CMS NOTES

Issue Contents

September 2021 : Vol. 53, No. 4

Greetings from the President-Elect

Cover Article

September 2021 (Vol. 53, No. 4)

David Pike (Memorial University)
President-Elect

I became the President-Elect of the CMS as an outcome of the election conducted by the society earlier this year. One of my executive duties will be to periodically write a cover article for the CMS Notes. Since this is the first such article in my new role, let me begin with a brief introduction for those who might not know me well. I obtained a BMath degree in 1992 from the University of Waterloo and then began graduate studies at Auburn University in Alabama. In 1998 I joined the faculty at Memorial University of Newfoundland where I am now a University Research Professor. My academic research interests are generally centred on combinatorial designs and graph theory, and often have computational aspects. My recreational interests include genealogical research, hiking, and curling with the Odds & Ends LGBTQ+ Curling League in St. John's. I am a gay man whose preferred personal pronouns are he/him/his.

My involvement with the CMS dates back to my time as a graduate student, for it was during my doctoral studies in the USA that I became a member of the CMS. A few years later I obtained a tenure-track position at Memorial University, thanks in part to a job advertisement that I saw published in the CMS Notes. This role that the CMS played in facilitating my return home to Canada has helped to give the society a special place in my heart. Certainly in those early years I could not have predicted that I would eventually be elected to the presidency of the society. I am honoured to have this opportunity to serve and to represent the Canadian mathematical community over the next several years.

The CMS has a rich history. This summer we celebrated with a 75th+1 Anniversary Meeting that was hosted online and had over 1,000 registrants. Beyond organising conferences and workshops, the society also has many other activities that engage various audiences. As but a few examples, we have our own scholarly journals as well as publications geared more towards grade-school students, we conduct annual mathematics competitions, we sponsor teams to represent Canada at events such as the European Girls' Mathematical Olympiad and the International Mathematical Olympiad, and we provide support for numerous math camps throughout the country.

As incoming president, a primary concern for me is ensuring that the CMS continues to thrive so that it can keep promoting mathematics. In short, I want to make sure that we are positioned to carry forward long into the future, well beyond my own time at the helm. In my mind the most critical requirement to ensuring the society's future is to ensure its financial well-being.

A quick look at the society's financial records from the past few years shows that we are paying more than $50,000 annually in occupancy costs. We have been renting our premises in Ottawa, and like any tenant this means that we have not acquired any equity in our home. In the past year the society decided to change this situation, and we are now looking to purchase a suitable building in Ottawa. Having our own "House of Mathematics" is a step towards establishing an improved long-term financial position, for not only will we come to own a substantial asset (which itself is likely to appreciate in value over time), but once it is paid for then the money that would otherwise have been spent on a mortgage or rent can instead be used to support other aspects of the society's mission. Stay tuned for more news on this matter in the coming months.

Meanwhile, let me also mention some of the components to the annual revenue of the CMS. We have published the Canadian Journal of Mathematics since 1949, the Canadian Mathematical Bulletin since 1958, and soon we hope to also publish a "Transactions" journal. For many years the subscriptions to our journals have generated a significant portion of our total income. How to maintain (or grow) this revenue stream amid a global environment that is shifting towards open access publishing is a conundrum that I am sure to learn much about in the coming months and years. Within days of my confirmation as President-Elect I was put onto both the Publications Committee and the Finance Committee of the CMS, each of which should help to educate me.

"

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Although I am fresh and a bit naïve in my new position, I reckon that fundraising is something else that I will also be learning about. Indeed, I've already had some society members approach me with advice and suggestions for how we might leverage our status as a registered charity to attract donations above and beyond what we currently receive. In 2016 we took in $11,624 in tax-receipted donations. While the following years were more fortunate for us (at the end of 2020 our 5-year average was $43,408), we can and should strive to further increase this aspect of our operations. For potential benefactors we have a variety of activities and initiatives that could attract their interest, including outreach and training activities, awards and prizes, and now also the expenses of buying a building and transforming it into our new home and headquarters. I invite anybody who has ideas on matters of fundraising to reach out and share their thoughts.

To offer a few closing words now, while I admit that I find it a bit scary to be taking on this new leadership role, I am approaching it and the coming years with excitement. The CMS operates as a large team and I am looking forward to working together with the staff and many volunteers who contribute to the successful operation and vibrancy of our society.

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Uncertain Times

Editorial

September 2021 (Vol. 53, No. 4)

Robert Dawson (Saint-Mary's University)
Editor-in-chief

As I write this in late July, the infectiousness of the delta COVID variant is being compared with that of chicken pox. Authorities are sure that the current vaccines are very effective at keeping people out of the hospital. However, they are less sure how much they protect people from being infected by the delta variant and passing it on. South of the border, some states are already deep in a new wave – Louisiana is announcing more new cases per day than their peak last October, and numbers are still increasing. Numbers from Alberta and British Columbia, while low, are showing the upward curve that signals an R number greater than 1, and the rest of Canada may not be far behind.

What are we doing to fight this – and what can we do? Vaccination uptake curves have flattened off across the country: if vaccinations are going to pick up again to the point where herd immunity is reliable, it will probably require a change of heart by people who have so far declined vaccination. Governments are showing signs of pandemic fatigue: in some provinces even people who know that they are carrying the coronavirus are now permitted to mingle in public unmasked. And most Canadian universities are not requiring vaccination for faculty or students who want to be on campus in the fall. (Is there a "right" to be in public places unvaccinated? Not according to the experts at the Canadian Museum of Human Rights. If you go there today, and are over the age of 12 and not fully vaccinated, you will not be allowed in.)

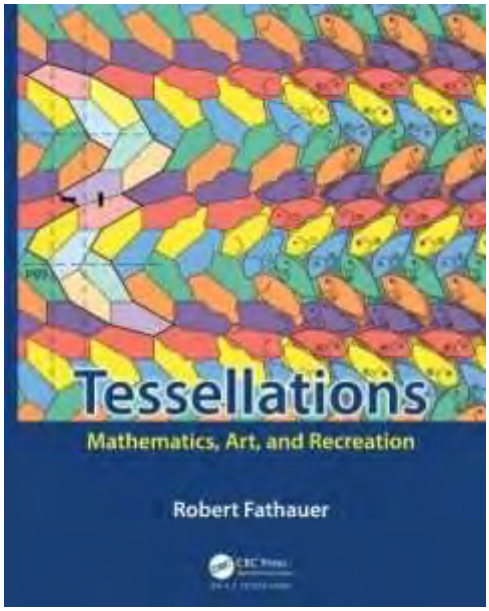
Universities across the country have been hoping to reopen in September, more or less as usual. There's no doubt about it: we're all tired of online teaching, we're all tired of online meetings, and we are all tired of examinations without effective invigilation. If things don't go well over the next month, we may be back to there for another semester or even year. Worse, if we do get back to campus and the virus starts to spread, some of us may be facing a repeat of March 2020 when in-person classes have to become online at a fes days' notice. Please be careful: it's up to us as individuals now, more than ever.

Stay well.

Robert Dawson (Saint-Mary's University)
Editor-in-Chief

Book *Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.*

Karl Dilcher, *Dalhousie University (notes-reviews@cms.math.ca)*



Tessellations: Mathematics, Art, and Recreation
by Robert Fathauer
A K Peters/CRC Press, 2020
ISBN: 978-0367185978

Robert Fathauer's *Tessellations: Mathematics, Art, and Recreation* is a gorgeous book. It's lavishly illustrated with photographs of tessellations and related patterns from nature and architecture; with reproductions of artwork by M. C. Escher and other artists who have found inspiration in tessellations; with the tilings of geometers such as Sir Roger Penrose, Robert Ammann, and Casey Mann; and, most of all, with the author's own creations.

The reader will need an interest in mathematics, but no great background; the level of rigor is only a little higher than what aficionados of the late Martin Gardner will remember. To take advantage of this, the book is liberally sprinkled with activities aimed at the K-12 classroom, including handouts, lists of vocabulary words, and (where relevant) CCSSM standards. These exercises are interesting and have solid mathematical content, but their level of difficulty varies widely. An appendix listing the activities by approximate grade level (or cross-tabulated with topic) might make it easier for a teacher dipping into the book to find an activity. In some cases (for instance, worksheet 8.1 on page 148) opportunities are missed for older students to use algebra rather than the too-ubiquitous hand calculator. The range of topics is wide, and each one is explored fairly deeply, with its relevant history. The second chapter, for instance, begins with regular tessellations of the plane. It goes on to the semiregular tessellations, pausing to consider the possible ways in which regular polygons can be arranged around a point. After that, it considers tilings with a single shape of polygonal tile, including the Laves tessellations, dual to the regular and semiregular tessellations. The fifteen classes of tilings with convex pentagons are shown (without proof, but with information about the discoverers.) We then consider the tessellations involving star polygons (the reader must be careful here: the definition on page 31 is more restrictive than what is implied by the tilings on pages 33-35.) Then there's a short look at non-edge-to-edge tessellations with squares and triangles, including "squared squares." Four pages on creating new tessellations from old, and a look at Apollonian packings, bring us to the end of the chapter. You will have to read the other twenty-four chapters yourself!

Chapter 8, on rosettes and spirals has a good introduction to Fibonacci spirals and phyllotaxis. I would argue with the author's decision (p. 122) to use "golden number" rather than "golden ratio" on the grounds that

> *"The term `golden ratio' implies a ratio of two objects, which is not how it will generally be used in this book."*

While I agree with him that

> *"[t]he number has a long history of being applied to objects that it doesn't really fit very well, such as the nautilus shell and the Acropolis,"*

the dimensionless number φ in a tiling is indeed a ratio of lengths (or of tile populations in the Penrose tiling) and this is not changed by other writers' over-eagerness to spot it in architecture and nature.

Chapter 9, on fractal tiles and fractal tilings, contains material of considerable beauty that will be new to most mathematicians. Many of these tilings have a deep connection to the hyperbolic plane that clearly cannot be explored at the level of this book! The author attempts on page 152 to draw a distinction between fractal and hyperbolic tilings, with Escher's "Square Limit" woodcut on one side of the line and his "Circle Limit" works on the other. I confess that I do not see the basis for this distinction. But this is a minor detail, that in no way detracts from the overall presentation.

Finally, Fathauer is not just a mathematician but also an artist. He shares his artistic tips freely here, including, in Chapter 13-19, some really good instructions on how to create an Escher-style tiling based on various symmetries. If you are a high school math teacher and you bring a copy of this book in to work, you may need to hide it from the art teacher!

Supporting Community and Communication through Online Learning Journals

Education Notes September 2021 (Vol. 53, No. 4)

Vanessa Radzimski (University of the Fraser Valley)

Kseniya Garaschuk (University of the Fraser Valley)

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Our motivation and goals

The COVID-19 pandemic has highlighted and magnified issues that have always been present in our classrooms: student course preparedness, the flexibility of students' study habits, and the ability to self-assess and reflect on course content and effectiveness of their learning, among others. One of the most prominent concerns of students in the remote environment has been self-regulation, time management and falling behind on their coursework [1, 2]; after all, if the lectures are recorded, it is easy to procrastinate engaging with the material. Once behind, the priority becomes catching up with the material, as opposed to cultivating deep understanding. The shortage of student collaboration — more present in live courses during in-class group work and outside of class time in informal study groups — also results in fewer opportunities for students to see each others' work, witness and discuss varying viewpoints. On the other hand, more structured and deliberate instructor-student online interactions lower the frequency and effectiveness of communication [3]. This affects the feedback loop for both parties involved: students get less input from the instructor regarding their progress, while the instructor gets less of an insight into students' process of learning and points of struggle. In our experience, students have few opportunities to reflect on their knowledge during the course of the semester, so as instructors we would like to implement more qualitative assessments for students to see the course as a whole and not just the sum of its parts.

Undergraduate curricula can be jam-packed with content and "doing": homework, quizzes, projects, and tests where students are (sometimes mindlessly) working through problem after problem. As instructors, we hope that our students have an appreciation for what they learn each week and why that content is important, but recognize that we may not provide assessments and reflection opportunities that align with this learning outcome. To this end, many institutions are now including *affective outcomes* in their course outlines, such as, "advocate for the value of *topic* in *context*". In their curriculum guidelines for undergraduate courses, the Mathematical Association of America suggests "mathematics faculty should deliver an unambiguous message concerning the importance of mathematical reasoning and communication skills and adopt instructional methods and curriculum content that develop these skills" [4]. Large scale, high-stakes projects that come at the end of a course have the potential to fit this bill, but are generally not intended for the gradual development of communication skills throughout the course of a semester. Ask yourself: do you have purposeful opportunities for your students to sit back and consider what was being done, why they did it, and what they learned (or didn't) throughout your course?

We present the idea of *learning journals* as a low-stakes, small-scale assessment for learning, which provides students a regular space to sit down and reflect on the simple question of "what did I do this week?" The structure of these online journals provide a virtual space for students to communicate with each other, answer each other's questions, and gain an alternative understanding of the content, as described by their peers. The journals also allow the instructor an insight into student processes, as well as the depth and richness of students' understanding beyond being able to carry out calculations or problem solve.

The What, How, and Why of Learning Journals

The learning journals used in our courses were done on a bi-weekly basis. The Learning Management System (LMS) at our institution is Blackboard, so we took advantage of the "Blogs" course tool built into the LMS. In creating these blogs, we chose the "course blog" option that allowed all students to view and comment on the posts of their peers.

The learning journals were employed in five distinct courses: Calculus I for Business, Calculus II for Life Sciences, Calculus II for Physical Sciences, Calculus III, and second-year Linear Algebra. Our courses varied in format (from fully asynchronous to virtual "flipped"), but the primary course content was always presented in pre-recorded videos scheduled for students to watch each week. Students were instructed to not write their learning journal until after they had watched all lecture videos and completed the associated homework assignment. As such, the learning journal was always due after the associated homework for the week's content was due. For example, the learning journal for weeks 2 and 3 would be due on the Saturday of week 4. This deadline is fundamental to the learning journal idea; the intent of these journals is to provide a space for students to regularly reflect on their learning process, so this deadline was integrated into their weekly "to-do's".

Each learning journal included the following instructions and rubric (credit and thanks to Cindy Blois (UofT) for her communication rubric).

Write between 400-500 words summarizing what you learned these past two weeks. The goal is to synthesize the content and your learning from this week. To help guide your writing, please respond to the following questions, with your peers being your target audience:

- **What?** Give an objective discussion of content. What was the focus of our study? What tools did we need? What ideas were developed?
- **So what?** Why are these ideas useful? How can we (or did we) use them? What is challenging?
- **Now what?** How can you better understand the content? What problems/examples were helpful? What do you need to do more of to further your understanding? What lingering questions do you have?

Once you are done with your post, answer one of your peer's questions from "So what?" If you don't have a complete answer, that's OK. The goal is to try to share some of your insight and understanding on a topic your peer (and possibly yourself) is having trouble with.

Marking of learning journal entries will be done on a scale of 0-3:

- 0 – (non-existent or needs work) Didn't complete or cheated. No full sentences are included. Writing is insufficient or mostly incomprehensible to an expert. Sentences are sparse or not understandable. The mathematics has been attempted to be described but many of the main points are either not described or are incorrect.
- 1 – (Satisfactory) Explanation is comprehensible but there are several sentences that are unclear, even to an expert. Most logical steps of the mathematics are shown, but there is at least one major error or gap in reasoning.
- 2 – (Good) Explanation is clear to an expert in the subject but may be confusing to peers in a few parts. One or two minor mathematical or logical errors have been made, such as improper notation or use of language. Main logical steps are correct.
- 3 – (Excellent) Explanation is clear and easy to understand. Visual aids such as pictures are included, if helpful. Mathematically, there are essentially no errors in reasoning or computation. All reasoning is shown. A deduction of 1 mark will be taken if no reply to a peer is posted.

The instructions and rubric for the learning journals were intentionally free-form. These journals were less an assessment of students' understanding of the content through specific problems, as they were an opportunity to reflect on their learning process. Students were explicitly asked to write with their peers as the audience, so as to avoid the use of overly technical language and jargon. In addition, students were also provided with a "sample" learning journal post, to give them a rough idea of the level of breadth and depth required in exposition.

In the first iteration of the learning journals, they were chosen to be done on a weekly basis, with entries being at least 250 words. The instructor quickly realized that this frequency was far too much of a workload for both students and instructor. For students, writing is a time-consuming task and not the one they are used to doing in mathematics. For the instructor, reading over 100 posts every week – on top of the other demands of preparing a remote course – was an unreasonable demand on time. Moreover, a week's worth of material didn't always provide fertile ground for deep connections within the material itself. As such, the learning journals were completed on a bi-weekly basis during the subsequent terms.

With the switch to a bi-weekly format, students were asked to write entries that were 400-500 words, to account for entries which covered two weeks worth of content. The bi-weekly format also encouraged students to consider how the two weeks of content were connected to each other. For example, in calculus courses, top marks were reserved for students who presented various representations of objects (analytic, numerical, graphical and verbal) and made meaningful connections to other course topics, other classes or areas of interest.

The open-ended nature of the rubric was pivotal in keeping the workload associated to this assignment in a reasonable range. While we wanted to give students reasonable feedback, this also needed to be balanced with the associated workload of reading and marking the entries. Short individual feedback was provided alongside to each student during grading. In a synchronous format, the instructor mentioned common themes and observations during class sessions. In an asynchronous format, the instructor sent out weekly digests with class-wide feedback on Learning Journals, including student quotes.

Our view of impact

The visibility of posts to the entire class serves to create a community of learning in the remote environment. With a requirement to reply to peer's posts, students get to see how their learning experience relates to their peers' and often see that they are not alone in their struggles and victories. The prompt also gives students an opportunity to try to respond to a students' question, which will challenge them to parse a question relative to their own understanding. This view of impact was valued by students, who remarked the following:

> *I learn so much from reading other's journals. Everyone has different background science knowledge, like there would be applications of calc in other study areas that I would never even think of but reading through journals is how I get to learn about other science applications. There's just so much we can learn from others.*
>
> *The journal entries are helpful because I think we can all reflect and see how all our peers are thinking in comparison to us. I think the conceptual part of learning math is the most important because "plugging and chugging" is not going to work anymore.*

The blog format where students could all see each other's entries also aids in the understanding that the intended audience consists of peers and not the instructor, avoiding the expectation that the reader is an expert. Articulating the material through writing allows students to organize their thoughts into one coherent logical sequence and fill gaps in their knowledge as they discover them through the writing process. It also highlights the importance of various representations of mathematical concepts — verbal, graphical, analytical and numerical. Writing these journals required students to engage with the material in a novel way and required them to go deeper, as evidenced by their testimonies:

> *I would say it was somewhat a good use of time as it allowed me to make sure I understood a concept enough that I coud (sic) write about it.*
>
> *We spend a lot of time learning to make calculations, however we don't always have a chance to explain how we did it or why we should learn it. These journals helped answer those questions!*
>
> *Doing the learning journals made me do additional research on multiple topics which added to my understanding of those topics.*

The additional benefit of the journals was in providing an easy, low-commitment forum to connect and create a sense of community, see what your peers struggled with and hear their voices in a remote environment. In both their journals and in comments, students often shared useful resources or study tips that they found to be helpful.

> *They are very useful to understand topic and get the extra help, they also allow you to learn shortcuts that other students found to help understand the topic even more.*
>
> *I believe that the learning journals were a great part of the course as they not only allowed for students to learn from teaching, but also interact with one another and view topics from the understanding of their peers.*

We ran a survey in a class of 44 students and received 33 responses. Of those who responded, over 70% of students said that they spent between over 30 minutes but under 2 hours on each learning journal entry, but none thought the time commitment was burdensome. In fact, some students specifically mentioned the reasonable workload and positive learning experience in their comments:

> *The learning journals weren't much of a workload especially since they were given every second week, giving students enough time to understand concepts. Other than that, these journals were a great learning experience especially since you could gain knowledge from reading other classmate's posts too. They were a great experience and fun too!*
>
> *[The learning journals were] not too too time consuming so there isn't any loss for students, more so much more gain in learning experiences. It is also easier for students to understand something that their peers wrote more clearly in some cases.*

Nearly half the students reported that on average they read 2-3 peers posts, while over a third of students said they read at least 4 peer posts. Over 80% of respondents advised that the instructor should keep learning journals for future offerings of this course. Interestingly, here are the comments from the only 2 students out of 30 said that they would not recommend Learning Journals as a course component:

> *I understand the reason why you implemented them in the course work but I don't think we need to have this in-depth understanding of each topic in order to be successful in the course. It feels like a burden and I don't think it's necessary, especially in a math course.*

> *I found I was choosing to stick to the materials I understood better rather than practicing skills that needed to be improved.*

We believe that students' positive experiences with Learning Journals as opposed to generally dreaded formal writing assignments can be attributed to the fact that the journals provided small, low-stakes writing opportunities. Coupled with the normalizing effect of seeing their peers' writings and getting instructor feedback, students were able to gradually build their communication skills with the support of the course community and with opportunity for growth.

An unexpected benefit to instructors was getting insight into students' experiences in their other courses. Specifically, in Calculus II for Life Sciences, students wrote about science applications in the fields of ecology, biology, forestry, agriculture, and geography. Examples were both authentic and relatable, which allowed the instructor to use them as inspiration for the design of additional class problems and course projects.

Concluding remarks

The implementation of online learning journals was born from the concern that the remote, online learning environment could have a detrimental impact on the course community. In their review of blogs in higher education, Williams and Jacobs [5] remark that "blogs have the potential, at least, to be a truly transformational technology in that they provide students with a high level of autonomy while simultaneously providing opportunity for greater interaction with peers" (p. 244). This sentiment is echoed by Ellison and Wu [6] who found that students enjoyed the blogging process and found their peers' blogs to aid in their own understanding of course material. Based on the feedback we have received from our students and presented above, we believe that our experiences in employing online learning journals further support these claims.

For both of us, having students come together and discuss their understandings and misunderstandings is fundamental to the teaching and learning process. As universities begin the transition from remote to in-person learning, both of us intend to keep online learning journals in our future courses. Cuhadar and Kuzu [7] observed that online blogs can positively impact community in face-to-face class meetings. They found that the online blog interactions supported students in feeling more comfortable communicating with both their peers and the instructor during in-person meetings. Beyond increasing community and collaboration, online learning journals have the potential to support a safe space for student learning. Indeed, students who may feel uncomfortable speaking up in a face-to-face class have an opportunity to find a voice in the online blog environment [8, 9]. The creation of a learning space that welcomes these students can foster a sense of belonging to the group [10], which brings us back to our initial goal of supporting community.

Online learning journals are small-scale, low-stakes writing assignments, which require students to reflect, write, read, and inquire. We hope that our remarks, as well as those of our students, persuade you to give learning journals a chance. Our students valued the experiences, struggles, and victories of their peers through learning journals, and just like them, we look forward to hearing your experiences in implementing learning journals for supporting community and communication.

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Kseniya Garaschuk is an Assistant Professor at the Department of Mathematics and Statistics at the University of the Fraser Valley. Her current scholarly activities revolve around facilitating opportunities for student-created content and effective peer feedback exchange.

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Bourbaki, Structuralism, and Categories

Jean-Pierre Marquis (Université de Montréal)

CSHPM Notes bring scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

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> Since Hilbert and Dedekind, we have known very well that large parts of mathematics can be developed logically and fruitfully from a small number of well-chosen axioms. That is to say, given the bases of a theory in an axiomatic form, we can develop the whole theory in a more comprehensible way than we could otherwise. This is what gave the general idea of the notion of mathematical structure. Let us say immediately that this notion has since been superseded by that of category and functor, which includes it under a more general and convenient form. It is certain that it will be the duty of Bourbaki … to incorporate the valid ideas of this theory in his works [1].

Jean Dieudonné was one of the most eminent French mathematicians of the 20th century. The foregoing quote was written in 1970, when Dieudonné and his colleagues were reflecting on Bourbaki's legacy. The claims made by Dieudonné are puzzling: on the one hand, mathematicians had by then a general idea of the notion of mathematical structure; on the other hand, category theory superseded that notion and Bourbaki, whoever that is, must incorporate the valid ideas of this theory in his work since they apparently give a better account of the notion of mathematical structure.

In the 1970s, every mathematician knew who Bourbaki was and what he had done. There were fans—big fans—and there were critics—virulent critics. Nowadays, not so much. Very few, I suspect, could say what the general idea of the notion of mathematical structure is precisely, or explain that Bourbaki defended a structuralist view of pure mathematics. Meanwhile, nowadays every mathematician, young and old, has at least heard of categories and functors. Was Dieudonné right? Did Bourbaki ever integrate categories and functors in his work? The short answer is 'no'. It is true that Bourbaki had a general idea of the notion of mathematical structure. But Bourbaki, although he knew about categories and functors from very early on, never found a way to integrate them in his work. With hindsight, this failure should not be surprising, for the issue is delicate and subtle. It brings us directly to the center of some of the most sensitive issues in the foundations of mathematics.

So, who is Bourbaki? What was his idea of the notion of mathematical structure? And what do categories and functors have to do with it? Let us take a quick look.

Bourbaki: a nano-historical sketch

Our story begins in Paris, December 1934, Boulevard Saint-Michel, in a small café [2]. A group of young university professors, mathematicians and almost all former students of the École Normale Supérieure, an elite school in France, sit around a table, discussing the appalling state of introductory textbooks in analysis available in French at the time. They decide that the only way to remedy the situation is to write a new textbook, a *modern* textbook, in the spirit of Van der Waerden's *Moderne Algebra* [3], a work they admire. So, they sketch the plan of the book and decide that they first need to write an introduction to the new "abstract" mathematics—that is, set theory, algebra, and topology—before moving on to analysis proper. They decide to join forces and meet in the following months to write the work. Thus, Bourbaki was born; although still nameless and yet to be fully organized, this improbable collaborative project was about to change the face of contemporary mathematics to this day.

Figure 1. Cartan, de Possel, Dieudonné, Weil (standing), Mirlès, Chevallez, and Mandelbrojt (seated) at first official meeting of the Bourbaki group in 1935. MacTutor

Around the table were: Jean Dieudonné, André Weil, Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Coulomb, René de Possel, Charles Ehresmann, and Szolem Mandelbrojt, nowadays called "the founding fathers" of Bourbaki [4]. They all had international careers as creative mathematicians. The first official meeting of the group happened in the summer of 1935. This is where they decided to adopt the name "N. Bourbaki" (later to become "Nicolas Bourbaki"), an absurd joke, typical of the humor practiced by the students of the École Normale Supérieure at the time.

The project, limited at first to an introductory volume on analysis, morphed into a vast and ambitious enterprise: the idea was to start from scratch, to present and develop the abstract axiomatic theories needed to get to classical analysis. Nothing is taken for granted, nothing is presupposed except for a certain "mathematical maturity", whatever that is. The mathematics should all be developed by purely logical means. It is, in a sense, an exercise in formalization, abstraction, and organization, although it is certainly not seen that way. Thus, the first book starts with an exposition of first order logic, then an axiomatic presentation of set theory and the general idea of structure. Subsequent volumes move on to the axioms of topology, the axioms for algebraic structures (e.g., monoids, groups, rings, fields). The next step consists in combining these structures (e.g., topological groups, topological vector spaces), and all this eventually brings us to analysis at last.

The written style is terse: definitions, theorems, proofs. Nothing else. No discussion, no image, no context. There are exercises and problems—a lot of them—and they are seen as an integral part of the presentation. The proofs are not written in the formal system—which the members see as a long and tedious exercise—but it is explicitly assumed that any competent mathematician should be able to translate the given proofs into the formal jargon. To this day—for Bourbaki still exists and still holds a seminar in Paris—more than 40 volumes have been published [5]. An introductory textbook? Absolutely not. An encyclopedia? The enterprise was never conceived as one and it certainly is not such a work. It is not quite clear what he ended up with. It is a singular object. But it influenced at least three generations of mathematicians worldwide. Emil Artin, Philip Hall, Samuel Eilenberg, and Saunders Mac Lane all praised the first volumes published in the 1940s. Michael Atiyah later said that his generation were all "bourbakistes". Be that as it may, at the core of the whole project is one key idea, one motivating *leitmotiv*: (pure) mathematics is about *abstract structures*.

Bourbaki's notion of (species of) structure and structuralist mathematics

What is, *in general*, a mathematical structure? Already in 1935, inspired by the recent developments in algebra, topology, and logic, Bourbaki decided that his project had to be based on the idea of mathematical structure. But how does one proceed to define *in general* the notion of a mathematical structure? How does one spell out concretely and rigorously the idea that all of mathematics is based on abstract structures? Bourbaki literally struggled for more than twenty years with these questions before he finally published his answer, even though at the end he knew very well that it did not work, in particular it could not accommodate categories and functors. But since he could not find a better way of doing it and many volumes that were based on his idea of mathematical structure had already been published, he had to let it go.

Nowadays, one thinks of a mathematical structure as a set with relations and/or operations satisfying certain conditions. Bourbaki's approach contains these elements, but his presentation is different. First, he defines what he called an echelon construction. Simplifying greatly, one can say that an echelon construction, denoted by E, is a basically a way to construct, from given basic sets, a new set by combining cartesian products and powersets on the given basic sets in a certain order. More precisely, one starts with basic sets $A_1,\ldots, A_n$, and parameter sets $B_1,\ldots, B_m$, and then proceeds by recursion, i.e., if X and Y have been constructed, that is, if they are in the echelon E, then the cartesian product $X \times Y$ and the powerset $\wp(X)$ are also in the echelon E, then the set obtained at the end of such a sequence of constructions is an echelon construction E.

The second step consists in defining a *species of structure* as one would expect, that is, by introducing relations and conditions on elements of a given echelon construction. We seem to get something akin to our original informal characterization, with the only difference that the underlying sets required are constructed appropriately. For instance, a topology on a set A, seen as a Bourbakian structure, is a set O_A in the echelon construction $\wp(\wp(A))$ satisfying the usual conditions on open sets. Thus, one defines relations $R_1,\ldots, R_k$ on elements of the echelon construction E to get to a species of structure. But that is not all. There is one missing ingredient that captures the essence of abstract mathematical structuralism: the relations must be *transportable*. This basically means that isomorphisms between two lists of basic sets should yield "the same" species of structures and thus that any theorem proven for one specific structure must hold for any structure isomorphic to the first one. Today, we would say that everything is done up to isomorphism [6].

Our foregoing presentation has been entirely informal. When one looks at Bourbaki's work, one immediately notices that he is firmly in a *formal* framework. More precisely, he is working in a formal language for set theory. He even says that a species of structure is a *text*. He does not give a *mathematical* definition of structure, but a *metamathematical* one. It was only natural for Bourbaki to use formal language for set theory to define species of structure in the 1930s and 1940s. However, by 1950, there was one type of mathematical structure that could not be reconstructed as a species of structure, namely categories.

Categories and Bourbaki's species of structure

In 1950, Samuel Eilenberg joined Bourbaki while he was collaborating with Henri Cartan on their then-forthcoming book *Homological Algebra* [7]. As is well-known, homological algebra treats certain functors between certain types of categories. There is no need to be more precise here. It was clear to everyone at the time that certain categories—e.g., the category of sets or the category of abelian groups or the category of modules over a ring R—cannot be sets without imposing some restrictions. Thus, categories in general cannot be set-based structures, for they do not have underlying sets, and their structure does not lend itself to the usual echelon construction and species of structure. One *could* move to a theory of classes, as Eilenberg and Mac Lane did in their original paper, but that was of no use to Bourbaki. Eilenberg was asked to help. Ralf Krömer found interesting passages in Eilenberg's *Nachlass* that exhibit the latter's struggle [8]:

> The method of functors and categories is in some sort of "competition" with the method of structures as developed at present. Unless this "competition" is resolved only one of these methods should be presented at the early stage. Bourbaki is committed by structures for all the material of part I at least [quoted in 8, p. 142].

It seems that Eilenberg's first idea to resolve the conflict was to subsume the notion of structure under the notions of categories. We read:

> The resolution of this "competition" is only possible through the definition of the notion of "structural homomorphism" which would convert each type of structure into a category [quoted in 8, p. 142].

But putting categories first also required that every type of structure comes with a notion of morphism, and some of the members of the group—especially André Weil—did not think this was justified. In a sense, it was simply too early to see clearly what the real problems were, for some of the basic concepts of category theory had still not seen the light! Indeed, the notions of equivalence of categories, adjoint functors, functor categories, representable functors, etc., were officially introduced between 1957 and 1961, after Bourbaki's volume on sets, structures, and isomorphisms.

Figure 2. Cartan in 1968, Eilenberg in 1970, and *Homological Algebra* in 1956.

Nonetheless, it was still possible to introduce categories and functors later in the enterprise, e.g., in the volumes on homological algebra, or algebraic topology, or in fact, in any other volume where categories came to play a central role. But Bourbaki did not do it and, in that respect, he failed.

By the late 1950s, Bourbaki was deeply divided, to the point that Alexandre Grothendieck, then a member of Bourbaki, left the group over the issue. Pierre Cartier has reported that Weil and Grothendieck were not even talking to each other [9].

Sets, categories and structures

However, to see a competition between Bourbaki's method of structures and categories is to confuse the issue, although unfortunately this is still a prevalent opinion, among certain philosophers of mathematics, for instance [10]. Bourbaki could not have articulated properly how his structuralist standpoint could accommodate category theory, for the latter *extends* his standpoint in a new conceptual direction. It does not *compete* with it, which is what most members seemed to believe even at the end of the last century.

To understand how Bourbaki's view on mathematical structures can accommodate categories, even higher-dimensional categories, is no trivial matter. One must rethink the foundations of mathematics from the ground up, from the formal apparatus to the semantics universe. Although we still do not have a complete picture of this new universe, we have candidates that point in precise directions, namely homotopy type theory and Michael Makkai's FOLDS [11]. One thing is sure: Bourbaki's metamathematical analysis of mathematical structuralism still stands, in the sense that within the forthcoming foundational framework, one will have to build in a requirement that the relations defining the structures have to be transportable with respect to an appropriate notion of isomorphism. This latter constraint being purely *metamathematical*, one does not have to give a general *mathematical* definition of structure; rather, one must specify the formal constraints within which such theories have to be constructed—and those were essentially identified by Bourbaki himself. It might still be possible to be a structuralist with respect to pure mathematics.

Jean-Pierre Marquis teaches logic, philosophy of science and philosophy of mathematics at the Université de Montréal. He has published a book on category theory and categorical logic as well as numerous articles in logic, philosophy of science, philosophy and the foundations of mathematics.

Notes

[1] Dieudonné, J. (1970) The Work of Nicholas Bourbaki. The American Mathematical Monthly 77(2), 134–145, on p. 138.

[2] For more on Bourbaki, see Mashaal, M. (2002) *Bourbaki: une société secrète de mathématiciens*, Les génies de la science no 2. Paris: Pour la Science. English translation by Anna Pierrehumbert (2006) American Mathematical Society.

[3] Waerden, B. L. van der. (1930–1931) *Moderne Algebra*. 2 vol. Berlin: Springer-Verlag.

[4] They might not all have been around the table at this first informal meeting. But they joined the group at one point or another and these people are nowadays considered to be the "founding fathers".

[5] The exact number depends on various parameters, e.g., language, editors, editions, etc.

[6] For the formal details, see Bourbaki, N. (2004) *Theory of Sets*, Springer. The relevant chapter is called "Structures and Isomorphisms"; it was originally published in 1957 and then integrated within a complete book in 1970.

[7] Cartan, Henri, and Samuel Eilenberg. (1956) *Homological Algebra*. Princeton Mathematical Series no 19. Princeton: Princeton University Press. Reprinted in 1999.

[8] For more details, see Krömer, R. (2006) La 'Machine de Grothendieck' se fonde-t-elle seulement sur des vocables métamathématiques? Bourbaki et les catégories au cours des années cinquante, *Revue d'histoire des mathématiques* 12, 119–162.

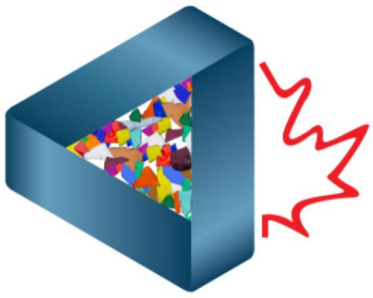
[9] Cartier, P. (2015) Alexander Grothendieck: A Country Known Only by Name. *Notices of the AMS* 62(4), 373–382, on p. 375.

[10] See Hellman, G. (2003) Does Category Theory Provide a Framework for Mathematical Structuralism? *Philosophia Mathematica*, Series III, 11(2), 129–157.

[11] For homotopy type theory, see Homotopy Type Theory: Univalent Foundations of Mathematics (2013) The Univalent Foundations Program, IAS, Princeton; for Makkai's FOLDS, see Makkai, M. (1998) Towards a Categorical Foundation of Mathematics, in *Logic Colloquium '95 (Haifa)*, edited by J.A. Makowsky & E.V. Ravve, Springer, 153–190.

Introducing MOSAIC: Asking the Tough but Simple Questions

Steven Rayan (he/him) (University of Saskatchewan)



MOSAIC, Outreach, Society, Accessibility, and Inclusiveness Column is directed by the CMS EDI committee and touches upon issues concerning equity, diversity, and inclusion in mathematics. Comments, suggestions and submissions are welcome.

Steven Rayan (he/him), University of Saskatchewan (rayan@math.usask.ca)

We are less than two years into the new decade and, already, it has presented formidable challenges. The Covid-19 pandemic has steamrolled through the world, costing lives and damaging the health of a great many. In keeping one another safe from this constantly-evolving virus, we have learned to live in sequestered ways that have tested our social bonds. At the same time, we continue to witness the unspeakable brutality of racism and hatred. A year ago, George Floyd's life was cut short by savage, hate-fueled violence, leading to renewed discussion of systemic and pervasive racism in nations around the world. This year, we have seen brutal killings of individuals of Asian descent in the US and of Muslims in Canada. In recent weeks, the discoveries of mass burial sites and unmarked graves in Canada at the sites of former residential schools has left the country shaken and horrified. The discoveries serve as a sad reminder of how much progress has yet to be made in the journey towards Reconciliation and how much healing has yet to happen.

It is a common folklore belief that mathematics, and more broadly science, is immune to what happens "out there in the world", and that the injustices faced by many in the population-at-large are experienced only by very few within what is surely a very welcoming and tolerant scientific community. It is erroneous beliefs such as this one that are part of the problem and part of the obstruction to positive change. The sooner we recognize that mathematics is a microcosm of the real world, the sooner we will be able to collectively take the steps needed to create the welcoming and tolerant community to which we aspire.

My writing that this belief is erroneous may not convince the casual reader who picks up this copy of the *CMS Notes*, at least depending on their lived experiences and how ingrained the philosophy of mathematics-as-utopia is in their consciousness. I encourage all readers to look beyond themselves and to ask themselves tough but simple questions about their professional environment — the questions that are easy to ask but whose answers may be challenging and indicative of uncomfortable truths. Let's start with: is your department diverse? Do you believe that every voice in your college or university is heard? Do you believe that every individual around you is rewarded in a fair and equitable way for their labour and expertise? Is the playing field level? In addition to asking these questions, I encourage readers to seek out the documentary Picture a Scientist, which details the horrifying injustices faced by women in the sciences. I applaud the Pacific Institute for the Mathematical Sciences (PIMS) and its Equity, Diversity, and Inclusivity Committee for making the documentary available to the community and holding a panel session in response to it on Women in Mathematics Day in May 2021.

Created a year ago, the CMS' Equity, Diversity, and Inclusivity Committee has been working to understand how to make mathematics in Canada more diverse and inclusive for all. The task is sweeping and finding a single place to start is

admittedly, a problem in and of itself. Take, just for instance, the CMS Meetings. The most recent occasion, in June 2021, was a celebration of the Society's 75(+1)-th anniversary. I was "there" — virtually, like everyone else — and I must say that it was an absolutely fantastic celebration. These meetings in general have become a staple of the Canadian mathematics calendar, occuring twice yearly. For many, these events are cherished: traveling to a quiet, picturesque campus in June or arriving at a hotel conference centre after braving the winter weather and impending December exams, and being rewarded either way with the opportunity to catch up with colleagues and friends from different corners of Canada and the world. But are these events inclusive? Is the cost of registration and travel prohibitive for students and non-tenure-track faculty? How about for attendees with accessibility needs, for whom travel may be very arduous if not impossible? Should future meetings be hybrid? Are the scientific sessions, which operate by direct invitation to speakers, diverse? When sessions occur repeatedly with no speakers at all from underrepresented groups, these questions have to be asked. Over the next few CMS meetings, you will see changes to a number of aspects of how the meetings operate, fueled by these observations and questions. For one, we will be experimenting with making it possible for any individual to apply to speak in scientific sessions. Please stay tuned.

> *It is a common folklore belief that mathematics, and more broadly science, is immune to what happens "out there in the world", and that the injustices faced by many in the population-at-large are experienced only by very few within what is surely a very welcoming and tolerant scientific community. The sooner we recognize that mathematics is a microcosm of the real world, the sooner we will be able to collectively take the steps needed to create the welcoming and tolerant community to which we aspire.*

Tweet

The meetings are just one example — one that arises without even approaching the much wider question of how mathematics departments operate, for better or for worse. We cannot be afraid to rethink how we do things, even those basic things that we perhaps consider immutable. We have to ask questions about all of the structures in play, many of which have been designed either consciously or unconsciously to favor the status quo and keep less powerful groups from achieving equity. We have to step outside the comfort of our own shoes and we have to listen. And then we have to act.

In order to facilitate this conversation, and to help channel ideas and questions into actions, a new column is being initiated here in the *CMS Notes*. This column, for which this article is the inaugural contribution, is titled ***MOSAIC***:

Mathematics, Outreach, Society, Accessibility, and Inclusiveness Column

This column is all about respect, honesty, learning, healing, welcoming, solving problems, acknowledging and listening to each other, moving forward, and building a stronger and richer Canadian mathematical community. You can expect to read about ongoing challenges on various fronts and proposed strategies for tackling them, successful examples of ally-ship, issues affecting students and early-career mathematicians, tips for work-life balance, advertisements and recaps of relevant events, and brainstorming around outreach.

The first few articles will come from the EDI Committee members, who are: Habiba Kadiri (Associate Professor, University of Lethbridge), Elana Kalashnikov (Assistant Professor, University of Waterloo), Karen Meagher (Professor, University of Regina and Chair, CMS Women in Mathematics Committee), Israel Ncube (Professor, Alabama A&M Univeristy), Monica Nevins (Professor, University of Ottawa and Member, CMS Executive Committee), and Reila Zheng (PhD Student, University of Toronto). We also plan to host various guest contributors as the series evolves.

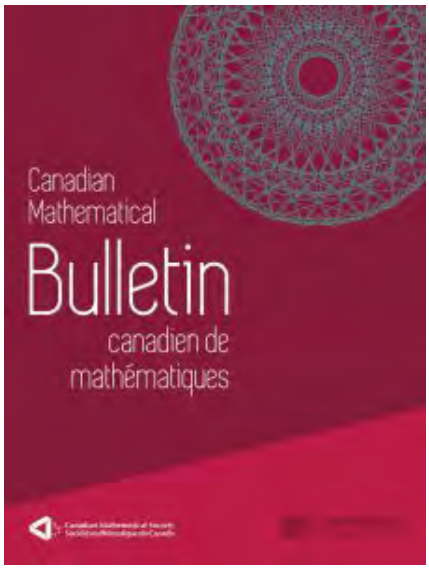
You too can be part of this series. If you have an article to propose, please do not hesitate to write to the EDI Committee at edic@cms.math.ca. We also welcome ideas for content to be featured in our Inclusive Mathematics project.

I would also like to acknowledge Denise Charron, Termeh Kousha, Zishad Lak, Yvette Roberts, Gosia Skrobutan, and Sarah Watson of the CMS for helping to facilitate and publicize EDI events and content across the CMS this past year.

So let's start the conversation: *ask*, *listen*, *act*. We can do this — together.

Steven Rayan (he/him) is the chair of CMS EDI Committee and Associate Professor at the Department of Mathematics and Statistics at University of Saskatchewan on Treaty 6 territory and the homeland of the Métis people.

2022 CJM/CMB Associate Editors



The Publications Committee of the CMS solicits nominations for Associate Editors for the *Canadian Journal of Mathematics* (CJM) and the *Canadian Mathematical Bulletin* (CMB). The appointment will be for five years beginning January 1, 2022. There are eight associate editors on the CJM/CMB Editorial Board whose mandates are ending at the end of December.

For over fifty years, the *Canadian Journal of Mathematics* (CJM) and the *Canadian Mathematical Bulletin* (CMB) have been the flagship research journals of the Society, devoted to publishing original research works of high standard. The CJM publishes longer papers with six issues per year and the CMB publishes shorter papers with four issues per year. CJM and CMB are supported by respective Editors-in-Chief and share a common Editorial Board.

Expressions of interest should include your curriculum vitae and your cover letter and sent electronically to: cjmcmb-ednom-2021@cms.math.ca **before September 15, 2021**.

The CMS Research Committee is inviting nominations for three prize lectureships. These prize lectureships are intended to recognize members of the Canadian mathematical community.

Coxeter-James Prize

The **Coxeter-James Prize** Lectureship recognizes young mathematicians who have made outstanding contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D. The selected candidate will deliver the prize lecture at the 2022 Winter Meeting.

Jeffery Williams Prize

The **Jeffery-Williams Prize** Lectureship recognizes mathematicians who have made outstanding and sustained contributions to mathematical research. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years. The prize lecture will be delivered at the 2022 Summer Meeting.

Krieger-Nelson Prize

The **Krieger-Nelson Prize** Lectureship recognizes outstanding research by a female mathematician. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years. The selected candidate will deliver the prize lecture at the 2022 Summer Meeting.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation. A candidate can be nominated for more than one research prize in the applicable categories; several candidates from the same institution can be nominated for the same research prize.

CMS research prizes are gender-neutral, except for the Krieger-Nelson prize, which is awarded to women only. Nominations of eligible women for the general research prizes in addition to the Krieger-Nelson Prize are strongly encouraged.

Nominations Requirements

The deadline for nominations, including at least three letters of reference, is **September 30, 2021**. Nomination letters should list the chosen referees and include a recent curriculum vitae for the nominee. Some arms-length referees are strongly encouraged. **New: the nominator most include a full citation of approximately 500 to 700 words.** Nominations and the reference letters from the chosen referees should be submitted electronically, preferably in PDF format, to the corresponding email address and **no later than September 30, 2021**:

Coxeter-James: cjprize@cms.math.ca

Jeffery-Williams: jwprize@cms.math.ca

Krieger-Nelson: knprize@cms.math.ca

2022 Cathleen Synge Morawetz Prize

Nominations are invited for the 2022 **Cathleen Synge Morawetz Prize** for an author(s) of an outstanding research publication. A series of closely related publications can be considered if they are clearly connected and focused on the same topic. The recipient(s) shall be a member of or have close ties to the Canadian mathematical community, and will receive a commemorative plaque.

The Cathleen Synge Morawetz Prize will be awarded according to the following 6-year rotation of subject areas:

1. Geometry and Topology *(2027, and every six years thereafter)*,
2. Combinatorics, Discrete mathematics, Logic and foundations, and Mathematical Aspects of Computer Science *(2022 , and every six years thereafter)*,
3. Applied mathematics, including but not limited to Numerical Analysis and Scientific Computing, Control Theory and Optimization, and Applications of Mathematics in Science and Technology *(2023, and every six years thereafter)*,
4. Probability and Mathematical Physics *(2024, and every six years thereafter)*,
5. Algebra, Number theory, Algebraic geometry (2025, and every six years thereafter),
6. Analysis and Dynamical systems *(2026, and every six years thereafter)*.

All of the above fields will be understood most broadly, to ensure that any outstanding publication can be considered under at least one of the categories. A paper (or a series of papers) which has significantly impacted more than one of the listed fields can be nominated more than once in the six-year rotation. The nomination must focus on a single topic, rather than a broad body of work by the nominee.

This call for nominations is for an author(s) of a publication or a series of closely related publications in the field of Combinatorics, Discrete Mathematics, Logic and Foundations, and Mathematical Aspects of Computer Science.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation.

The nomination letter should highlight the research paper(s) being nominated, providing evidence of its impact and significance. The nomination letter should list the chosen referees, and should include a recent curriculum vitae of the nominee(s), if available. Up to three reference letters in support of the nomination should be sent directly to the CMS. All documents should be submitted electronically, preferably in PDF format and **no later than September 30, 2021**, to **csmprize@cms.math.ca**.

About the Award

The prize was established in 2020 in honour of Cathleen Synge Morawetz (1923-2017), to reflect the remarkable breadth and influence of her research achievements in pure and applied mathematics. Professor Morawetz completed her undergraduate studies at the University of Toronto. She was encouraged to pursue a PhD in Mathematics by Cecilia Krieger (of Krieger-Nelson Prize). She went to MIT for a master's degree, and then got her PhD at NYU, where she would spend the bulk of her career, becoming the director of Courant Institute in 1984. Her main research contributions were in the field of partial differential equations. Cathleen Synge Morawetz was a recipient of the Jeffery-Williams Prize in 1984 (the only woman to win the Prize up to date), the National Medal of Science (1998), the Leroy P. Steele Prize for Lifetime Achievement (2004) and the George David Birkhoff Prize in Applied Mathematics (2006). Through its explicit rotation among subject areas, this prize highlights the enormous spectrum of research in the Canadian mathematical sciences community.

2022 David Borwein Distinguished Career Award

The Canadian Mathematical Society (CMS) invites nomination for the 2022 **David Borwein Distinguished Career Award**. This prize recognizes mathematicians who have made exceptional, broad, and continued contributions to Canadian mathematics and is awarded every four years.

The award presentation will take place at the CMS Winter Meeting in December 2022 and a plenary lecture given by the recipient.

A complete nomination dossier consists of:

- A signed nomination statement from a present or past colleague, or collaborator (no more than three pages) having direct knowledge of the nominee's contribution;
- A short curriculum vitae, no more than five pages;
- Two to four letters of support in addition to the nomination;
- Other supporting material may be submitted, no more than 10 pages.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues regardless of race, gender, ethnicity or sexual orientation.

The complete nomination and all documentation should be submitted electronically, preferably in PDF format, **no later than November 15, 2021**, to **dbaward@cms.math.ca**.

2018 David Borwein Distinguished Career Award Recipient

Anthony To-Ming Lau
University of Alberta

Prof. Lau is the most recent recipient of the award. Please read the Media Release. For a list of past recipients and to read their citations, please visit the official David Borwein Distinguished Award page.

2022 Excellence in Teaching Award

The CMS Excellence in Teaching Award Selection Committee invites nominations for the **2022 Excellence in Teaching Award**.

The **Excellence in Teaching Award** focuses on the recipient's proven excellence as a teacher at the undergraduate level, including at universities, colleges and cégeps, as exemplified by unusual effectiveness in the classroom and/or commitment and dedication to teaching and to students. The dossier should provide evidence of the effectiveness and impact of the nominee's teaching. The prize recognizes sustained and distinguished contributions in teaching at the post-secondary undergraduate level at a Canadian institution. Only full-time teachers or professors who have been at their institution for at least five years will be considered. The nomination will remain active for three years, with a possibility to update.

The CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues regardless of race, gender, ethnicity or sexual orientation.

A nomination will consist of:

- a signed nominating statement from a present or past colleague, or collaborator (no more than three pages) having direct knowledge of the nominee's contribution;
- a curriculum vitae (maximum five pages);
- three letters of support, at least one from a former student (who has followed a course more than a year ago) and one from the chair of the nominee's unit. The letter of the Chair of the nominee's unit could include a one-page summary on information from student evaluations, or similar information;
- other supporting material (maximum 10 pages).

Nominations and reference letters should be submitted electronically, preferably in PDF format, to: **etaward@cms.math.ca** no later than the deadline of **November 15, 2021**.

2021 Excellence in Teaching Award Recipient

Alfonso Gracia-Saz
University of Toronto

Dr. Gracia-Saz is the most recent recipient of the award. For a list of past recipients and to read their citations, please visit the official Excellence in Teaching Award page.

2021 Endowment Grants

The Canadian Mathematical Society is pleased to announce the ***2021 Endowment Grants Competition***. The CMS Endowment Grants fund projects that contribute to the broader good of the mathematical community. Projects funded by the Endowment Grants must be consistent with the interests of the CMS: to promote the advancement, discovery, learning and application of mathematics.

An applicant may be involved in only one proposal per competition as a principal applicant. Proposals must come from CMS members, or, if joint, at least one principal applicant must be a CMS member.

The deadline for applications is **September 30, 2021.** Successful applicants will be informed in January 2022 and the grants issued in February 2022.

Further details about the endowment grants and the **application process are available on the CMS website here**.

The Endowment Grants Committee (EGC) administers the distribution of the grants and adjudicates proposals for projects. The EGC welcomes questions or suggestions you may have on the program. Please contact the Committee by e-mail at chair-egc@cms.math.ca.

2022 CMS Math Competition Grants

The CMS is now accepting applications for the ***2022 CMS Math Competition Grants*** program. The CMS supports activities that promote the learning of mathematics among Canadian youth. In addition to the Society's math competitions, the CMS offers math competition grants for activities at the elementary and secondary school levels.

The deadline for submissions is **November 15, 2021**. Successful applicants will be informed in January 2022 and the grants issued in February 2022.

Further details and guidelines about the math competitions grants and the **application process are available on the CMS website here**.

The Committee on Grants for Provincial Competitions (CGPC) adjudicates proposals for support. Should you have further questions or comments, please contact the Committee by e-mail at **chair-grants-pc@cms.math.ca**

Applications should be submitted electronically using the online application form and additional documents preferably in PDF format, **no later than November 15, 2021** to **mathgrants@cms.math.ca**.

CMS 75th+1 Anniversary Summer Meeting, Online

Sarah Watson

Meetings Manager

Over 1000 mathematicians were welcomed to the online platform for the 2021 CMS 75th+1 Anniversary Summer Meeting from June 7-11. This was the third online meeting the CMS has hosted. Participants attended over 40 scientific sessions; five plenary lectures; three prize lectures and one public lecture over the course of the meeting. The plenary lecturers were Henri Darmon (McGill University; Moon Duchin (Tufts University); Matilde Marcolli (University of Toronto); Aaron Naber (Northwestern University); and Ian Putnam (University of Victoria).

This summer the CMS continued to offer three-hour mini courses on Friday June 4. There were eight mini courses in diverse areas of mathematics including a student mini course on Career Diversity in mathematics.

The official conference opened on Monday morning with an opening and welcome given by CMS President, Javad Mashreghi followed by a plenary lecture by Ian Putnam.

The CMS Student Committee (STUDC) hosted a student social giving students a chance to network and interact in a non-academic setting in a relaxed atmosphere buoyed by icebreaker activities. Mathematically themed games were also played.

At the end of Wednesday, CMS hosted a well attended Public Lecture featuring Anne Broadbent (University of Ottawa) who gave a talk titled *The apple of my i*.

The meeting was bookended by two related events with integrated registration.iThe first was the Canadian Women in Mathematics Across Canada (CWiMAC) meeting from June 2-3. The purpose of the CWiMAC workshops is to support the career development of junior female-identified academics in the Canadian mathematics community.

The second was the annual CMESG meeting from June 11th to June 13th. The 2021 meeting included special joint sessions with the CMS as well as GDM (Groupe de didactique des mathématiques du Québec). Additionally, CMESG co-hosted a panel on June 7, 2021 that focused on Data Literacy in light of world events of the past year.

During the 2021 Summer Meeting attendees were able to network with peers and like-minded individuals on the online community boards and in our networking sessions.

The Women in Mathematics committee invited all participants to participate in a discussion on balancing mathematics and life in COVID times during one of the breaks. Additionally, there was a break on mathematics and film moderated by Brett Stevens (Carleton).

There was not an awards banquet at the 2021 75th anniversary meeting, but the CMS recognized our 2021 prize winners during the opening remarks and prize winners also prepared a lecture. Those recipients include: Joel Kamnitzer (University of Toronto) recipient of the Jeffery-Williams Prize who gave a lecture on *Perfect bases in the representation theory of semisimple Lie algebras* and Anita Layton (University of Waterloo) recipient of the Krieger-Nelson Prize who gave a prize lecture on *His or Her Mathematical Models – Understanding Sex Differences in Physiology*. This year, the Excellence in Teaching Prize was awarded to Alfonso Gracia-Saz. For the last 7 years, Alfonso Gracia-Saz worked tirelessly in [MN1] [SW2] his course Calculus with Proofs (called "Calculus!"). had a huge impact on students, undergraduate and graduate, and professors. This talk was prepared by his colleague Bernardo Galvao-Sousa (Toronto) in memory of Alfonso who passed away from COVID-19. In the talk Bernardo guided the audience through the story of Calculus! and Alfonso's development as an educator.

The AARMS – CMS Student Poster Awards were also presented during the online event. The winners were AARMS Prize: Yuying Li (Western University) with *Modelling and pricing cyber security risk*; CMS President's Prize: Axel Turnquist (New Jersey Institute of Technology) with *Optimal transport on the sphere*; and CMS Student Committee Prize: Stéphanie Abo (University of Waterloo) with *Modelling the circadian regulation of the immune system: sexually dimorphic effects of shift work*.

Putting on such a meeting requires much dedication and hard work and would not be possible without the efforts of the Scientific Organizing Committee, the Scientific Directors, the Session Organizers, and the CMS staff. Ailana Fraser (University of British Columbia), Monica Nevins and Mateja Šajna (University of Ottawa) the Scientific Directors, worked tirelessly on a large scientific program originally for 2020 and then for 2021 when the meeting was postponed and moved online[MN3] . Their dedication and flexibility helped to make this 75th +1 anniversary meeting one to remember.

The CMS would like to acknowledge financial support from the University of Ottawa, MITACS, PIMS, FIELDS, CRM, AARMS, MapleSoft, and Bolster Academy.

The Canadian Mathematical Society (CMS) welcomes and invites session proposals and mini-course proposals for the 2021 CMS Winter Meeting held online on **December 3-7**.

Call for Sessions

Proposals should include

1. names, affiliations, and contact information for two (or more) session co-organizers,
2. a title and brief description of the focus and purpose of the session,
3. a preliminary list of potential speakers with their affiliations and if they have agreed to participate, along with a total number of expected speakers.

Sessions will take place December 4, 5, and 6. Sessions will be advertised in the *CMS Notes*, on the website and in the *AMS Notices*. Speakers will be requested to submit abstracts, which will be published on the website and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific directors and copy the CMS office. Potential organizers are encouraged to consider diversity in their selection of session invitees.

Proposals should be submitted by **September 30, 2021.**

Call for Mini-Courses

The CMS is organizing three-hour mini-courses to add more value to meetings and make them attractive for students and researchers to attend. The Mini-courses will be held on Friday afternoon, December 3rd, before the public lecture, and include topics suitable for graduate students, postdocs and other interested parties.

Proposals should include names, affiliations, and contact information for all the mini-course organizers and title and brief description of the focus of the mini-course.

Scientific Directors:

Nils Bruin (Simon Fraser University) nbruin@sfu.ca
Nilima Nigam (Simon Fraser University) nigam@math.sfu.ca

CMS Office: meetings@cms.math.ca

June 3-6, 2022, St. John's, Newfoundland

The Canadian Mathematical Society (CMS) welcomes and invites session proposals and mini course proposals for the 2022 CMS Summer Meeting in St. John's from **June 3-6, 2022**.

CALL FOR SESSIONS

Proposals should include (1) names, affiliations, and contact information for all session co-organizers, (2) title and brief description of the focus and purpose of the session, (3) a preliminary list of potential speakers, with their affiliations and if they have agreed to participate, along with a total number of expected speakers.

Sessions will take place June 4-6. Sessions will be advertised in the CMS Notes, on the web site and in the AMS Notices. Speakers will be requested to submit abstracts, which will be published on the web site and in the meeting program. Those wishing to organize a session should send a proposal to the Scientific Directors and copy the CMS office. Those submitting proposals are encouraged to pay attention to the diversity of both the session invitees and the proposed session organizers.

Proposals should be submitted by **January 31, 2022.**

CALL FOR MINI COURSES

The CMS is organizing three-hour mini-courses to add more value to meetings and make them attractive for students and researchers to attend.

The mini-courses will be held on Friday, June 3, before the public lecture, and include topics suitable for graduate students, postdocs and other interested parties.

Proposals should include names, affiliations, and contact information for all the mini course co-organizers and title and brief description of the focus of the mini course

Scientific Directors:

Chunhua Ou (Memorial)
Marco Merkli (Memorial)

CMS Office:

meetings@cms.math.ca

SAVE THE DATE

Canadian Mathematical Society
Société mathématique du Canada

WINTER 2021

Online Meeting - December 2-7

SUMMER 2022

St. John's, Newfoundland - Memorial University - June 3-7

WINTER 2022

Toronto, Ontario - December 2-5

SUMMER 2023

Halifax, Nova Scotia - June 2-5

WINTER 2023

Montreal, Quebec - Hilton Double Tree - December 1-4

SUMMER 2024

Saskatoon, Saskatchewan, University of Saskatoon - June 2-5

CMS MEETINGS

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Canadian Mathematical Society
Société mathématique du Canada

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CMS Notes

The editors welcome articles, letters and announcements. Indicate the section chosen for your article, and send it to *CMS Notes* at the appropriate email address indicated above.

No responsibility for the views expressed by authors is assumed by the *CMS Notes*, the editors or the CMS.

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