions for which we may establish a uniform bound like that proposed in the conjecture? One positive answer comes from the arithmetic of elliptic curves. The *Lattès examples* are a special class of rational functions constructed from elliptic curves, and uniform bounds on the torsion subgroup of the group of rational points on an elliptic curve, due to Mazur and Merel [7, 8], lead quickly to uniform bounds on the number of periodic points for the corresponding Lattès functions.

Another class of examples where uniform bounds are available are certain covers of the family $z^d + c$. For instance, it is shown in [5] that if $P(t), Q(t) \in \mathbb{Q}[t]$ have no common factor, and $Q(t)$ has at least 5 distinct roots of odd order, then there is a bound on the number of periodic points for $z^2 + P(t)/Q(t)$, uniform as $t \in \mathbb{Q}$ varies (with $Q(t) \neq 0$). Here the main ingredient is a deep diophantine theorem of Darmon and Granville [1].

In both cases, we must work fairly hard even to contrive a family of examples for which a uniform bound can be established.

References

[1] H. Darmon and A. Granville. On the equations $x^p + y^q = z^r$ and $z^m = f(x, y)$. *Bull. London Math. Soc.* 27 no. 129 (1995), pp. 513–544.

[2] G. Faltings. Endlichkeitssätze für abelsche Varietäten über Zahlkörpern. *Invent. Math.* 73 no. 3, pp. 349–366.

[3] E. V. Flynn, B. Poonen, and E. Schaefer. Cycles of quadratic polynomials and rational points on a genus 2 curve. *Duke Math. J.*} 90 (1997), pp. 435–463.

[4] B. Hutz and P. Ingram. On Poonen's conjecture concerning rational preperiodic points of quadratic maps. *Rocky Mountain J. Math.* 43 no. 1 (2013), pp. 193–204.

[5] P. Ingram. Canonical heights and preperiodic points for subhomogeneous families of polynomials. arXiv:1510.08807

[6] M. Kisaka. On Some Exceptional Rational Maps. *Proc. Japan Acad.* 71 Ser. A (1995), pp. 35–38.

[7] Mazur. Modular curves and the Eisenstein ideal. *Pub. Math. de l'IHÉS* 47 no. 1 (1977), pp. 33–186.

[8] L. Merel. Bornes pour la torsion des courbes elliptiques sur les corps de nombres. *Invent. Math.* 124 no. 1 (1996), pp. 437-–449.

[9] P. Morton. Arithmetic properties of periodic points of quadratic maps, II. *Acta Arith.* 87 (1998), pp. 89–102.

[10] D. G. Northcott. Periodic points of an algebraic variety. *Annals of Math.* 51 (1950), pp. 167–177.

[11] B. Poonen. The classification of rational preperiodic points of quadratic polynomials over $\mathbb{Q}$: A refined conjecture. *Math. Z.* 228 (1998), pp. 11–29.

[12] M. Stoll. Rational 6-cycles under iteration of quadratic polynomials. *Lond. Math. Soc. J. Comput. Math.* 11 (2008), pp. 367–380.

Corrections to CMS Notes Vol. 48 No. 6 (December 2016)

Page 2, Column 1, Paragraph 4, Lines 2-3: "$f(x)$" has been corrected to "$f^t(x)$".

Page 2, Column 2, Paragraph 4, Lines 3-4: "$f(x)$" has been corrected to "$f^t(x)$".

Page 5: "FIELDS" has been corrected to "Fields" throughout the calendar.

Page 8, Column 2, Paragraph 2, Line 8: Equation has been corrected to $e^{ix} = \cos x + i \sin x.$

Page 11, Column 2, Paragraph 5: Equation has been corrected to "...∞ as does $n/\log n \ldots$"

Page 15, Column 1, 3rd Equation: Equation has been corrected to

$$E_n := 2\int_0^1 \cdots \int_0^1 \left(\prod_{1 \le j < k \le n} \frac{u_k - u_j}{u_k + u_j} \right)^2 \mathrm{d}t_2\, dt_3 \cdots \mathrm{d}t_n,$$

Corrections aux Notes de la SMC Tome 48, numéro 6 (décembre 2016)

Page 2, colonne 1, paragraphe 4, lignes 2-3 : « $f(x)$ » a été corrigé comme suit « $f^t(x)$ »

Page 2, colonne 2, paragraphe 4, lignes 3-4 : « $f(x)$ » a été corrigé comme suit « $f^t(x)$ »

Page 5 : « FIELDS » a été corrigé comme suit « Fields » dans l'ensemble du calendrier

Page 8, colonne 2, paragraphe 2, ligne 8 : L'équation a été corrigée comme suit $e^{ix} = \cos x + i \sin x.$

Page 11, colonne 2, paragraphe 5 : L'équation a été corrigée comme suit « ...∞ as does $n/\log n \ldots$ »

Page 15, colonne 1, 3e équation : L'équation a été corrigée comme suit

$$E_n := 2\int_0^1 \cdots \int_0^1 \left(\prod_{1 \le j < k \le n} \frac{u_k - u_j}{u_k + u_j} \right)^2 \mathrm{d}t_2\, dt_3 \cdots \mathrm{d}t_n,$$

CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *Independent Scholar (aackerbe@verizon.net)*

Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

Les articles de la SCHPM présente des travaux de recherche en histoire et en philosophie des mathématiques à la communauté mathématique élargie. Les auteurs sont membres de la Société canadienne d'histoire et de philosophie des mathématiques (SCHPM). Vos commentaries et suggestions sont le bienvenue; ils peuvent être adressées à l'une des co-rédacteurs:

Amy Ackerberg-Hastings, *Chercheuse indépendente (aackerbe@verizon.net)*

Hardy Grant, *York University [retraité] (hardygrant@yahoo.com)*

Bonaventura Cavalieri and the CSHPM Logo

Michael Molinsky, *University of Maine at Farmington*

For the first two years after the creation of the CSHPM in 1974, the society had no official emblem. The society's logo made its debut at the annual business meeting in 1976, where it was unveiled on new letterheads designed by the Secretary-Treasurer, Charles V. Jones (who had also been the society's first President, as well as the chair of the meeting that officially formed the CSHPM). The minutes from the 1976 meeting state that the new logo was originally suggested to Jones by Stillman Drake, the Canadian historian of science, and that the image from Bonaventura Cavalieri's *Geometrica Indivisibilibus Continuorum* (see Figure 1) was selected "because of the significance of the methodology of Cavalieri, for both the history and the philosophy of mathematics, i.e. the breaking away from the method of exhaustion and the ushering in of infinite processes."

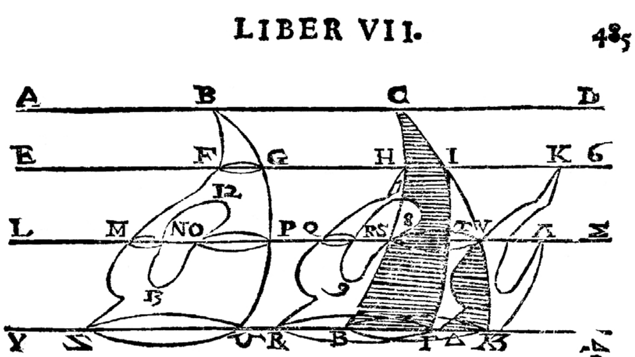


Figure 1: The CSHPM/SCHPM Logo

Although Jones cheerfully admitted in the minutes that the only response from the audience was "mystification," his selection has remained the CSHPM logo through the present day. The following paragraphs will explore the history and context from which our society's logo originated.

By the early 17th century, European mathematicians had begun exploring methods of calculating areas and volumes involving the use of both infinitesimals (for example, splitting areas up into infinitely many rectangles) and indivisibles (for example, splitting areas up into a set of parallel line segments). When compared to the ancient Greek method of exhaustion, which involved filling a given area with polygons and using a double proof-by-contradiction to show that the area could not be smaller or larger than a given quantity, methods involving infinitesimals or indivisibles could often provide simpler, more efficient arguments.

Unfortunately, those arguments could also potentially yield contradictory results. For example, consider the scalene triangle *ABC* with height *AD* shown in Figure 2. It is possible to envision that the two smaller triangles *ABD* and *ACD* are made up of vertical, "indivisible" line segments, and those two sets of line segments can easily be placed into a one-to-one correspondence: for each point on the segment *AD*, a line drawn perpendicular to *AD* intersects the triangle at the points *P* and *Q*, and therefore uniquely pairs the indivisibles *PR* and *QS* that have equal heights. So both *ABD* and *ACD* can be shown to contain exactly the same (infinite) set of indivisible line segments, but obviously it is not safe to conclude that the areas of the two triangles are identical in this case.

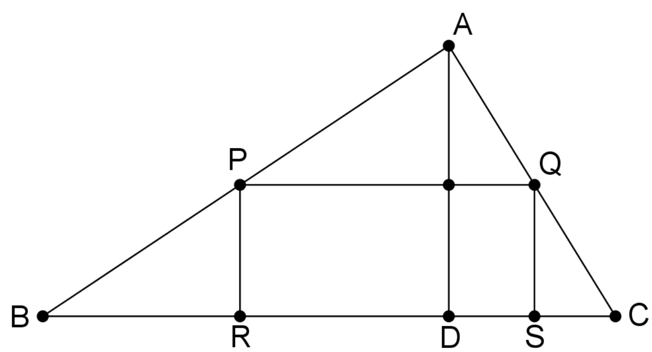


Figure 2

It is in this context that we find Bonaventura Cavalieri, the 17th-century mathematician who held the chair in mathematics at the University of Bologna from 1629 until his death in 1647. In 1635 Cavalieri published the first complete theory of indivisibles, in his work *Geometrica Indivisibilibus Continuorum nova quadam ratione promota* (*Geometry, Advanced by a New Method through Indivisibles of the Continua*). At the start of the seventh book is the following theorem for comparing areas and volumes (now called Cavalieri's principle):

> If between the same parallels any two plane figures are constructed, and if in them, any straight lines being drawn equidistant from the parallels, the included portions of any one of these lines are equal, the plane figures are also equal to one another; and, if between the same parallel planes any

> solid figures are constructed, and if in them, any planes being drawn equidistant from the parallel planes, the included plane figures out of any one of the planes so drawn are equal, the solid figures are likewise equal to one another (Evans, 448).

Note that this theorem rules out the example shown earlier in Figure 2, since the principle requires that two areas can be compared only if they have equal heights and if the equal indivisible "slices" of the area are made at right angles to this height (in the triangles *ABD* and *ACD*, while the triangles did have equal heights, the indivisibles were formed parallel to that height). Cavalieri presented justifications of special cases of this theorem using the method of exhaustion, but he first offered a general proof using superposition—which included the diagram that eventually became the CSHPM logo.

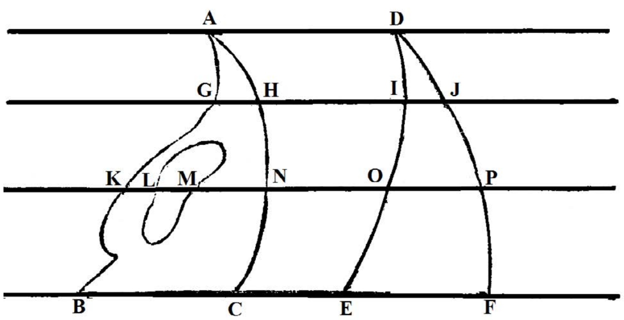


Figure 3

To briefly outline Cavalieri's argument, consider the simplified version of his diagram shown in Figure 3, containing the figures *ABC* and *DEF* (where *ABC* has an ovoid gap in the interior). The figures are included in the same parallels (that is, they have the same heights), and it is also assumed that the "included portions" are equal, so the line segments *BC* and *GH* have the same lengths as the segments *EF* and *IJ* respectively, and the sum of the lengths of *KL* and *MN* would equal the length of *OP*.

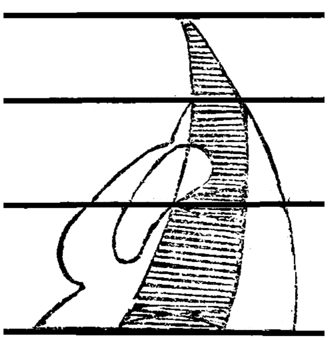


Figure 4

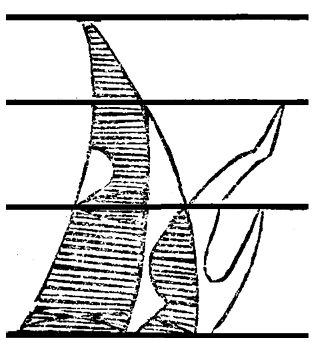


Figure 5

If you slide the figure *ABC* over *DEF* until some of the included portions overlap, you get the diagram in Figure 4. The shaded region shows the overlapping areas of the two figures, which must be equal. The process is then repeated, translating the non-overlapping portion on the left side over so that more areas are matched up (see Figure 5). Cavalieri argues that since the "included portions" (that is, the parallel indivisible segments) in both figures are equal in length, additional translations of the non-overlapping portions of the figure will eventually lead to the two areas overlapping perfectly (although such a process might require infinitely many steps).

Cavalieri's work was certainly not universally lauded in his time, and he attempted to answer some criticisms of his indivisible methods in *Exercitationes Geometricae Sex* (*Six Geometrical Exercises*), published in 1647. But despite his critics, many other contemporary mathematicians respected his accomplishments and built upon the foundation Cavalieri created. As the mathematician and scientist Evangelista Torricelli stated in his work on the quadrature of the parabola using indivisibles, "The geometry of indivisibles was, indeed, in the mathematical briar bush, the so-called royal road, and one that Cavalieri first opened and laid out for the public as a device of marvelous invention" (Carruccio, 447).

References

[1] Carruccio, E. (1991). Cavalieri, Bonaventura. In *Biographical Dictionary of Mathematicians.* (Vol. 1, pp. 446–450). New York, NY: Charles Scribner's Sons.

[2] Evans, G. W. (1917). Cavalieri's Theorem in His Own Words. *The American Mathematical Monthly*, *24(10)*, 447–451.

Michael Molinsky (michael.molinsky@maine.edu) is a Professor of Mathematics at the University of Maine at Farmington, where he encourages his students (particularly those preparing to be K-12 teachers) to explore the history of mathematics. He currently serves as both Webmaster and Archivist for the CSHPM.

CMS Notes
Volume 49 No. 1, February 2017

Melissa Bingol-Phillips, *Special Projects Coordinator – Canadian Mathematical Society (commsp@cms.math.ca)*

This year, the CMS welcomed over 400 mathematicians to Niagara Falls for the 2016 Winter Meeting from December 2nd through the 5th. Attendees participated in twenty-seven scientific sessions and eleven lectures during the four-day meeting.

The CMS Awards Banquet recognized individuals with exceptional performance in the area of mathematical research and education. The event included the CRM-Fields-PIMS Prize announcement of Henri Darmon (McGill University) and as well as the presentation of CMS research and education awards including the Adrien Pouliot Award, the Coxeter-James Prize, the Doctoral Prize and the Jeffery-Williams Prize.

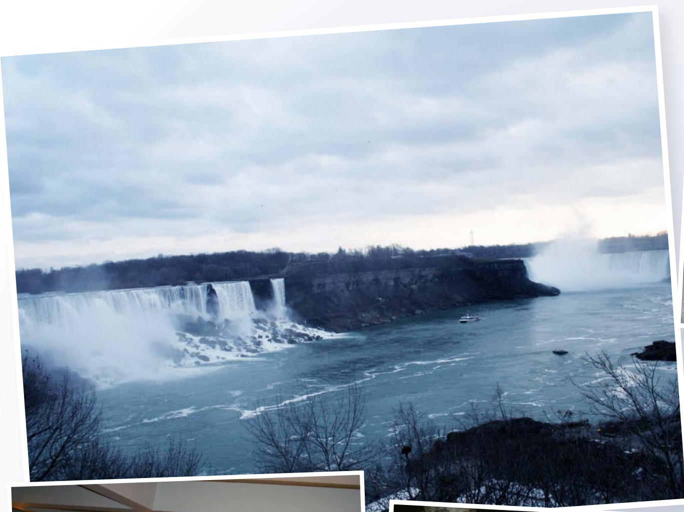
The photos below capture just some of the CMS Winter Meeting highlights including the CMS Welcome Reception, the Women in Math Committee Luncheon, and the CMS Awards Banquet.

Melissa Bingol-Phillips, *Coordonnatrice des projets spéciaux – Société mathématique du Canada (commsp@smc.math.ca)*

Cette année, la SMC a accueilli plus de 400 mathématiciens à Niagara Falls pour la Réunion d'hiver 2016, du 2 au 5 décembre. Les participants ont assisté à 27 sessions scientifiques et à 11 conférences pendant les quatre jours de la Réunion.

Lors du banquet, la SMC a récompensé plusieurs personnes pour des réalisations exceptionnelles en recherche et en enseignement des mathématiques. Le banquet a aussi donné lieu à l'annonce du lauréat du prix CRM-Fields-PIMS, Henri Darmon (Université McGill), et à la remise des prix de recherche et d'enseignement de la SMC, soit les prix Adrien-Pouliot, Coxeter-James et Jeffery-Williams ainsi que le Prix de doctorat.

Les photos ci-dessous illustrent quelques-uns des grands moments de la Réunion d'hiver de la SMC, notamment la réception de bienvenue, un lunch et panel des mathématiciennes organisée par le Comité des femmes en mathématiques de la SMC, et le banquet de la SMC.



CMS Winter Meeting by the Numbers:

- Over 420 speakers providing a public, plenary lecture or session talk
- 27 scientific sessions
- 13 poster presenters at the AARMS-CMS Student Poster Session
- 8 prizes and awards presented
- 1 public lecture

La Réunion d'hiver de la SMC en chiffres :

- Plus de 420 conférenciers présentant une conférence publique, plénière ou une communication
- 27 sessions scientifiques
- 13 présentations par affiches lors de la session pour étudiants de l'AARMA-SMC
- 8 prix remis
- 1 conférence publique

CMS Awards Banquet / Banquet de la SMC

The following research and education awards were presented at the CMS banquet to honour and recognize exceptional performance: / La SMC a remis les prix suivants lors de son banquet pour souligner des réalisations exceptionnelles en recherche ou en enseignement :

The *StudC Student Poster Award* was presented to **Sylvie Bronsard** (McMaster University) by the Graduate Student Poster Session organizer, Svenja Huntemann (Dalhousie University);

Le *prix de la meilleure présentation par affiche du Comité des étudiants* a été remis à **Sylvie Bronsard** (Université McMaster) par l'organisatrice de la session, Svenja Huntemann (Université Dalhousie).

The *CMS President's Poster Award* was presented to **Sara Maghdoori** (York University) by CMS Past-President Lia Bronsard (McMaster University);

Le *prix de la meilleure présentation par affiche du président de la SMC* a été remis à **Sara Maghdoori** (Université York) par la présidente sortante de la SMC, Lia Bronsard (Université McMaster).

The *AARMS Student Poster Award* was presented to **Emilia Alvarez** (Concordia University) by Sanjeev Seahra, Director of AARMS;

Le *prix de la meilleure présentation par affiche de l'AARMS* a été remis à **Emilia Alvarez** (Université Concordia) par Sanjeev Seahra, directeur de AARMS.

The *Adrien Pouliot Award* was presented to **Donald Violette** (Université de Moncton) by Malgorzata Dubiel (Simon Fraser University), Chair of CMS Education Committee;

Le *prix Adrien-Pouliot* a été remis à **Donald Violette** (Université de Moncton) par Malgorzata Dubiel (Université Simon Fraser), présidente du Comité d'éducation de la SMC.

The *Coxeter James Prize* was presented to **Louigi Addario-Berry** (McGill University) by Ailana Fraser (University of British Columbia), Chair of the CMS Research Committee;

Le *prix Coxeter-James* a été remis à **Louigi Addario-Berry** (Université McGill) par Ailana Fraser (Université de la Colombie-Britannique), présidente du Comité de recherche de la SMC.

The *Doctoral Prize* was presented to **Vincent X. Genest** (Massachusetts Institute of Technlogy) by Ailana Fraser (University of British Columbia), Chair of the CMS Research Committee;

Le *Prix de doctorat* a été remis à **Vincent X. Genest** (Massachusetts Institute of Technlogy) par Ailana Fraser (Université de la Colombie-Britannique), présidente du Comité de recherche de la SMC.

The *G. de B. Robinson Award* was presented to **Jim Agler** (University of California at San Diego) and **John E. McCarthy** (Washington University at St. Louis) by Karl Dilcher (Dalhousie University), Chair of the CMS Publications Committee; and

Le *prix G. de B.Robinson* a été remis à **Jim Agler** (Université de la Californie à San Diego) et à **John E. McCarthy** (Université de Washington à St. Louis) par Karl Dilcher (Université Dalhousie), président du Comité des publications de la SMC.

The *Jeffery-Williams Prize* was presented to **Daniel Wise** (McGill University) by Ailana Fraser (University of British Columbia), Chair of the CMS Research Committee.

Le *prix Jeffery-Williams* a été remis à **Daniel Wise** (Université McGill) par Ailana Fraser (Université de la Colombie-Britannique), présidente du Comité de recherche de la SMC.

CMS Notes
Volume 49 No. 1, February 2017

2017 Adrien Pouliot Award

Nominations of individuals or teams of individuals who have made significant and sustained contributions to mathematics education in Canada are solicited. Such contributions are to be interpreted in the broadest possible sense and might include: community outreach programs, the development of a new program in either an academic or industrial setting, publicizing mathematics so as to make mathematics accessible to the general public, developing mathematics displays, establishing and supporting mathematics conferences and competitions for students, etc.

Nominations must be received by the CMS Office **no later than April 30, 2017.**

Please submit your nomination electronically, preferably in PDF format, to **apaward@cms.math.ca**.

Nomination requirements

- Include contact information for both nominee and nominator.
- Describe the nominated individual's or team's sustained contributions to mathematics education. This description should provide some indication of the time period over which these activities have been undertaken and some evidence of the success of these contributions. This information must not exceed four pages.
- Two letters of support from individuals other than the nominator should be included with the nomination.
- Curricula vitae should not be submitted since the information from them relevant to contributions to mathematics education should be included in the nomination form and the other documents mentioned above.
- If nomination was made in the previous year, please indicate this.
- Members of the CMS Education Committee will not be considered for the award during their tenure on the committee.

Renewals

Individuals who made a nomination last year can renew this nomination by simply indicating their wish to do so by the deadline date. In this case, only updating materials need be provided as the original has been retained.

Prix Adrien Pouliot 2017

Nous sollicitons la candidature de personne ou de groupe de personnes ayant contribué d'une façon importante et soutenue à des activités mathématiques éducatives au Canada. Le terme « contributions » s'emploie ici au sens large; les candidats pourront être associés à une activité de sensibilisation, un nouveau programme adapté au milieu scolaire ou à l'industrie, des activités promotionnelles de vulgarisation des mathématiques, des initiatives spéciales, des conférences ou des concours à l'intention des étudiants, etc.

Les mises en candidature doivent parvenir au bureau de la SMC **avant le 30 avril 2017**.

Veuillez faire parvenir votre mise en candidature par voie électronique, de préférence en format PDF, à **prixap@smc.math.ca**.

Conditions de candidature

- Inclure les coordonnées du/des candidat(s) ainsi que du/des présentateur(s).
- Décrire en quoi la personne ou le groupe mis en candidature a contribué de façon soutenue à des activités mathématiques. Donner un aperçu de la période couverte par les activités visées et du succès obtenu. La description ne doit pas être supérieure à quatre pages.
- Le dossier de candidature comportera deux lettres d'appui signées par des personnes autres que le présentateur.
- Il est inutile d'inclure des curriculums vitae, car les renseignements qui s'y trouvent et qui se rapportent aux activités éducatives visées devraient figurer sur le formulaire de mise en candidature et dans les autres documents énumérés ci- dessus.
- Si la candidature a été soumise l'année précédente, veuillez l'indiquer.
- Les membres du Comité d'éducation de la SMC ne pourront être mis en candidature pour l'obtention d'un prix pendant la durée de leur mandat au Comité.

Renouveler une mise en candidature

Il est possible de renouveler une mise en candidature présentée l'année précédente, pourvu que l'on en manifeste le désir avant la date limite. Dans ce cas, le présentateur n'a qu'à soumettre des documents de mise à jour puisque le dossier original a été conservé.

2017 Graham Wright Award for Distinguished Service

In 1995, the Society established this award to recognize individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society. The award was renamed in 2008, in recognition of Graham Wright's 30 years of service to the Society as the Executive Director and Secretary.

Nominations should include a reasonably detailed rationale and be submitted by **March 31, 2017**.

All documentation should be submitted electronically, preferably in PDF format, by the appropriate deadline, to **gwaward@cms.math.ca**.

CMS Research Prizes

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

The **Coxeter-James Prize** Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. For more information, visit: **https://cms.math.ca/Prizes/cj-nom**

The **Jeffery-Williams Prize** Lectureship recognizes mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. For more information: **https://cms.math.ca/Prizes/jw-nom**

The **Krieger-Nelson Prize** Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. For more information: **https://cms.math.ca/Prizes/kn-nom**

The deadline for nominations, including at least three letters of reference, is **September 30, 2017**. Nomination letters should list the chosen referees and include a recent curriculum vitae for the nominee. Some arms-length referees are strongly encouraged. Nominations and the reference letters from the chosen referees should be submitted electronically, preferably in PDF format, to the corresponding email address and **no later than September 30, 2017**:

Coxeter-James: **cjprize@cms.math.ca**
Jeffery-Williams: **jwprize@cms.math.ca**
Krieger-Nelson: **knprize@cms.math.ca**

Prix de recherche de la SMC

Le Comité de recherche de la SMC lance un appel de mises en candidatures pour trois de ses prix de conférence. Ces prix ont tous pour objectif de souligner l'excellence de membres de la communauté mathématique canadienne.

Le **Prix Coxeter-James** rend hommage aux jeunes mathématiciens qui se sont distingués par l'excellence de leur contribution à la recherche mathématique. Cette personne doit etre membre de la communauté mathématique canadienne. Les candidats sont admissibles jusqu'à dix ans après l'obtention de leur doctorat. Toute mise en candidature est modifiable et demeurera active l'année suivante, à moins que la mise en candidature originale ait été faite la 10e année suivant l'obtention du doctorat. Pour les renseignements, voir : **https://cms.math.ca/Prix/cj-nom**

Le **Prix Jeffery-Williams** rend hommage aux mathématiciens ayant fait une contribution exceptionnelle à la recherche mathématique. Cette personne doit être membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant trois ans. Pour les renseignements, voir : **https://cms.math.ca/Prix/jw-nom**

Le **Prix Krieger-Nelson** rend hommage aux mathématiciennes qui se sont distinguées par l'excellence de leur contribution à la recherche mathématique. La laureate doit etre membre de la communauté mathématique canadienne. Toute mise en candidature est modifiable et demeurera active pendant deux ans. Pour les renseignements, voir : **https://cms.math.ca/Prix/info/kn**

La date limite pour déposer une candidature, qui comprendra au moins trois lettres de référence, est **le 30 septembre 2017**. Le dossier de candidature doit comprendre le nom des personnes données à titre de référence ainsi qu'un curriculum vitae récent du candidat ou de la candidate. Veuillez faire parvenir les mises en candidature et lettres de référence par voie électronique, de préférence en format PDF, avant la date limite, à l'adresse électronique correspondante et **au plus tard le 30 septembre 2017 :**

Coxeter-James : **prixcj@smc.math.ca**
Jeffery-Williams : **prixjw@smc.math.ca**
Krieger-Nelson : **prixkn@smc.math.ca**

Prix Graham-Wright pour service méritoire 2017

En 1995, la Société mathématique du Canada a créé un prix pour récompenser les personnes qui contribuent de façon importante et soutenue à la communauté mathématique canadienne et, notamment, à la SMC. Ce prix était renommé à compter de 2008 en hommage de Graham Wright pour ses 30 ans de service comme directeur administratif et secrétaire de la SMC.

Pour les mises en candidature prière de présenter des dossiers avec une argumentation convaincante et de les faire parvenir, **le 31 mars 2017** au plus tard.

Veuillez faire parvenir tous les documents par voie électronique, de préférence en format PDF, avant la date limite à **prixgw@smc.math.ca**.

Mathematical Congress of the Americas
Congrès Mathématique des Amériques

MCA 2017

July 24-28 juillet Montréal, Canada

The second Mathematical Congress of the Americas (MCA) will take place from July 24-28, 2017, at Centre Mont-Royal and McGill University in Montreal, Canada. MCA 2017 will highlight mathematical achievements of the Americas and will foster collaboration between the continents' mathematical communities.

There will be a large number of special sessions. Follow this link to view the confirmed scientific sessions: **https://mca2017.org/program/scientific-program**

Registration is now open!

mca2017.org

Le deuxième Congrès mathématiques des Amériques (CMA) aura lieu du 24 au 28 juillet 2017 au Centre Mont-Royal et l'Université McGill à Montréal, Canada. Le CMA 2017 met en lumière les accomplissements mathématiques des Amériques et encourage la collaboration entre les différentes communautés mathématiques du continent.

Un grand nombre de sessions spéciales seront au programme. Consultez la page **https://mca2017.org/program/scientific-program** pour voir les sessions confirmées.

La période d'inscription est ouverte!

mca2017.org/fr

If undelivered, please return to:
Si NON-LIVRÉ, veuillez retourner à :

CMS Notes / Notes de la SMC
209 - 1725 St. Laurent Blvd Ottawa, ON K1G 3V4 Canada