Search Problems

(Where reasoning consists of exploring alternatives)

R&N: Chap. 3, Sect. 3.1–2 + 3.6

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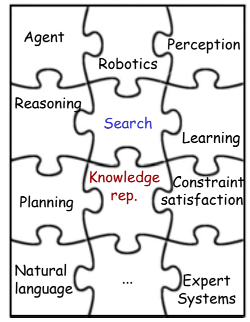

- Declarative knowledge creates alternatives:
  - Which pieces of knowledge to use?
  - How to use them?
- Search is a about exploring alternatives. It is a major approach to exploit knowledge

2

Example: 8-Puzzle

8	2	
3	4	7
5	1	6

Initial state

1	2	3
4	5	6
7	8	

Goal state

State: Any arrangement of 8 numbered tiles and an empty tile on a 3x3 board

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8-Puzzle: Successor Function

8	2	7
3	4	
5	1	6

SUCC(state) → subset of states

The successor function is knowledge about the 8-puzzle game, but it does not tell us which outcome to use, nor to which state of the board to apply it.

8	2	
3	4	7
5	1	6

8	2	7
3	4	6
5	1	

8	2	7
3		4
5	1	6

Search is about the exploration of alternatives

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Across history, puzzles and games requiring the exploration of alternatives have been considered a challenge for human intelligence:

- Chess originated in Persia and India about 4000 years ago
- Checkers appear in 3600-year-old Egyptian paintings
- Go originated in China over 3000 years ago

So, it's not surprising that AI uses games to design and test algorithms

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(n^2-1)-puzzle

8	2	
3	4	7
5	1	6

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

....

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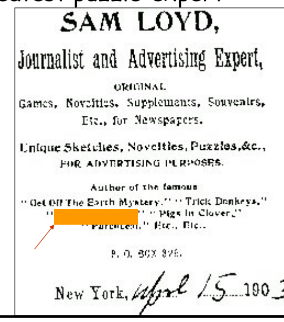
15-Puzzle

Introduced (?) in 1878 by Sam Loyd, who dubbed himself "America's greatest puzzle-expert"



15-Puzzle

Sam Loyd offered $1,000 of his own money to the first person who would solve the following problem:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

? →

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

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But no one ever won the prize !!

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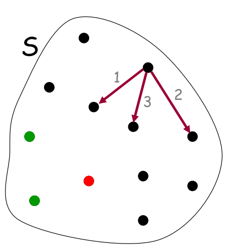
Stating a Problem as a Search Problem



- State space S
- Successor function: $x \in S \rightarrow \text{SUCCESSORS}(x) \in 2^S$
- Initial state s_0
- Goal test: $x \in S \rightarrow \text{GOAL?}(x) = \text{T or F}$
- Arc cost

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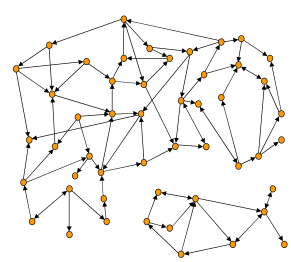
State Graph

- Each state is represented by a distinct node
- An arc (or edge) connects a node s to a node s' if s' $\in$ SUCCESSORS(s)
- The state graph may contain more than one connected component



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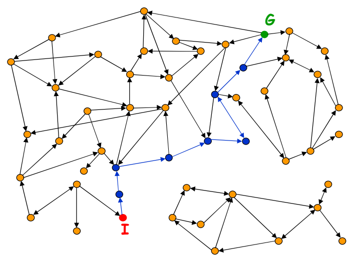
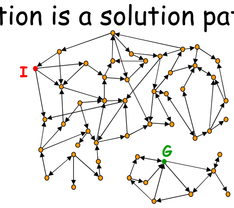
Solution to the Search Problem

- A solution is a path connecting the initial node to a goal node (any one)



Solution to the Search Problem

- A solution is a path connecting the initial node to a goal node (any one)
- The cost of a path is the sum of the arc costs along this path
- An optimal solution is a solution path of minimum cost
- There might be no solution !



How big is the state space of the (n^2-1)-puzzle?

- 8-puzzle → ?? states

How big is the state space of the (n^2-1)-puzzle?

- 8-puzzle → 9! = 362,880 states
- 15-puzzle → 16! ~ 2.09 x 10^{13} states
- 24-puzzle → 25! ~ 10^{25} states

But only half of these states are reachable from any given state
(but you may not know that in advance)

Permutation Inversions

- Wlg, let the goal be:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

- A tile j appears after a tile i if either j appears on the same row as i to the right of i, or on another row below the row of i.
- For every i = 1, 2, ..., 15, let n_i be the number of tiles j < i that appear after tile i (permutation inversions)
- N = $n_2 + n_3 + \ldots + n_{15}$ + row number of empty tile

1	2	3	4
5	10	7	8
9	6	11	12
13	14	15	

$n_2 = 0$ $n_3 = 0$ $n_4 = 0$
$n_5 = 0$ $n_6 = 0$ $n_7 = 1$
$n_8 = 1$ $n_9 = 1$ $n_{10} = 4$
$n_{11} = 0$ $n_{12} = 0$ $n_{13} = 0$
$n_{14} = 0$ $n_{15} = 0$

→ N = 7 + 4

- Proposition: (N mod 2) is invariant under any legal move of the empty tile
- Proof:
  - Any horizontal move of the empty tile leaves N unchanged
  - A vertical move of the empty tile changes N by an even increment ($\pm 1 \pm 1 \pm 1 \pm 1$)

s =

1	2	3	4
5	6		7
9	10	11	8
13	14	15	12

s' =

1	2	3	4
5	6	11	7
9	10		8
13	14	15	12

N(s') = N(s) + 3 + 1

- Proposition: (N mod 2) is invariant under any legal move of the empty tile
- → For a goal state g to be reachable from a state s, a necessary condition is that N(g) and N(s) have the same parity
- It can be shown that this is also a sufficient condition
- → The state graph consists of two connected components of equal size

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15-Puzzle

Sam Loyd offered $1,000 of his own money to the first person who would solve the following problem:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

? →

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

N = 4 N = 5

So, the second state is not reachable from the first, and Sam Loyd took no risk with his money ...

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What is the Actual State Space?

a) The set of all states?
[e.g., a set of 16! states for the 15-puzzle]

b) The set of all states reachable from a given initial state?
[e.g., a set of 16!/2 states for the 15-puzzle]

In general, the answer is a)
[because one does not know in advance which states are reachable]

But a fast test determining whether a state is reachable from another is very useful, as search techniques are often **inefficient** when a problem has no solution

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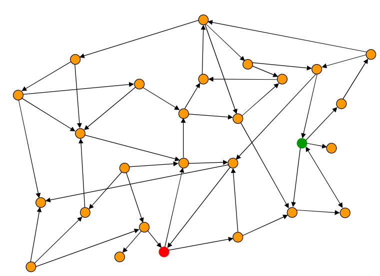
Searching the State Space



- It is often not feasible (or too expensive) to build a complete representation of the state graph

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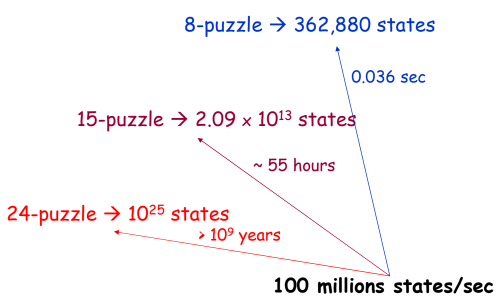
8-, 15-, 24-Puzzles



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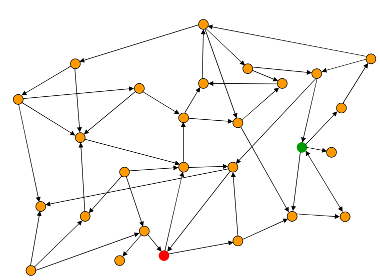
Searching the State Space



- Often it is not feasible (or too expensive) to build a complete representation of the state graph
- A problem solver must construct a solution by exploring a small portion of the graph

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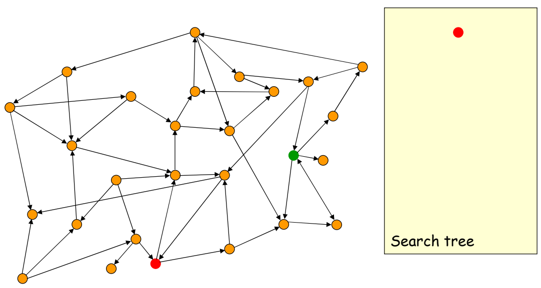
Searching the State Space



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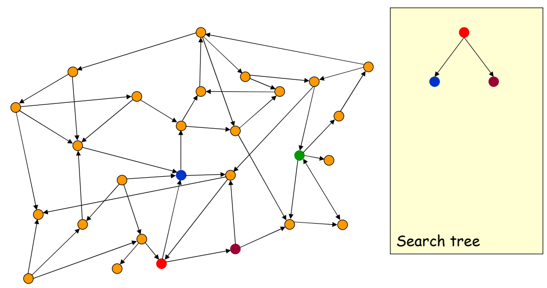
Searching the State Space



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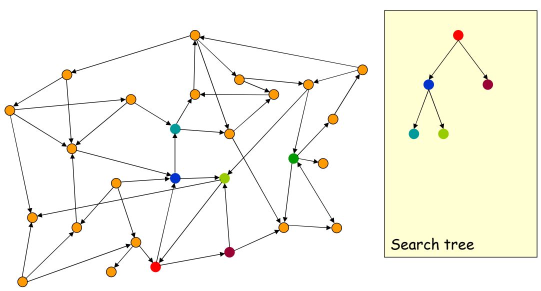
Searching the State Space



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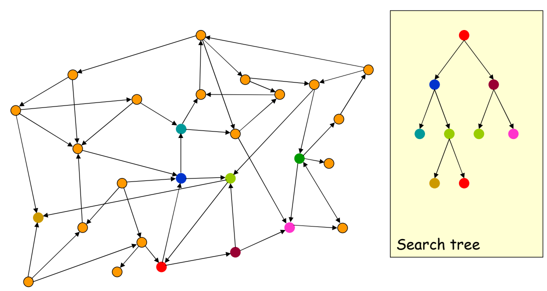
Searching the State Space



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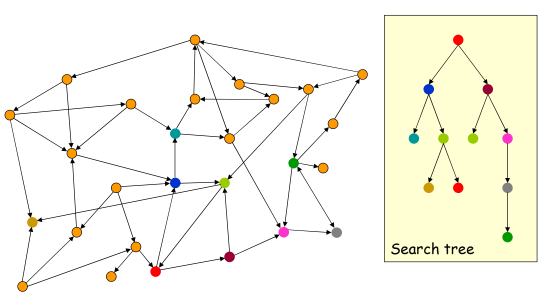
Searching the State Space



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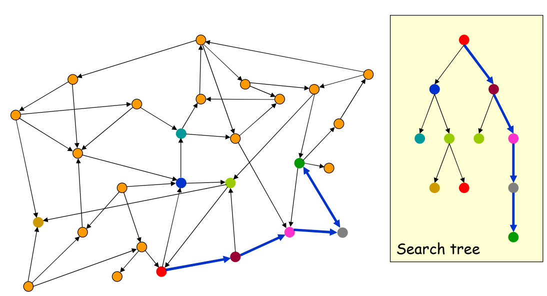
Searching the State Space



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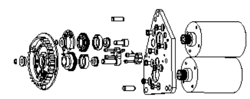
Simple Problem-Solving-Agent Algorithm

1. I ← sense/read initial state
2. GOAL? ← select/read goal test
3. Succ ← select/read successor function
4. solution ← **search**(I, GOAL?, Succ)
5. perform(solution)

State Space

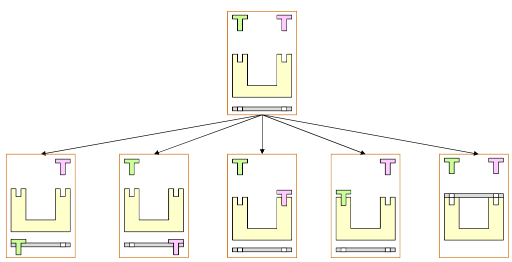
- Each state is an abstract representation of a collection of possible worlds sharing some crucial properties and differing on non-important details only
  E.g.: In assembly planning, a state does not define exactly the absolute position of each part



- The state space is discrete. It may be finite, or infinite

Successor Function

- It implicitly represents all the actions that are feasible in each state



Successor Function

- It implicitly represents all the actions that are feasible in each state
- Only the results of the actions (the successor states) and their costs are returned by the function
- The successor function is a "black box": its content is unknown
  E.g., in assembly planning, the successor function may be quite complex (collision, stability, grasping, ...)

Path Cost

- An arc cost is a positive number measuring the "cost" of performing the action corresponding to the arc, e.g.:
  - 1 in the 8-puzzle example
  - expected time to merge two sub-assemblies
- We will assume that for any given problem the cost c of an arc always verifies: $c \geq \varepsilon > 0$, where ε is a constant

Path Cost

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  - 1 in the 8-puzzle example
  - expected time to merge two sub-assemblies
- We will assume that for any given problem the cost c of an arc always verifies: $c \geq \varepsilon > 0$, where ε is a constant
  [This condition guarantees that, if path becomes arbitrarily long, its cost also becomes arbitrarily large]

Why is this needed?

Goal State

- It may be explicitly described:

1	2	3
4	5	6
7	8	

- or partially described:

1	a	a
a	5	a
a	8	a

("a" stands for "any" other than 1, 5, and 8)

- or defined by a condition,
  e.g., the sum of every row, of every column, and of every diagonal equals 30

15	1	2	12
4	10	9	7
8	6	5	11
3	13	14	

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Other examples

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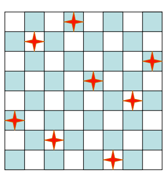
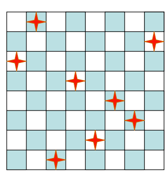
8-Queens Problem

Place 8 queens in a chessboard so that no two queens are in the same row, column, or diagonal.



A solution



Not a solution

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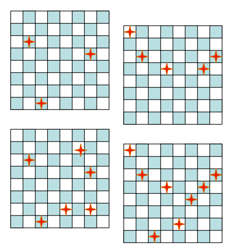
Formulation #1



- **States:** all arrangements of 0, 1, 2, ..., 8 queens on the board
- **Initial state:** 0 queens on the board
- **Successor function:** each of the successors is obtained by adding one queen in an empty square
- **Arc cost:** irrelevant
- **Goal test:** 8 queens are on the board, with no queens attacking each other

→ ~ 64x63x...x57 ~ 3×10^{14} states

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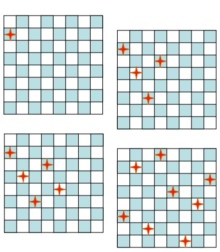
Formulation #2



- **States:** all arrangements of k = 0, 1, 2, ..., 8 queens in the k leftmost columns with no two queens attacking each other
- **Initial state:** 0 queens on the board
- **Successor function:** each successor is obtained by adding one queen in any square that is not attacked by any queen already in the board, in the leftmost empty column
- **Arc cost:** irrelevant
- **Goal test:** 8 queens are on the board

→ 2,057 states

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n-Queens Problem

- A solution is a goal node, not a path to this node (typical of design problem)
- Number of states in state space:
  - 8-queens → 2,057
  - 100-queens → 10^{52}
- But techniques exist to solve n-queens problems efficiently for large values of n
  They exploit the fact that there are many solutions well distributed in the state space

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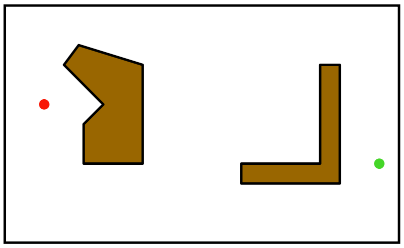
Path Planning



What is the state space?

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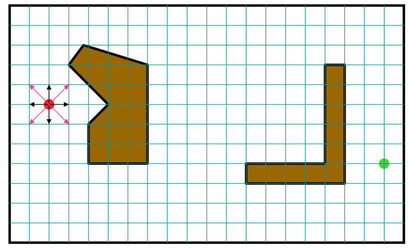
Formulation #1



Cost of one horizontal/vertical step = 1
Cost of one diagonal step = $\sqrt{2}$

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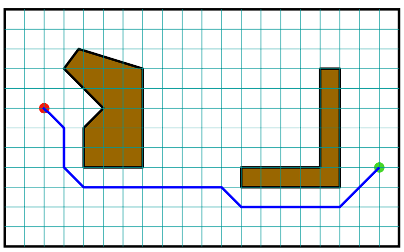
Optimal Solution



This path is the shortest in the discretized state space, but not in the original continuous space

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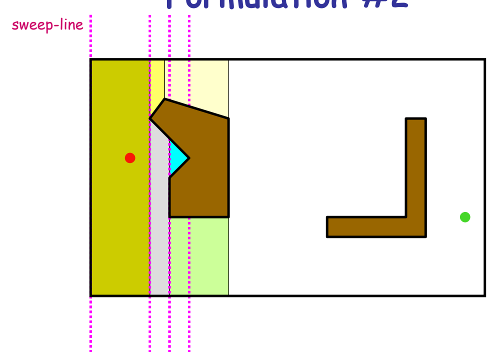
Formulation #2



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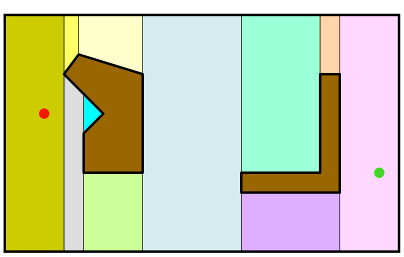
Formulation #2



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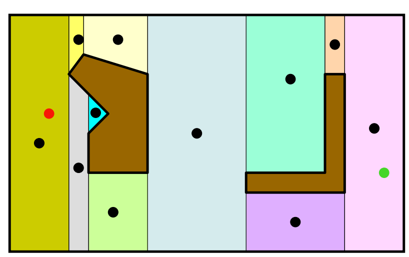
States



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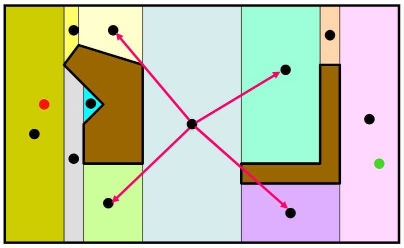
Successor Function



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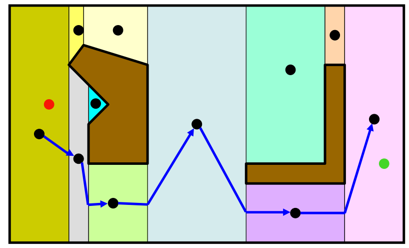
Solution Path



A path-smoothing post-processing step is usually needed to shorten the path further

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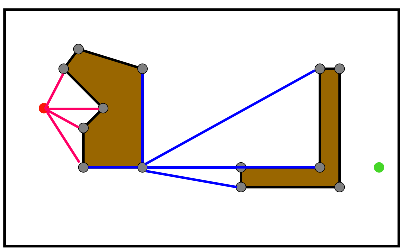
Formulation #3



Cost of one step: length of segment

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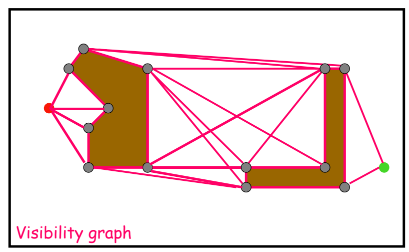
Formulation #3



Cost of one step: length of segment

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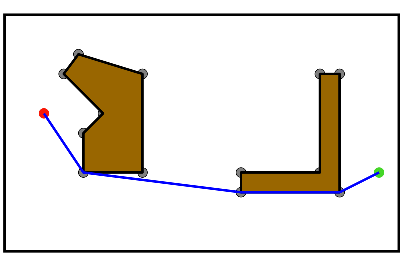
Solution Path



The shortest path in this state space is also the shortest in the original continuous space

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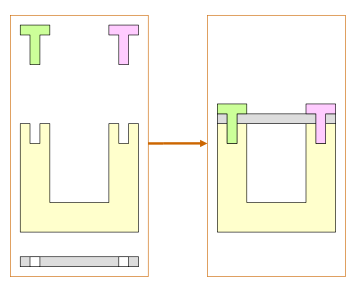
Assembly (Sequence) Planning



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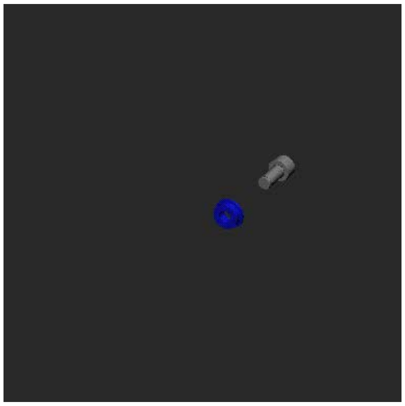

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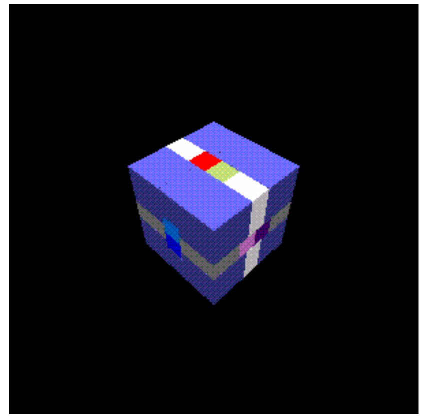

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Possible Formulation

- **States:** All decompositions of the assembly into subassemblies (subsets of parts in their relative placements in the assembly)
- **Initial state:** All subassemblies are made of a single part
- **Goal state:** Un-decomposed assembly
- **Successor function:** Each successor of a state is obtained by merging two subassemblies (the successor function must check if the merging is feasible: collision, stability, grasping, ...)
- **Arc cost:** 1 or time to carry the merging

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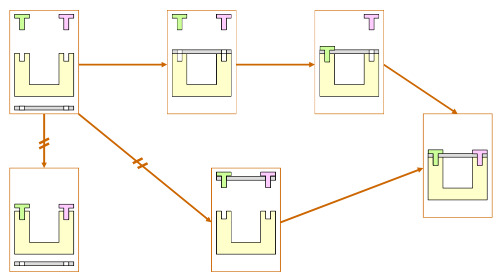
A Portion of State Space



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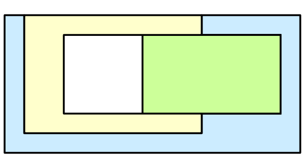
But the formulation rules out "non-monotonic" assemblies



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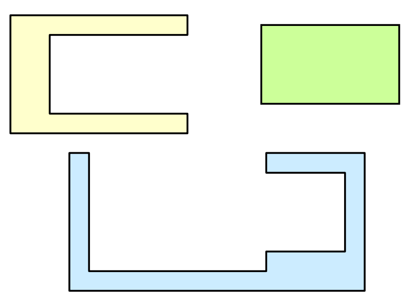

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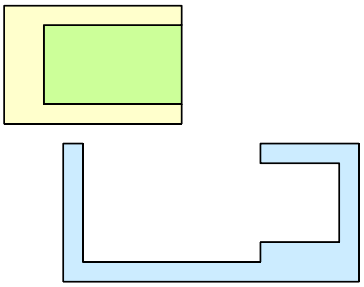

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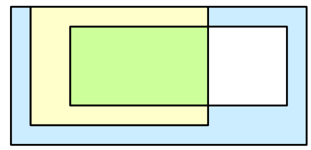

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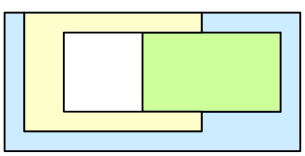

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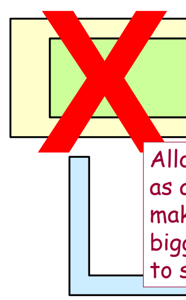

This "subassembly" is not allowed in the definition of the state space: the 2 parts are not in their relative placements in the assembly

Allowing any grouping of parts as a valid subassembly would make the state space much bigger and more difficult to search

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Assumptions in Basic Search

- The world is static
- The world is discretizable
- The world is observable
- The actions are deterministic

But many of these assumptions can be removed, and search still remains an important problem-solving tool

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Search and AI

- Search methods are ubiquitous in AI systems. They often are the backbones of both core and peripheral modules
- An autonomous robot uses search methods:
  - to decide which actions to take and which sensing operations to perform,
  - to quickly anticipate collision,
  - to plan trajectories,
  - to interpret large numerical datasets provided by sensors into compact symbolic representations,
  - to diagnose why something did not happen as expected,
  - etc...
- Many searches may occur concurrently and sequentially

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Applications

Search plays a key role in many applications, e.g.:

- Route finding: airline travel, networks
- Package/mail distribution
- Pipe routing, VLSI routing
- Comparison and classification of protein folds
- Pharmaceutical drug design
- Design of protein-like molecules
- Video games

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