REINFORCEMENT LEARNING FOR LEGGED ROBOTS

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November 28, 2025

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QUICK HISTORY OF ROBOT LEARNING

2024: BIPEDAL LOCOMOTION



Policy trained with current RL techniques [Lia+24]

Video: https://youtu.be/8pR1HE-wMHw

2020: QUADRUPEDAL LOCOMOTION



Teacher-student residual reinforcement learning [Lee+20]

Video: https://youtu.be/oPNkeoGMvAE

2018: IN-HAND REORIENTATION



LSTM policy with domain randomization [And+20]

Video: https://youtu.be/jwSbzNHGflM

2010: HELICOPTER STUNTS



Helicopter aerobatics through apprenticeship learning [ACN10]

Video: https://youtu.be/M-QUkgk3HyE

1997: PENDULUM SWING UP

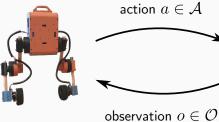


Swinging up an inverted pendulum from human demonstrations [AS97]

Video: https://youtu.be/g3I2VjeSQUM?t=294

INTRO TO REINFORCEMENT LEARNING

Agent



reward $r \in \mathbb{R}$

Environment



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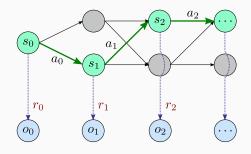






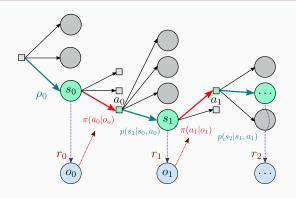
Image credit: L. M. Tenkes, source: https://araffin.github.io/post/sb3/

PARTIALLY OBSERVABLE MARKOV DECISION PROCESS (1/2)



- State: s_t , ground truth of the environment
- Action: a_t , decision of the agent (discrete or continuous)
- Transitions: only depend on current state and action (Markov property)
- Observation: o_t , partial estimation of the state from sensors
- Reward: $r_t \in \mathbb{R}$, scalar feedback, often $r_t = r(s_t, a_t)$ or $r(s_t, a_t, s_{t+1})$

PARTIALLY OBSERVABLE MARKOV DECISION PROCESS (2/2)



	Deterministic	Stochastic	
Transitions:	$s_{t+1} = f(s_t, a_t)$	$s_{t+1} \sim p(\cdot s_t, a_t)$	how the environment evolves
Initial state:	s_0	$s_0 \sim \rho_0(\cdot)$	where we start from
Observation:	$o_t = h(s_t)$	$o_t \sim z(\cdot s_t)$	how sensors measure the world
Policy:	$a_t = g(s_t)$	$a_t \sim \pi(\cdot o_t)$	what the agent decides

EXAMPLE: THE GYMNASIUM API



```
import gymnasium as gym

with gym.make("CartPole-v1", render_mode="human") as env:
    observation, _ = env.reset()
    action = env.action_space.sample()
    for step in range(1_000_000):
        observation, reward, terminated, truncated, _ = env.step(action)
        if terminated or truncated:
            observation, _ = env.reset()
        position = observation[0]
        action = 0 if position > 0.0 else 1
```

SAME API FOR SIMULATION AND REAL ROBOTS



```
import gymnasium as gym

with gym.make("Upkie-PyBullet-Pendulum", frequency=200.0) as env:
    observation, _ = env.reset()
    action = env.action_space.sample()
    for step in range(1_000_000):
        observation, reward, terminated, truncated, _ = env.step(action)
        if terminated or truncated:
            observation, _ = env.reset()
        pitch = observation[0]
        action[0] = 10.0 * pitch # action is [ground_velocity]
```

GOAL OF REINFORCEMENT LEARNING

Two last missing pieces:

- Episode: $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots)$ truncated or infinite¹
- Return: $R(\tau) = \sum_{t \in \tau} r_t$ or with discount $\gamma \in]0,1[:R(\tau) = \sum_{t \in \tau} \gamma^t r_t$

We can now state what reinforcement learning is about:

Goal of reinforcement learning

The goal of reinforcement learning is to find a policy that maximizes returns.

 $^{^{1}}$ In practice episodes contain o_t rather than s_t . In RL, we implicitly assume that observations contain enough information to be in bijection with their corresponding states. See also Augmenting observations thereafter.

STOCHASTIC REINFORCEMENT LEARNING

In the stochastic setting, the goal of reinforcement learning is:

```
\begin{aligned} \max_{\pi} & \mathbb{E}_{\tau}[R(\tau)] \\ \text{s.t. } & R(\tau) = \sum_{t} r_{t} \\ & \tau = (s_{0}, a_{0}, r_{0}, s_{1}, a_{1}, r_{1}, \ldots) \\ & s_{0} \sim \rho_{0}(\cdot) \\ & o_{0} \sim z(\cdot|s_{0}) \\ & a_{0} \sim \pi(\cdot|o_{0}) \\ & s_{1} \sim p(\cdot|s_{0}, a_{0}) \\ & \vdots \end{aligned}
```

The value $V(s) \in \mathbb{R}$ of a state s is the expected return achieved when starting from s.

On-policy value function

Expected return from s following a given policy:

$$V^{\pi}(s) = \mathbb{E}_{\tau \sim \pi}(R(\tau)|s_0 = s)$$

Optimal value function

Best return we can expect from s:

$$V^*(s) = \max_{\pi} \mathbb{E}_{\tau \sim \pi}(R(\tau)|s_0 = s)$$

15	14	13	14	15	22
16	A	12	17	16	21
—		-	18	19	20
11	V	10	9	8	7
10	14	15	18	5	6
9	8	16	17	4	3
9	0	. 0	1 /		
10	7	17	3	2	2

Figure 1: Labyrinth with discrete NSEW actions. Orange: distance to the exit.

Value functions satisfy the Bellman equation:

Bellman equation

For the on-policy value function, we have recursively:

$$V^{\pi}(s) = \mathbb{E}_{a \sim \pi(\cdot|s), (r,s') \sim p(s'|s,a)}[r + \gamma V^{\pi}(s')]$$

And similarly for the optimal value function:

$$V^{*}(s) = \max_{\pi} \mathbb{E}_{a \sim \pi(\cdot|s), (r,s') \sim p(s'|s,a)} [r + \gamma V^{*}(s')]$$

An optimal policy π^* can thus be derived from an optimal value function V^* .

This gives rise to algorithms like Q-learning that search for optimal value functions.

COMPONENTS OF AN RL ALGORITHM

A reinforcement-learning algorithm may include any of the following:

- · Policy: compute the agent's action from an observation
- · Value function: estimate the policy/best return from a state
- · Model: an internal function approximating the (unknown) environment dynamics

An algorithm with a policy (actor) and a value function (critic) is called actor-critic.

An algorithm with an explicit model is called *model-based* (without: *model-free*).

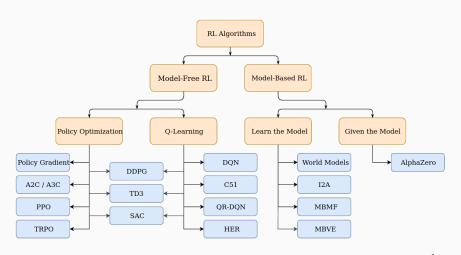


Figure 2: There are several taxonomies, none of them fully works. This one is from [Ach18].

A TAXONOMY OF RL ALGORITHMS

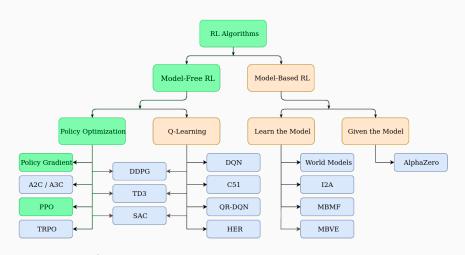


Figure 3: We will focus on this branch in what follows.

POLICY OPTIMIZATION

PARAMETERIZED POLICY

We parameterize our policy π_{θ} by a vector $\theta \in \mathbb{R}^n$.

For continuous actions, it is common to use a diagonal Gaussian policy:

$$a \sim \pi_{\theta}(\cdot|s) \iff a = \mu_{\theta}(s) + \operatorname{diag}(\sigma_{\theta}(s))z, \ z \sim \mathcal{N}(0, I_m)$$

where $\mu_{ heta}$ and $\sigma_{ heta}$ are neural networks mapping states to means and standard deviations.²

²In practice, σ often does not depend on s and we store $\log \sigma \in \mathbb{R}^m$ rather than $\sigma \in \mathbb{R}^m_+$ in θ .

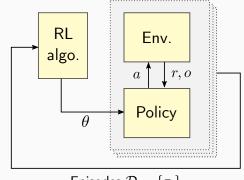
POLICY-BASED ALGORITHMS

At each iteration *k*:

- Collect a batch of episodes $\mathcal{D}_k = \{\tau\}$
- Update policy parameters

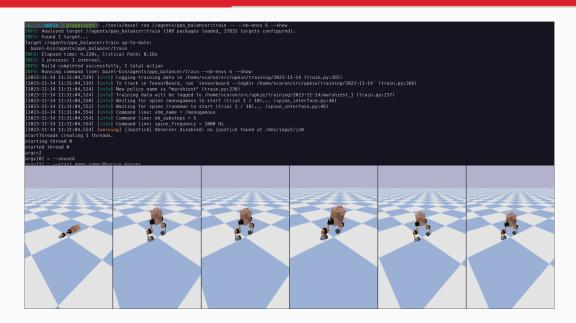
$$\theta_{k+1} = \text{update}(\theta_k, \mathcal{D}_k)$$

to get a new policy $\pi_{\theta_{k+1}}$



Episodes $\mathcal{D} = \{\tau_i\}$

ROLLING OUT EPISODES WITH A SIMULATOR



REINFORCEMENT LEARNING FOR LEGGED ROBOTS

POLICY OPTIMIZATION

The goal of RL is to find a policy that maximizes the expected return. In terms of θ :

$$J(\theta) = \mathbb{E}_{\tau \sim \pi_{\theta}}[R(\tau)]$$

In policy optimization, we seek an optimum by gradient ascent:

$$\theta_{k+1} = \theta_k + \alpha \nabla_{\theta} J(\theta_k)$$

The gradient $\nabla_{\theta}J$ with respect to policy parameters θ is called the *policy gradient*.

POLICY GRADIENT THEOREM

Policy gradient theorem

The policy gradient can be computed from returns and the log-policy gradient $\nabla_{\theta} \log \pi_{\theta}$ as:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\tau \sim \pi_{\theta}} \left(R(\tau) \sum_{s_t, a_t \in \tau} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right)$$

LHS: the graal. RHS: things we observe $(R(\tau))$ or know by design $(\nabla_{\theta} \log \pi_{\theta})$.

POLICY GRADIENT THEOREM: PROOF SKETCH

$$\begin{split} \nabla_{\theta} J(\theta) &= \nabla_{\theta} \mathbb{E}_{\tau \sim \pi_{\theta}}(R(\tau)) & \text{definition} \\ &= \nabla_{\theta} \int_{\tau} R(\tau) \mathbb{P}(\tau|\theta) \mathrm{d}\tau & \text{expectation as integral} \\ &= \int_{\tau} R(\tau) \nabla_{\theta} \mathbb{P}(\tau|\theta) \mathrm{d}\tau & \text{Leibniz integral rule} \\ &= \int_{\tau} R(\tau) \mathbb{P}(\tau|\theta) \nabla_{\theta} \log \mathbb{P}(\tau|\theta) \mathrm{d}\tau & \text{log-derivative trick} \\ &= \int_{\tau} R(\tau) \sum_{s_{t}, a_{t} \in \tau} \nabla_{\theta} \log \pi_{\theta}(a_{t}|s_{t}) \mathbb{P}(\tau|\theta) \mathrm{d}\tau & \text{expand } \mathbb{P}(\tau|\theta) \text{ as product} \\ &= \mathbb{E}_{\tau \sim \pi_{\theta}} \left(R(\tau) \sum_{s_{t}, a_{t} \in \tau} \nabla_{\theta} \log \pi_{\theta}(a_{t}|s_{t}) \right) & \text{integral as expectation} \end{split}$$

REINFORCEMENT LEARNING FOR LEGGED ROBOTS

With a diagonal Gaussian policy $\mu_{\theta}(s)$, σ_{θ} :

$$\pi_{\theta}(a|s) = \prod_{i=1}^{\dim(a)} \frac{1}{\sqrt{2\pi\sigma_{\theta,i}^2}} \exp\left(\frac{-(a_i - \mu_{\theta,i}(s))^2}{\sigma_{\theta,i}^2}\right)$$

$$\log \pi_{\theta}(a|s) = -\frac{1}{2} \sum_{i=1}^{\dim(a)} \left[\frac{(a_i - \mu_{\theta,i}(s))^2}{\sigma_{\theta,i}^2} + 2\log\sigma_{\theta,i} + \log 2\pi\right]$$

$$\nabla_{\theta} \log \pi_{\theta}(a|s) = \sum_{i=1}^{\dim(a)} \left[\frac{a_i - \mu_{\theta,i}(s)}{\sigma_{\theta,i}^2} \nabla_{\theta} \mu_{\theta,i}(s) + \left(\frac{(a_i - \mu_{\theta,i}(s))^2}{\sigma_{\theta,i}^3} - \frac{1}{\sigma_{\theta,i}}\right) \nabla_{\theta} \sigma_{\theta,i}\right]$$

where $s \mapsto \mu_{\theta}(s)$ is typically a neural network, from which we get $\nabla_{\theta} \mu_{\theta}(s)$.

REINFORCE (1/2)

REINFORCE algorithm [SB18]

REINFORCE (2/2)

Gradient ascent:

$$\theta_{k+1} = \theta_k + \alpha \nabla_{\theta} J(\theta_k)$$

From the policy gradient theorem, this is equivalent to:

$$\theta_{k+1} = \theta_k + \alpha \mathbb{E}_{\tau \sim \pi_{\theta}} \left(R(\tau) \sum_{s_t, a_t \in \tau} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right)$$

REINFORCE drops the expectation:

$$\theta_{k+1} = \theta_k + \alpha R(\tau_k) \sum_{s_t, a_t \in \tau_k} \nabla_{\theta} \log \pi_{\theta}(a_t|s_t)$$

Vanilla policy gradient [Ach18]

Data: initial policy parameters θ_0 , initial value function parameters ϕ_0 , learning rate α for $k=0,1,2,\ldots$ do

Collect episodes $\mathcal{D}_k = \{\tau_i\}$ by running $\pi_{\theta} = \pi(\theta_k)$;

Compute returns \hat{R}_t and advantage estimates \hat{A}_t based on V_{ϕ_k} ; Estimate the policy gradient as

$$\hat{g}_k = \frac{1}{|\mathcal{D}_k|} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^T \nabla_\theta \log \pi_\theta(a_t|s_t)|_{\theta_k} \hat{A}_t$$

Update policy parameters by e.g. gradient ascent, $\theta_{k+1}=\theta_k+\alpha\hat{g}_k$; Fit value function by regression on mean-square error:

$$\phi_{k+1} = \arg\min_{\phi} \frac{1}{T|\mathcal{D}_k|} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(\hat{R}_t - V_{\phi}(s_t)\right)^2$$

end

Proximal policy optimization [Sch+17]

Data: initial policy parameters θ_0 , initial value function parameters ϕ_0

for
$$k = 0, 1, 2, ...$$
 do

Collect episodes $\mathcal{D}_k = \{\tau_i\}$ by running $\pi_{\theta} = \pi(\theta_k)$;

Compute returns \hat{R}_t and advantage estimates \hat{A}_t based on V_{ϕ_k} ;

Clipping: Update policy parameters by maximizing the clipping objective:

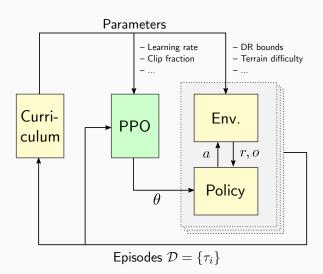
$$\theta_{k+1} = \arg\max_{\theta} \frac{1}{|\mathcal{D}_k|T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \min\left(\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta_k}(a_t|s_t)} A^{\pi_{\theta_k}}(s_t, a_t), \operatorname{clip}(\epsilon, A^{\pi_{\theta_k}}(s_t, a_t))\right)$$

where $\operatorname{clip}(\epsilon, A) = (1 + \epsilon)A$ if $A \ge 0$ else $(1 - \epsilon)A$

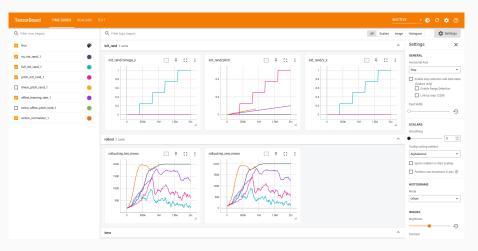
Fit value function by regression on mean-square error:

$$\phi_{k+1} = \arg\min_{\phi} \frac{1}{T|\mathcal{D}_k|} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(\hat{R}_t - V_{\phi}(s_t)\right)^2$$

end

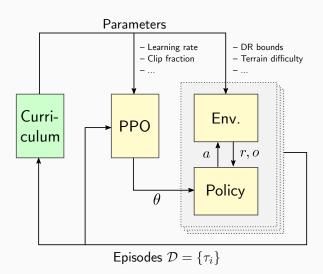


MONITORING TRAINING



Monitor the average return ep_rew_mean and length ep_rew_len of episodes. If training goes well, both eventually plateau at their maximum values.

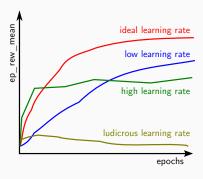
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OPTIMIZER PARAMETERS: STEPS, EPOCHS, MINI-BATCHING

The optimizer behind PPO, usually Adam [KB14], comes with parameters:

- learning_rate: step size parameter, typically decreasing with a linear schedule.
- n_epochs : number of uses of the rollout buffer while optimizing the surrogate loss.
- batch_size: mini-batch size, same as in stochastic gradient descent.



APPLICATION TO ROBOTICS

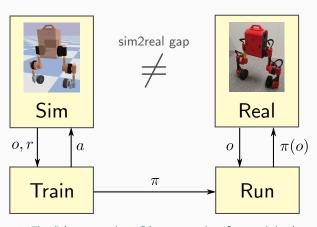


Figure 4: The "sim-to-real gap" is a metaphor for model mismatch.

CROSSING THE GAP

General reinforcement learning techniques:

- · Normalize observations and actions
- Augment observations with history
- Curriculum learning
- Reward shaping

For the sim-to-real gap in robotics:

- · Domain randomization
- · Data-based simulation
- · Teacher-student distillation

OBSERVATION-ACTION NORMALIZATION

Unnormalized actions don't work well on actors with Gaussian policies:

- \cdot Bounds too large \Rightarrow sampled actions cluster around zero.
- Bounds too small \Rightarrow sampled actions saturate all the time, bang-bang behavior.

Good practice: bound observations/states, rescale actions to [-1, 1].

AUGMENTING OBSERVATIONS WITH HISTORY

We assumed a Markovian system, but real systems have lag:

Definition

The *lag of a system* is the number of observations required to estimate its state.

Counter-measure: augment observations with history to restore the Markov property.

DOMAIN RANDOMIZATION

Randomize selected environment parameters:

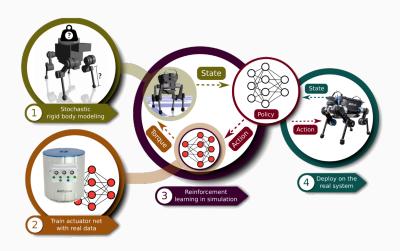
- · Robot geometry: limb lengths, wheel diameters, ...
- · Inertias: masses, mass distributions
- Initial state: $s_0 \sim \rho_0(\cdot)$
- · Actuation models: delays, bandwidth, ...
- Perturbations: send $(1\pm\epsilon)\tau$ torques...

There is a tradeoff to this:

- Pro: closer/may include real-robot distribution.
- · Con: makes policies more conservative.

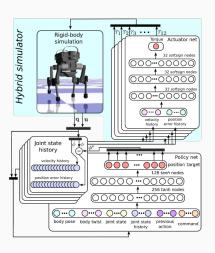


DATA-BASED ACTUATION MODELS



³Jemin Hwangbo et al. "Learning agile and dynamic motor skills for legged robots". In: *Science Robotics* 4.26 (2019).

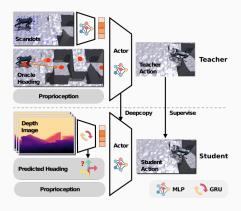
POLICY WITH HISTORY AND HYBRID SIMULATION



⁴Jemin Hwangbo et al. "Learning agile and dynamic motor skills for legged robots". In: *Science Robotics* 4.26 (2019).

TEACHER-STUDENT DISTILLATION

- Train a **teacher policy** in simulation with privileged information
- Train a **student policy** in simulation with observations and teacher action

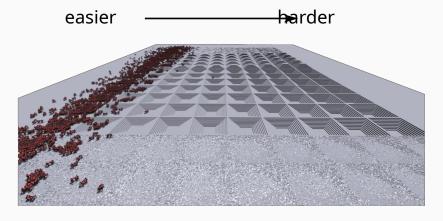


⁵Xuxin Cheng et al. **"Extreme parkour with legged robots".** In: 2024 IEEE International Conference on Robotics and Automation (ICRA). IEEE. 2024, pp. 11443–11450.

CURRICULUM LEARNING

Randomization and task difficulty vary based on policy performance.

Example: terrain curriculum for quadrupedal locomotion [Lee+20]:



REWARD SHAPING

Let r_e denote the reward associated with an error function e:

Motivation:

• Exponential: $r_e = \exp(-e^2)$

Penalization:

- Absolute value $r_e = -|e|$
- Squared value: $r_e = -e^2$

Making an RL pipeline work can lead to complex rewards, e.g. in [Lee+20]:

- \cdot Linear velocity tracking: $r_{lv}=\exp(-2.0(v_{pr}-0.6)^2)$, or 1, or 0
- · Angular velocity tracking: $r_{av} = \exp(-1.5(\omega_{pr} 0.6)^2)$, or 1
- Base motion tracking: $r_b = \exp(-1.5v_o^2) + \exp(-1.5\|(B_{IB}\omega)_{xy}\|^2)$
- Foot clearance: $r_{fc} = \sum_{i \in I_{swing}} 1_{fclear}(i) / |I_{swing}|$
- Body-terrain collisions: $r_{bc} = -|I_{c,body} \backslash I_{c,foot}|$
- Foot acceleration smoothness: $r_s = -\|(r_{f,d})_t 2(r_{f,d})_{t-1} + (r_{f,d})_{t-2}\|$
- Torque penalty: $r_{ au} = -\sum_{i} | au_{i}|$

Final reward: $r = 0.05r_{lv} + 0.05r_{av} + 0.04r_b + 0.01r_{fc} + 0.02r_{bc} + 0.025r_s + 2 \cdot 10^{-5}r_{\tau}$

KEEP IN MIND THAT WE ARE IN A STOCHASTIC WORLD

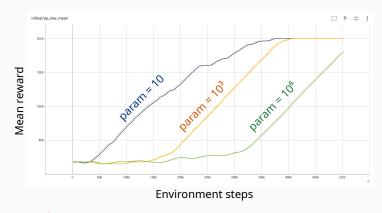


Figure 5: We may be observing the effect of our parameter.

KEEP IN MIND THAT WE ARE IN A STOCHASTIC WORLD

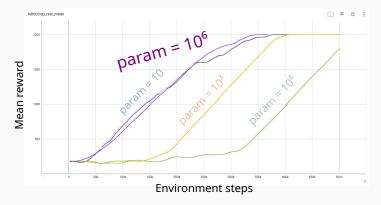


Figure 5: Or we may be observing the variance of the training process.

WHAT DID WE SEE?

Introduction to policy optimization:

- Partially-observable Markov decision process (POMDP)
- The goal of reinforcement learning
- Model, policy and value function
- · Policy optimization: REINFORCE, policy gradient, PPO

Application to robotics:

- · Sim-to-real gap: domain randomization, hybrid simulation
- Techniques: curriculum, distillation, history, "RewArt"

RL is not magic: great results, possibly going to great lengths!

THANK YOU FOR YOUR ATTENTION!6

These slides have received feedback from E. Chane-Sane, T. Flayols, N. Perrin-Gilbert, R. P. Singh, P. Souères, and the 2023-2025 classes at ENS-PSL, Mines Paris-PSL and Master MVA. Thank you all!

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Qiayuan Liao et al. "Berkeley humanoid: A research platform for learning-based control". In:



BONUS SLIDES ON PPO

INTUITION BEHIND CLIPPING IN PPO

When the advantage is positive:

$$L(s, a, \theta_k, \theta) = \min\left(\frac{\pi_{\theta}(a|s)}{\pi_{\theta_k}(a|s)}, (1+\epsilon)\right) A^{\pi_{\theta_k}}(s, a)$$

The objective increases if the action becomes more likely $\pi_{\theta}(a|s) > \pi_{\theta_k}(a|s)$, but no extra benefit as soon as $\pi_{\theta}(a|s) > (1+\epsilon)\pi_{\theta_k}(a|s)$.

When the advantage is negative: idem mutatis mutandis.

PPO LOSS FUNCTION

Surrogate loss of PPO

```
loss = policy_gradient_loss + ent_coef * entropy_loss + vf_coef * value_loss
```

- policy_gradient_loss: regular loss resulting from episode returns.
- entropy_loss: negative of the average policy entropy. It should increase to zero over training as the policy becomes more deterministic.
- value_loss: value function estimation loss, *i.e.* error between the output of the function estimator and Monte-Carlo or TD(GAE lambda) estimates.

PPO HYPERPARAMETERS

The PPO implementation in Stable Baselines3 has >25 parameters, including:

- clip_range: clipping factor in policy loss.
- ent_coef: weight of entropy term in the surrogate loss.
- gae_lambda: parameter of Generalized Advantage Estimation.
- net_arch_pi : policy network architecture.
- net_arch_vf: value network architecture.
- normalize_advantage: use advantage normalization?
- vf_coef: weight of value-function term in the surrogate loss.

PPO HEALTH METRICS

Some metrics indicate whether training is going well:

- approx_kl : approximate KL divergence between the old policy and the new one.
- clip_fraction: mean fraction of policy ratios that were clipped.
- clip_range : value of the clipping factor for policy ratios.
- explained_variance : ≈ 1 when the value function is a good predictor for returns.

