# Heuristic (Informed) Search

(Where we try to choose smartly)

R&N: Chap. 4, Sect. 4.1-3



Recall that the ordering of FRINGE defines the search strategy

## Search Algorithm #2

SEARCH#2
1. INSERT(initial-node,FRINGE)
2. Repeat:
   a. If empty(FRINGE) then return failure
   b. N ← REMOVE(FRINGE)
   c. s ← STATE(N)
   d. If GOAL?(s) then return path or goal state
   e. For every state s' in SUCCESSORS(s)
      i. Create a node N' as a successor of N
      ii. INSERT(N',FRINGE)



## Best-First Search

- It exploits state description to estimate how "good" each search node is
- An evaluation function f maps each node N of the search tree to a real number $f(N) \geq 0$
  [Traditionally, f(N) is an estimated cost; so, the smaller f(N), the more promising N]
- Best-first search sorts the FRINGE in increasing f
  [Arbitrary order is assumed among nodes with equal f]



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  [Arbitrary order is assumed among nodes with equal f]

"Best" does not refer to the quality of the generated path
Best-first search does not generate optimal paths in general



## How to construct f?

- Typically, f(N) estimates:
  - either the cost of a solution path through N
    Then f(N) = g(N) + h(N), where
    - g(N) is the cost of the path from the initial node to N
    - h(N) is an estimate of the cost of a path from N to a goal node
  - or the cost of a path from N to a goal node
    Then f(N) = h(N) → Greedy best-search
- But there are no limitations on f. Any function of your choice is acceptable.
  But will it help the search algorithm?



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Heuristic function

- But there are no limitations on f. Any function of your choice is acceptable.
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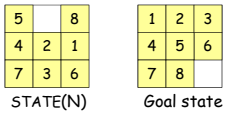
## Heuristic Function

- The heuristic function h(N) ≥ 0 estimates the cost to go from STATE(N) to a goal state

  Its value is **independent of the current search tree**; it depends only on STATE(N) and the goal test GOAL?

- Example:



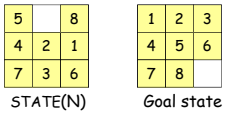

STATE(N) Goal state

$h_1(N)$ = number of misplaced numbered tiles = 6

[Why is it an estimate of the distance to the goal?]



## Other Examples



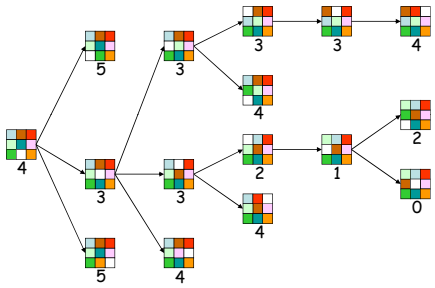

STATE(N) Goal state

- $h_1(N)$ = number of misplaced numbered tiles = 6
- $h_2(N)$ = sum of the (Manhattan) distance of every numbered tile to its goal position
  = 2 + 3 + 0 + 1 + 3 + 0 + 3 + 1 = 13
- $h_3(N)$ = sum of permutation inversions
  = $n_5 + n_8 + n_4 + n_2 + n_1 + n_7 + n_3 + n_6$
  = 4 + 6 + 3 + 1 + 0 + 2 + 0 + 0
  = 16



## 8-Puzzle

f(N) = h(N) = number of misplaced numbered tiles

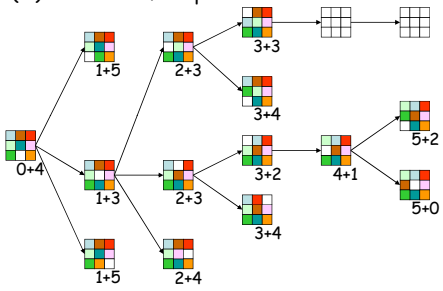


The white tile is the empty tile



## 8-Puzzle

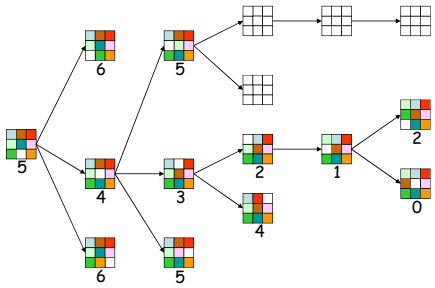
f(N) = g(N) + h(N)
with h(N) = number of misplaced numbered tiles





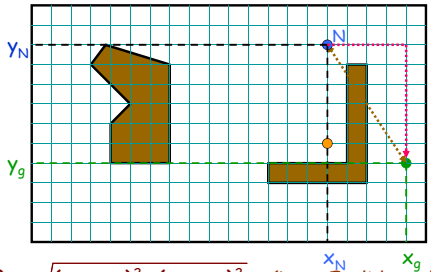
## 8-Puzzle

f(N) = h(N) = Σ distances of numbered tiles to their goals





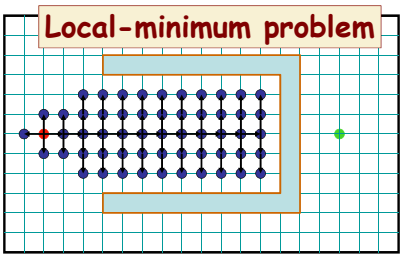
## Robot Navigation




$h_1(N) = \sqrt{(x_N - x_g)^2 + (y_N - y_g)^2}$ ($L_2$ or Euclidean distance)

$h_2(N) = |x_N - x_g| + |y_N - y_g|$ ($L_1$ or Manhattan distance)

## Best-First ↛ Efficiency




f(N) = h(N) = straight distance to the goal

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## Can we prove anything?

- If the state space is infinite, in general the search is not complete
- If the state space is finite and we do not discard nodes that revisit states, in general the search is not complete
- If the state space is finite and we discard nodes that revisit states, the search is complete, but in general is not optimal

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## Admissible Heuristic

- Let h*(N) be the cost of the optimal path from N to a goal node
- The heuristic function h(N) is admissible if:

$$0 \leq h(N) \leq h^*(N)$$

- An admissible heuristic function is always optimistic !

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G is a goal node ➔ h(G) = 0

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## 8-Puzzle Heuristics

| | | |
|---|---|---|
| 5 | | 8 |
| 4 | 2 | 1 |
| 7 | 3 | 6 |

STATE(N)

| | | |
|---|---|---|
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| 7 | 8 | |

Goal state

- $h_1(N)$ = number of misplaced tiles = 6
  is ???

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- $h_1(N)$ = number of misplaced tiles = 6
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  is not admissible

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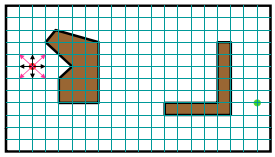
## Robot Navigation Heuristics



Cost of one horizontal/vertical step = 1
Cost of one diagonal step = $\sqrt{2}$

$h_1(N) = \sqrt{(x_N - x_g)^2 + (y_N - y_g)^2}$ is admissible

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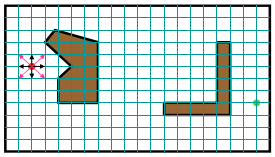
## Robot Navigation Heuristics



Cost of one horizontal/vertical step = 1
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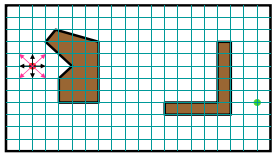
## Robot Navigation Heuristics



Cost of one horizontal/vertical step = 1
Cost of one diagonal step = $\sqrt{2}$

$h_2(N) = |x_N - x_g| + |y_N - y_g|$ is admissible if moving along diagonals is not allowed, and not admissible otherwise

$h^*(I) = 4\sqrt{2}$
$h_2(I) = 8$

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## How to create an admissible h?

- An admissible heuristic can usually be seen as the cost of an optimal solution to a relaxed problem (one obtained by removing constraints)
- In robot navigation:
  - The Manhattan distance corresponds to removing the obstacles
  - The Euclidean distance corresponds to removing both the obstacles and the constraint that the robot moves on a grid
- More on this topic later

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## A* Search
## (most popular algorithm in AI)

1) f(N) = g(N) + h(N), where:
   - g(N) = cost of best path found so far to N
   - h(N) = **admissible** heuristic function
2) for all arcs: $c(N,N') \geq \varepsilon > 0$
3) SEARCH#2 algorithm is used

➔ Best-first search is then called A* search

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## Result #1

A* is complete and optimal

[This result holds if nodes revisiting states are not discarded]

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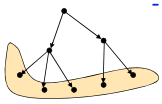
## Proof (1/2)

**1) If a solution exists, A* terminates and returns a solution**



- For each node N on the fringe, $f(N) = g(N)+h(N) \geq g(N) \geq d(N)\times\varepsilon$, where d(N) is the depth of N in the tree

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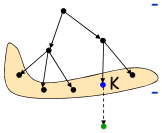


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- As long as A* hasn't terminated, a node K on the fringe lies on a solution path

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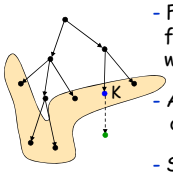


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- As long as A* hasn't terminated, a node K on the fringe lies on a solution path
- Since each node expansion increases the length of one path, K will eventually be selected for expansion, unless a solution is found along another path

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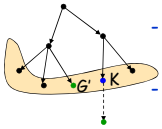
## Proof (2/2)

**2) Whenever A* chooses to expand a goal node, the path to this node is optimal**




- $C^*$= cost of the optimal solution path
- G': non-optimal goal node in the fringe
  $f(G') = g(G') + h(G') = g(G') > C^*$
- A node K in the fringe lies on an optimal path:
  $f(K) = g(K) + h(K) \leq C^*$
- So, G' will not be selected for expansion

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## Time Limit Issue

- When a problem has no solution, A* runs for ever if the state space is infinite. In other cases, it may take a huge amount of time to terminate
- So, in practice, A* is given a time limit. If it has not found a solution within this limit, it stops. Then there is no way to know if the problem has no solution, or if more time was needed to find it
- When AI systems are "small" and solving a single search problem at a time, this is not too much of a concern.
- When AI systems become larger, they solve many search problems concurrently, some with no solution. **What should be the time limit for each of them?** More on this in the lecture on Motion Planning ...

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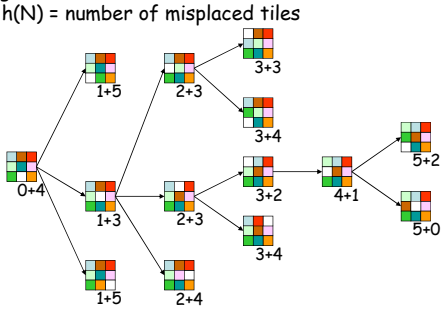
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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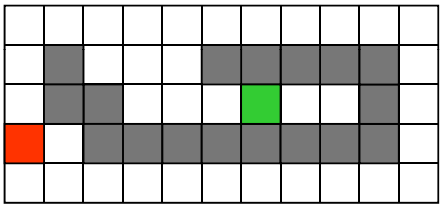
## Robot Navigation



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## Robot Navigation

f(N) = h(N), with h(N) = Manhattan distance to the goal
(not A*)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 4 | 5 | 6 |
| 7 | | 5 | 4 | 3 | | | | | | 5 |
| 6 | | | 3 | 2 | 1 | 0 | 1 | 2 | | 4 |
| 7 | 6 | | | | | | | | | 5 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 4 | 5 | 6 |

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## Robot Navigation

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| 7 | | 5 | 4 | 3 | | | | | | 5 |
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| 7 | 6 | | | | | | | | | 5 |
| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 3 | 4 | 5 | 6 |

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## Robot Navigation

f(N) = g(N)+h(N), with h(N) = Manhattan distance to goal
(A*)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 8+3 | 7+4 | 6+3 | 5+6 | 4+7 | 3+8 | 2+9 | 3+10 | 4 | 5 | 6 |
| 7+2 | | 5+6 | 4+7 | 3+8 | | | | | | 5 |
| 6+1 | | | 3 | 2+9 | 1+10 | 0+11 | 1 | 2 | | 4 |
| 7+0 | 6+1 | | | | | | | | | 5 |
| 8+1 | 7+2 | 6+3 | 5+4 | 4+5 | 3+6 | 2+7 | 3+8 | 4 | 5 | 6 |

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## Best-First Search

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## A* Search

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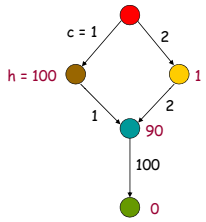
## Result #1

A* is complete and optimal

[This result holds if nodes revisiting states are not discarded]



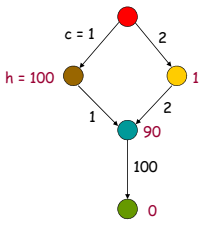
## What to do with revisited states?

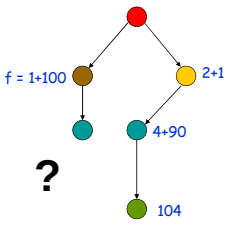



The heuristic h is clearly admissible



## What to do with revisited states?

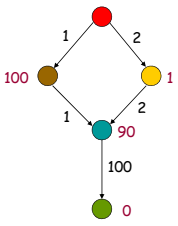


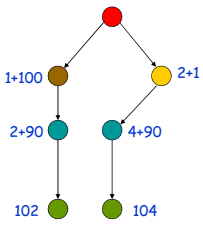




If we discard this new node, then the search algorithm expands the goal node next and returns a non-optimal solution



## What to do with revisited states?

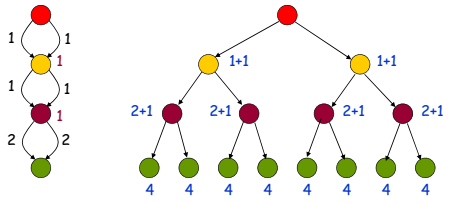



Instead, if we do not discard nodes revisiting states, the search terminates with an optimal solution

## But ...

If we do not discard nodes revisiting states, the size of the search tree can be exponential in the number of visited states



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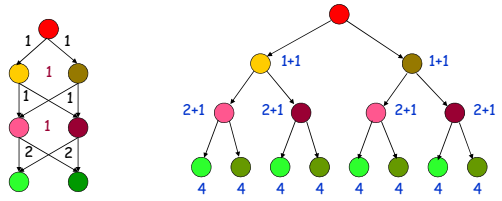



2n+1 states

O(2ⁿ) nodes



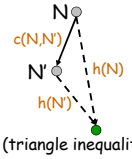
- It is not harmful to discard a node revisiting a state if the cost of the new path to this state is ≥ cost of the previous path
  [so, in particular, one can discard a node if it re-visits a state already visited by one of its ancestors]
- A* remains optimal, but states can still be re-visited multiple times
  [the size of the search tree can still be exponential in the number of visited states]
- Fortunately, for a large family of admissible heuristics – consistent heuristics – there is a much more efficient way to handle revisited states



## Consistent Heuristic

An admissible heuristic h is consistent (or monotone) if for each node N and each child N' of N:

$h(N) \leq c(N,N') + h(N')$

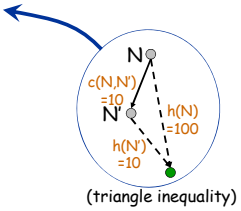



(triangle inequality)

→ Intuition: a consistent heuristics becomes more precise as we get deeper in the search tree

## Consistency Violation

If h tells that N is 100 units from the goal, then moving from N along an arc costing 10 units should **not** lead to a node N' that h estimates to be 10 units away from the goal

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## Consistent Heuristic
### (alternative definition)

A heuristic h is consistent (or monotone) if

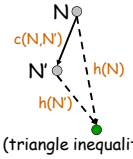
1) for each node N and each child N' of N:

$h(N) \leq c(N,N') + h(N')$

2) for each goal node G:

$h(G) = 0$

A consistent heuristic is also admissible

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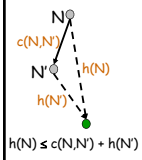
## Admissibility and Consistency

- A consistent heuristic is also admissible
- An admissible heuristic may not be consistent, but many admissible heuristics are consistent



## 8-Puzzle




| | | |
|---|---|---|
| 5 | | 8 |
| 4 | 2 | 1 |
| 7 | 3 | 6 |

STATE(N)

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | |

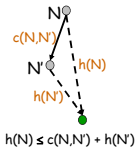
goal

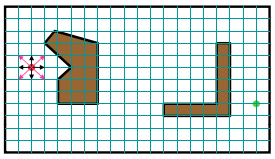
- $h_1(N)$ = number of misplaced tiles
- $h_2(N)$ = sum of the (Manhattan) distances of every tile to its goal position

are both consistent (why?)



## Robot Navigation



Cost of one horizontal/vertical step = 1
Cost of one diagonal step = $\sqrt{2}$

$h_1(N) = \sqrt{(x_N - x_g)^2 + (y_N - y_g)^2}$ is consistent

$h_2(N) = |x_N - x_g| + |y_N - y_g|$ is consistent if moving along diagonals is not allowed, and not consistent otherwise



## Result #2

If h is consistent, then whenever A* expands a node, it has already found an optimal path to this node's state



## Proof (1/2)


1) Consider a node N and its child N'
Since h is consistent: $h(N) \le c(N,N')+h(N')$

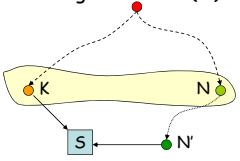
$f(N) = g(N)+h(N) \le g(N)+c(N,N')+h(N') = f(N')$

So, f is non-decreasing along any path



## Proof (2/2)

2) If a node K is selected for expansion, then any other node N in the fringe verifies $f(N) \ge f(K)$



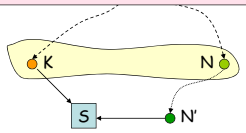

If one node N lies on another path to the state of K, the cost of this other path is no smaller than that of the path to K:

$f(N') \ge f(N) \ge f(K)$ and $h(N') = h(K)$

So, $g(N') \ge g(K)$

**Result #2**

If h is consistent, then whenever A* expands a node, it has already found an optimal path to this node's state



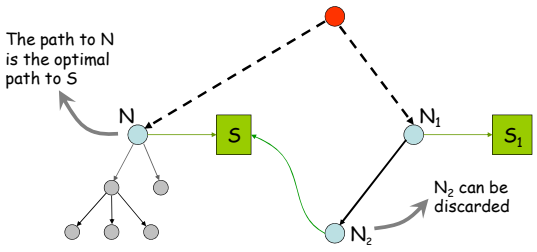

If one node N lies on another path to the state of K, the cost of this other path is no smaller than that of the path to K:

$f(N') \geq f(N) \geq f(K)$ and $h(N') = h(K)$

So, $g(N') \geq g(K)$



## Implication of Result #2






## Revisited States with Consistent Heuristic

- When a node is expanded, store its state into CLOSED
- When a new node N is generated:
  - If STATE(N) is in CLOSED, discard N
  - If there exists a node N' in the fringe such that STATE(N') = STATE(N), discard the node – N or N' – with the largest f (or, equivalently, g)



## Is A* with some consistent heuristic all that we need?

No !

There are **very dumb** consistent heuristic functions



## For example: $h \equiv 0$

- It is consistent (hence, admissible) !
- A* with $h \equiv 0$ is uniform-cost search
- Breadth-first and uniform-cost are particular cases of A*



## Heuristic Accuracy

Let $h_1$ and $h_2$ be two consistent heuristics such that for all nodes N:

$$h_1(N) \leq h_2(N)$$

$h_2$ is said to be more accurate (or more informed) than $h_1$

| | | |
|---|---|---|
| 5 | | 8 |
| 4 | 2 | 1 |
| 7 | 3 | 6 |

STATE(N)

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | |

Goal state

- $h_1(N)$ = number of misplaced tiles
- $h_2(N)$ = sum of distances of every tile to its goal position
- $h_2$ is more accurate than $h_1$

## Result #3

- Let $h_2$ be more accurate than $h_1$
- Let $A_1^*$ be A* using $h_1$ and $A_2^*$ be A* using $h_2$
- Whenever a solution exists, all the nodes expanded by $A_2^*$, except possibly for some nodes such that $f_1(N) = f_2(N) = C^*$ (cost of optimal solution) are also expanded by $A_1^*$

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## Proof

- $C^* = h^*(\text{initial-node})$ [cost of optimal solution]
- Every node N such that $f(N) < C^*$ is eventually expanded. No node N such that $f(N) > C^*$ is ever expanded
- Every node N such that $h(N) < C^* - g(N)$ is eventually expanded. So, every node N such that $h_2(N) < C^* - g(N)$ is expanded by $A_2^*$. Since $h_1(N) \le h_2(N)$, N is also expanded by $A_1^*$
- If there are several nodes N such that $f_1(N) = f_2(N) = C^*$ (such nodes include the optimal goal nodes, if there exists a solution), $A_1^*$ and $A_2^*$ may or may not expand them in the same order (until one goal node is expanded)

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## Effective Branching Factor

- It is used as a measure the effectiveness of a heuristic
- Let n be the total number of nodes expanded by A* for a particular problem and d the depth of the solution
- The effective branching factor b* is defined by $n = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$

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## Experimental Results

(see R&N for details)

- 8-puzzle with:
  - $h_1$ = number of misplaced tiles
  - $h_2$ = sum of distances of tiles to their goal positions
- Random generation of many problem instances
- Average effective branching factors (number of expanded nodes):

| d | IDS | $A_1^*$ | $A_2^*$ |
|---|---|---|---|
| 2 | 2.45 | 1.79 | 1.79 |
| 6 | 2.73 | 1.34 | 1.30 |
| 12 | 2.78 (3,644,035) | 1.42 (227) | 1.24 (73) |
| 16 | -- | 1.45 | 1.25 |
| 20 | -- | 1.47 | 1.27 |
| 24 | -- | 1.48 (39,135) | 1.26 (1,641) |

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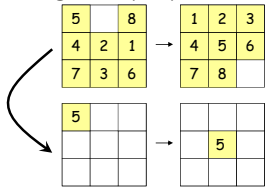
## How to create good heuristics?

- By solving relaxed problems at each node
- In the 8-puzzle, the sum of the distances of each tile to its goal position ($h_2$) corresponds to solving 8 simple problems:



$d_i$ is the length of the shortest path to move tile i to its goal position, ignoring the other tiles, e.g., $d_5 = 2$

$h_2 = \Sigma_{i=1,\ldots 8}\, d_i$

- It ignores negative interactions among tiles

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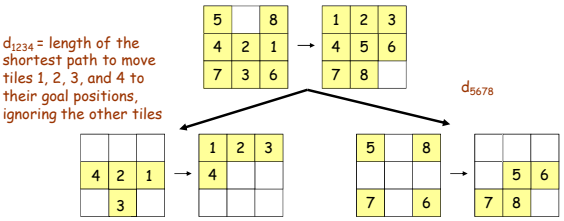
## Can we do better?

- For example, we could consider two more complex relaxed problems:




- → $h = d_{1234} + d_{5678}$ [disjoint pattern heuristic]

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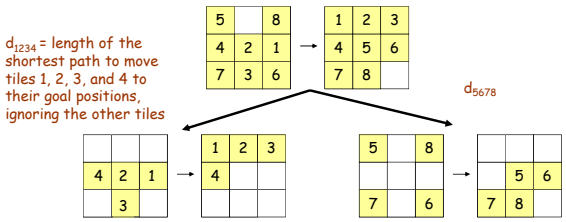
## Can we do better?

- For example, we could consider two more complex relaxed problems:




- → h = $d_{1234}$ + $d_{5678}$ [disjoint pattern heuristic]
- How to compute $d_{1234}$ and $d_{5678}$?



## Can we do better?

- For example, we could consider two more complex relaxed problems:

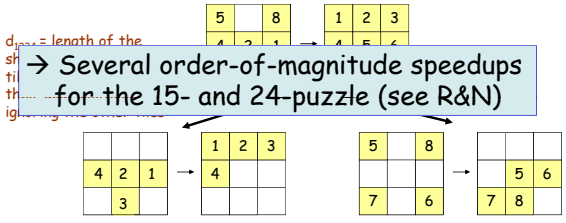



- → h = $d_{1234}$ + $d_{5678}$ [disjoint pattern heuristic]
- These distances are pre-computed and stored
  [Each requires generating a tree of 3,024 nodes/states (breadth-first search)]



## Can we do better?

- For example, we could consider two more complex relaxed problems:



→ Several order-of-magnitude speedups for the 15- and 24-puzzle (see R&N)

- → h = $d_{1234}$ + $d_{5678}$ [disjoint pattern heuristic]
- These distances are pre-computed and stored
  [Each requires generating a tree of 3,024 nodes/states (breadth-first search)]



## On Completeness and Optimality

- A* with a consistent heuristic function has nice properties: completeness, optimality, no need to revisit states
- Theoretical completeness does not mean "practical" completeness if you must wait too long to get a solution (remember the time limit issue)
- So, if one can't design an accurate consistent heuristic, it may be better to settle for a non-admissible heuristic that "works well in practice", even through completeness and optimality are no longer guaranteed



## Iterative Deepening A* (IDA*)

- Idea: Reduce memory requirement of A* by applying cutoff on values of f
- Consistent heuristic function h
- Algorithm IDA*:
  1. Initialize cutoff to f(initial-node)
  2. Repeat:
     a. Perform depth-first search by expanding all nodes N such that f(N) ≤ cutoff
     b. Reset cutoff to smallest value f of non-expanded (leaf) nodes



## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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f(N) = g(N) + h(N)
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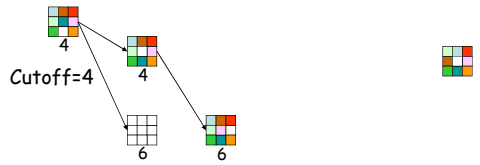


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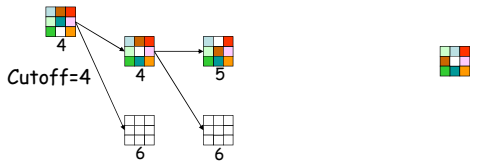
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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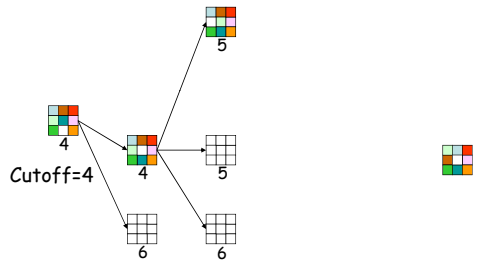
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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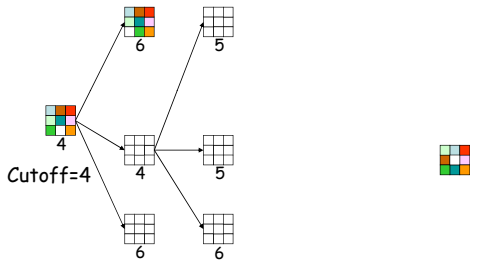
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles


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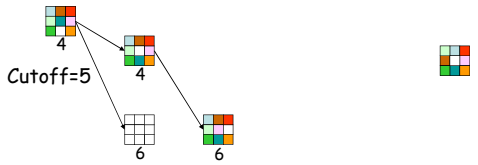
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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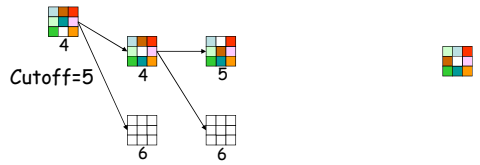
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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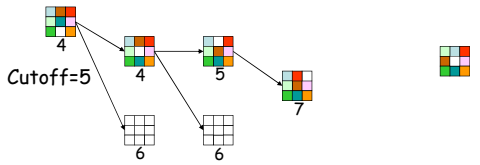
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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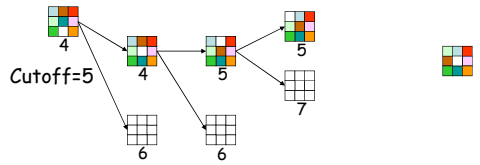
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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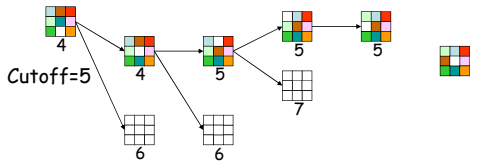
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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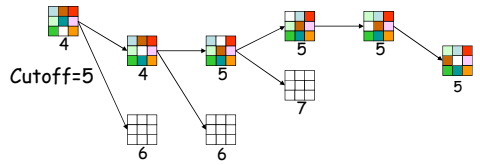
## 8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles

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## Advantages/Drawbacks of IDA*

- Advantages:
  - Still complete and optimal
  - Requires less memory than A*
  - Avoid the overhead to sort the fringe
- Drawbacks:
  - Can't avoid revisiting states not on the current path
  - Available memory is poorly used
    (→ memory-bounded search, see R&N p. 101-104)

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## Local Search

- Light-memory search method
- No search tree; only the current state is represented!
- Only applicable to problems where the path is irrelevant (e.g., 8-queen), unless the path is encoded in the state
- Many similarities with optimization techniques

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## Steepest Descent

1) S ← initial state
2) Repeat:
   a) S' ← arg $\min_{S' \in SUCCESSORS(S)}\{h(S')\}$
   b) if GOAL?(S') return S'
   c) if h(S') < h(S) then S ← S' else return failure

Similar to:
- hill climbing with –h
- gradient descent over continuous space

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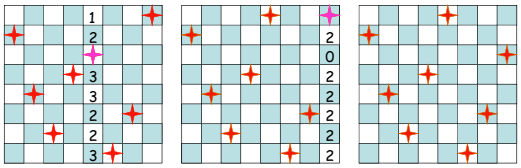
## Application: 8-Queen

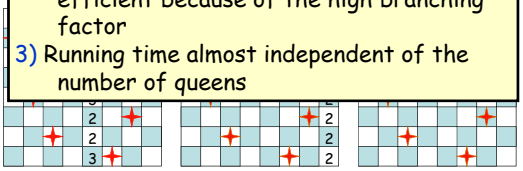
1) Pick an initial state S at random with one queen in each column
2) Repeat k times:
   a) If GOAL?(S) then return S
   b) Pick an attacked queen Q at random
   c) Move Q in its column to minimize the number of attacking queens → new S [min-conflicts heuristic]
3) Return failure




## Application: 8-Queen

**Why does it work ???**

1) There are **many** goal states that are well-distributed over the state space
2) If no solution has been found after a few steps, it's better to start it all over again. Building a search tree would be much less efficient because of the high branching factor
3) Running time almost independent of the number of queens




## Steepest Descent

1) S ← initial state
2) Repeat:
   a) S' ← arg $\min_{S' \in SUCCESSORS(S)}\{h(S')\}$
   b) if GOAL?(S') return S'
   c) if h(S') < h(S) then S ← S' else return failure

may easily get stuck in local minima
→ Random restart (as in n-queen example)
→ Monte Carlo descent

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## Monte Carlo Descent

1) S ← initial state
2) Repeat k times:
   a) If GOAL?(S) then return S
   b) S' ← successor of S picked at random
   c) if h(S') ≤ h(S) then S ← S'
   d) else
      - Δh = h(S')-h(S)
      - with probability ~ exp(–Δh/T), where T is called the "temperature", do: S ← S' [Metropolis criterion]
3) Return failure

Simulated annealing lowers T over the k iterations.
It starts with a large T and slowly decreases T

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## "Parallel" Local Search Techniques

They perform several local searches concurrently, but not independently:

- Beam search
- Genetic algorithms

See R&N, pages 115-119

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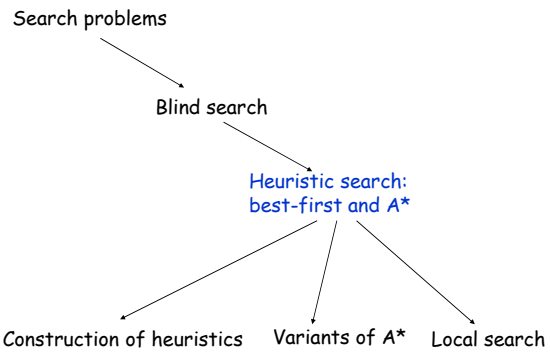


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## When to Use Search Techniques?

1) The search space is small, and
   - No other technique is available, or
   - Developing a more efficient technique is not worth the effort
2) The search space is large, and
   - No other available technique is available, and
   - There exist "good" heuristics

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