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The Ethics of Ancient Mathematics: The Case of Claudius Ptolemy

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Ptolemy, the 2nd-century mathematician, is remembered most for his astronomy. He composed the *Almagest*, a thirteen-book treatise comprised of astronomical models and tables that explain the movements of the stars and planets and predict where any celestial body will be on any given date. The *Almagest* was so influential that it eclipsed the astronomical texts that preceded it and became authoritative. In medieval Islam and Renaissance Europe, mathematicians studied it as the premier text in astronomy.

Although astronomy was first in importance for Ptolemy, it was just one of the mathematical sciences he studied. He also composed texts on harmonics and geography, for instance, and he approached these fields in a mathematical way. He additionally studied what were then considered physical sciences, including element theory, cosmology, and astrology, but most of his contributions were in the mathematical sciences.

Why was Ptolemy so dedicated to the study of the mathematical sciences? Today many of us take for granted that mathematics and the mathematical sciences are worthy of study, but the high-level study of mathematics was rare in antiquity. At any given time in the ancient Mediterranean, at most a few dozen individuals pursued it. Moreover, we have no evidence of mathematical schools at least until the 4th century CE. Much more common was the study of philosophy, especially in the traditions established by Plato and Aristotle. Advanced mathematics, then, was not an obvious choice, and it is reasonable to ask why anyone would devote his or her life to its study.

I argue in my book, *Ptolemy's Philosophy: Mathematics as a Way of Life*, that the answer to why Ptolemy devoted his life to the mathematical sciences lies in his philosophy [1]. Scattered among his more technical discussions in the mathematical and physical sciences are references to a fully developed and unique philosophical system. He engages with the most fundamental areas of philosophy—metaphysics, epistemology, and ethics—and he even makes claims that would have been highly controversial at the time.

What most motivates Ptolemy's study of mathematics is his ethics, his theory of what it means to be excellent and live the best life possible. He adapts a type of ethical commitment endorsed by Plato, where the principal goal of life is to be as much as possible like the gods. Even though Plato and Ptolemy were Greek, the idea is not to be like any of the Olympian gods we are familiar with from Greek mythology. Instead, the goal is to emulate the divine, where the divine is anything that is eternal. For Ptolemy, the divine entities we specifically are meant to emulate reside in the heavens. In ancient Greek cosmology, a spherical Earth lies at the center of the cosmos and is surrounded by a series of spheres, on which are situated the stars and planets. The cosmos, from the Moon outwards to the sphere of fixed stars, is the heavens.

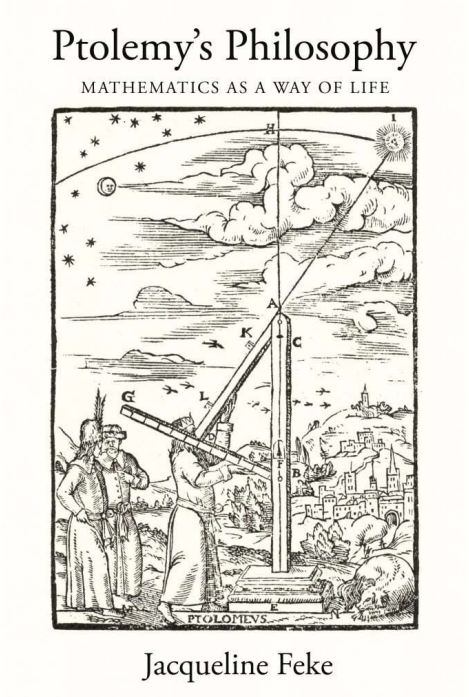

Figure 1. The cover of *Ptolemy's Philosophy: Mathematics as a Way of Life*. Provided by the author.

What makes Ptolemy's ethical theory different from those of Plato and his other predecessors is how mathematical it is. For Ptolemy, the ethical exemplars are not the stars and planets, or even the heavenly spheres themselves. Rather, the exemplars are specifically the movements and configurations of the stars and planets. We can begin to glean what mathematical objects Ptolemy has in mind from his definition of mathematics in the first chapter of the *Almagest*. He lists a number of objects which are the subject matter of mathematics, including movements from place to place, shape, number, size, place, and time. Astronomy, in particular, studies the movements of the stars and planets.

What about these movements is meant to be emulated? Ptolemy indicates that certain qualities of these movements are exemplary. He explains near the beginning of the *Almagest*:

> With regard to virtuous conduct in actions and character, [mathematics], above all, could make clearsighted men; from the constancy, good order, commensurability, and calm that are contemplated in the case of the divine, it, on the one hand, makes its followers lovers of this divine beauty, and, on the other hand, accustoms and, as it were, reforms their natures to a similar state of the soul [2].

By studying the stars and planets, the mathematician becomes aware of certain qualities of their movements and configurations: their constancy, good order, commensurability, and calm. In ancient philosophy of mathematics, the first three are specifically mathematical terms. In Ptolemy's ethics, one is meant to transform one's soul, the very essence of a person, so that it has these same mathematical qualities. The best condition of the soul, therefore, is mathematical. We are our best selves when we ourselves have the properties of divine mathematical objects. By modeling our souls after the movements and configurations of the celestial bodies, we become as much as possible like the divine.

Ptolemy also discusses his ethics in the *Harmonics*, his treatise on music theory. In the final chapters, he explores how the arithmetic ratios that define the relations among musical pitches exist in the heavens and in human souls. He defines the science of harmonics in a way that is particularly illuminating:

> The theoretical science of [harmonia] is a form of mathematics, the [form] concerned with the ratios of differences between things that are heard, this [form] itself contributing to the good order that arises out of the theory and understanding to people habituated in it [3].

There isn't only one mathematical science that can lead to the ethical transformation of the soul. In addition to astronomy, harmonics can put one's soul in good order. What is interesting in this quote is that Ptolemy indicates that harmonics is not simply a theoretical area of inquiry. It is something that people do; people become habituated to it. In the case of harmonics, one does not simply study the ratios; one furthermore creates them on musical instruments, especially the monochord, which allows for the more precise measurement and creation of musical relations. The term 'habit', however, indicates that there is more to doing harmonics than playing musical pitches, since it was a keyword in ancient virtue ethics. The habit that mathematicians acquire in doing harmonics is to act in a well-ordered way not just when studying harmonics and the other mathematical sciences, but also when engaging in the ordinary affairs of life. As the quote from the *Almagest* indicates, the study of the mathematical sciences transforms one's soul so that one's conduct in general becomes virtuous, or excellent.

How does Ptolemy's ethics relate to the work of today's mathematicians? Perhaps it invites us to explore why we choose to study mathematics and what is involved in its study. Why did you decide to study mathematics? Do you admire the content of mathematics, as Ptolemy loved the divine beauty of the movements and configurations of the stars and planets? What habits have you acquired in studying mathematics? How does its study affect your engagement in the ordinary affairs of life? Ptolemy thought deeply about these sorts of questions, and perhaps we should, too.

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References

[1] Feke, Jacqueline. (2018) *Ptolemy's Philosophy: Mathematics as a Way of Life.* Princeton University Press.

[2] Ptolemy, *Almagest* 1.1, H7. Translation by the author.

[3] Ptolemy, *Harmonics* 3.4, D94-5. Translation by the author.