Welcome to the June Issue of CMS Notes

Issue Contents

June 2022 : Vol. 54, No. 3

CMS Activities and Engagement

Cover Article June 2022 (Vol. 54, No. 3)

Dr. David Pike (Memorial University)
President of the CMS

I want to begin this note by saying what a delight it was to attend the CMS Summer Meeting in St. John's at the beginning of June. This was the society's first in-person gathering in over two years, and despite a few hiccoughs (such as a shipment of conference materials being lost by a courier) the conference overall was a great success. It was especially heartwarming to see people engaging in discussion and collaboration, the kind that happens when people come together and are able to freely chat and share ideas. For many student attendees, it was their first chance to experience the joy and benefit of such networking and to have the opportunity of serendipitous interaction.

The Society held its Annual General Meeting during this conference, at which time I transitioned to being President, having spent the previous twelve months as President-Elect.[1] In these two capacities I have been (and still am) gaining a much better awareness of the many facets of the society and its operations. Some of these I was only peripherally aware of, and as I suspect this is the case for others as well, I want to let you know about them.

One key observation that I will circle back to near the end is the profound impact that volunteers have within the CMS. At our headquarters in Ottawa2 we have a small group of paid staff who fill some of the core roles needed to maintain the society's functionality. The remainder of what the CMS does is largely dependent on the contributions of people who generously volunteer their time and expertise. At the governance level, the society has a President, five Vice-Presidents, a Treasurer and 19 members on its Board of Directors, all of whom serve without compensation. Moreover, the society has several standing committees, which again are comprised of people who donate their time and knowledge. The main committee list is as follows

- Education
- Endowment Grants
- Equity, Diversity and Inclusiveness
- Finance

Shortly before the pandemic struck, we set out to find and purchase a suitable building in Ottawa for our new headquarters. Thinking that we could accomplish this fairly quickly, we left the space that we had previously been renting. As a result, our office staff have had to work remotely from their homes, not because of the pandemic, but because we have not had office space in which to employ them.[2] We owe them a huge debt of gratitude for the work they have done under these difficult and prolonged circumstances.

- Fellows Selection
- International Affairs
- Invested Funds
- Mathematical Competitions
- Nominating
- Publications
- Reconciliation in Mathematics
- Research
- Student
- Women in Mathematics
- Distinguished Awards Selection

Additionally, two new CMS committees are in the process of being established (for Human Rights of Mathematicians, and International Prize Nominations). There are also several editorial boards for the society's various journals and other publications, although in this note I will focus on the society's committees.

Some committees (and their subcommittees) generally act behind the scenes and people may not be aware of their function or impact. For example, the Nominating Committee plays a key role in identifying and recommending people to serve on many of these committees. The Finance Committee and Invested Funds Committee play critical roles in maintaining the society's financial future by developing budgets, suggesting fees, and overseeing our financial assets. Frequently in the background are the Scientific Organizing Committees that are formed for each of the society's semi-annual conferences, and whose efforts lay important foundations for our conferences to succeed.

Among the society's standing committees, the Student Committee is unique in that it is primarily made up of students, whereas most other committees are predominantly populated by faculty members. The Student Committee is very active. Among other things, it oversees the poster competitions that are held at regular CMS meetings, along with hosting workshops and student socials at these meetings. The Student Committee also organizes the annual Canadian Undergraduate Mathematics Conference, the next instance of which will take place in July at l'Université Laval in Québec.

The Education Committee does many things, as evidenced by its multitude of subcommittees. Among its more visible roles is that of overseeing the CMS Math Camps Program which supports summer camps that offer mathematical introductions and enrichments to students. Less obvious to the casual observer but nevertheless significant in terms of promoting the beauty and importance of mathematics, particularly to the upcoming generation, is that the committee provides grant funding in support of provincial competitions.

The Endowments Grant Committee supports a myriad of initiatives that contribute to the greater good of the Canadian mathematical community. Although the available funding is modest, the impact that it has is wide and diverse. Projects that have been supported include focussed professional development workshops, outreach activities, and other novel initiatives. There is an annual competition; the current call for proposals has a deadline of September 30th.

Conducting mathematical competitions and supporting students who represent Canada at international competitions are CMS activities that serve to promote mathematical awareness and appreciation, as well as giving students the opportunity to shine and do us all proud. These activities fall within the purview of the CMS Mathematical Competitions Committee. It runs four annual competitions: the Canada Jay Mathematical Competition, the Canadian Open Mathematics Challenge, the Canadian Junior Mathematical Olympiad and the Canadian Mathematical Olympiad. In 2021, over 8000 students participated in these competitions. The CJMC is relatively new (it was first held in 2020) and is open to students in grades K-8. Performance in the COMC is used to help select which students are invited to participate in the CMO as well as to select students who will represent Canada at the International Mathematical Olympiad and the European Girls' Mathematical Olympiad. The Mathematical Competitions Committee also conducts training camps to help prepare our teams for international competitions.

I don't want to tire readers with an exhaustive list of the activities of every committee. Instead, by highlighting a selection, my hope is that I have increased awareness of some of the things that our society does and supports. And I want to reiterate that these activities are primarily driven by volunteers and a collective joy of mathematics.

Our society simply could not function without these volunteer efforts, and I want to emphatically offer thanks and praise to everybody who contributes towards our common goals and vision. No matter how big or small the effort, we are all pulling together as a team and as a family.

If anybody is interested in reading more about our committees, editorial boards, and what they do, let me suggest taking a look at the comments given in some of the society's online Annual Reports. Full statements of mandates and terms and reference for standing committees are also available on our website. If you know of somebody (possibly even yourself) who might want to get involved in some way, then there is a Volunteer Application form on our website that can be filled out.

On a closing note, I also want to point out that many of the society's activities benefit greatly from financial sponsors, to whom I also express our collective gratitude. The CMS has been a registered Canadian charity since 1967 and as such we are able to provide income tax receipts for donations that we receive. We welcome contributions that help us to carry on our mission of promoting mathematics.

[1] Let me pause to express my sincere thanks to Javad Mashreghi, who has now transitioned from being President to being Past-President. The COVID pandemic presented unprecedented challenges during Javad's presidency. Javad led us through this storm, during which the society remained active and ac- complished many things. One particular legacy of Javad is the purchase of a new home for the society's headquarters in Ottawa (about which more news should be coming soon). Javad, you have inspired us, and I hope to live up to your example.

[2] The society has actually been operating for about two years now without a physical office location.

Alchemy

CMS NOTES

June 2022 (Vol. 54, No. 3)

Robert Dawson (Saint Mary's University)
Editor-in-Chief

A little while ago I was reading a book about the history of alchemy. What I found most surprising was how single-minded the alchemists were: for centuries, almost all their research efforts were dedicated to one or two goals: transmutation of base metals into gold or silver usually via the Philosopher's Stone, which could also grant immortality. Some useful techniques (notably the distillation of alcohol) were developed along the way, but overall alchemy had few spinoffs, given the labor and time involved. One major impediment was undoubtedly the backward-looking nature of the alchemical tradition: most alchemists believed that they were rediscovering an already-known secret. The vague outlines of this secret were generally agreed upon: there was little point looking elsewhere!

After a while, I got to wondering about what the corresponding episode, if any, in the history of mathematics might be. Astronomy spent a long time held back by the Ptolemaic system; biology was impeded by the Four Humors and various other theories. Aristotle's views on mechanics did little to aid the progress of physics. What of our own subject?

If you look back far enough, math simply wasn't studied much. When did people start? It's hard to tell – it's probable that the vast majority of Babylonian mathematical writings have been lost forever. We do know, however, that they had some idea of trigonometry and algebra, and applied it to astronomy. As far as I can find out, we don't know of anything that they (as a culture) got wrong mathematically.

And then (runs the oversimplified popular account) along came Pythagoras, around 500 BCE, and suddenly we had number theory and geometry. Well, it wasn't quite that simple: but there's little evidence that the splash of mysticism that the Pythagoreans flavored their math with actually did much harm. Calling odd numbers "male" and even numbers "female" seems unproductive and pointless, but it doesn't seem to have got in the way of their math. Euclid, working around 300 BCE, had the occasional lapse in rigor, but his results, interpreted with a little charity, are almost unfailingly correct.

So has mathematics ever gone off the rails? One might think of the "problems of antiquity" – trisecting the angle, duplicating the cube, and squaring the circle using compass and straightedge. These were once considered respectable but difficult, much as we consider the Riemann conjecture or the "P=NP" problem. Over the years, the suspicion grew that they were actually impossible, and that those attempting to solve them were wasting their time. In 1837 Wantzel proved that this was indeed the case. Needless to say, that didn't stop the trisectors! A good analogy in the natural sciences might be the dream of building a perpetual motion machine, which went through a similar evolution from plausible idea, to eccentricity, to "outsider science" attempted only by those unable to understand why it couldn't be done. It's worth noting that neither perpetual motion nor the "problems of antiquity" ever dominated the research efforts of their respective communities.

The point, of course, is not that early mathematicians were less error-prone than their counterparts in other fields. Rather, they were (and we are) fortunate in that mathematics only has to be self-consistent; this is much easier than being consistent with reality.

Arabic arithmetic in context: al-Hawārī's Essential Commentary

CSHPM Notes June 2022 (Vol. 54, No. 3)

Jeffrey Oaks (University of Indianapolis)

Jeff Oaks received his PhD in mathematics from the University of Rochester in 1991. Since 1992 he has been a professor in the math department at the University of Indianapolis, and in 1999 he abandoned differential geometry to take up history of mathematics. His translation with conceptual, historical, and mathematical commentary of the Arithmetica *of Diophantus, coauthored with Jean Christianidis, will be published later this year by Routledge.*

CSHPM Notes bring scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *Independent Scholar (aackerbe@verizon.net)*
Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

In 1999, eight years after finishing my Ph.D. in mathematics, I was looking to change the direction of my research. Having the freedom afforded by a teaching institution to make that change truly drastic, I returned to an interest in medieval Arabic mathematics that I had developed during an undergraduate course in History of Science. Given that I nearly majored in history in college and that I had recently finished writing a 560-page book on the history of railroad tie preservation (no room to explain that here!), history of mathematics was a natural choice. My first step in this direction was to build a website that grew to list over 1,300 books and articles on Arabic mathematics, arranged by topic, that had been published since about 1950 [5]. Then, with the secondary literature under control and without really knowing where it would lead, I turned my attention to algebra. I began with al-Khwārazmī's early-ninth-century *Book of algebra* (*Kitāb al-jabr wa l-muqābala*), the earliest extant Arabic book on the topic, but because I could not read Arabic I had to work from Latin and English translations [2; 3]. I soon discovered a couple of interesting features that no one had written about, so to really understand the matter I enlisted my Palestinian colleague Haitham Alkhateeb to teach me Arabic. With his help, and with what seemed to be unwarranted determination, I was soon able to read the texts myself.

At first, I naively approached the algebra as if it were modern algebra written in Arabic prose. But observing the ways that certain operations are worded and taking into account the overall procedures of many solutions, I discovered almost immediately that such a reading is untenable. Those curious yet consistent differences between Arabic algebra and the algebra we practice today turned out to be signals of a radically different way of understanding monomials, polynomials, and equations. And as I learned only later, the algebraic expressions are themselves grounded in the practical Arabic understanding of 'number'.

With this approach of paying careful attention to the wording of the texts, and with sensitivity to the Arabic authors' potentially different ways of conceiving of mathematical objects, I became a specialist in history of mathematics. I have since published over twenty research articles, not just on Arabic mathematics but also on Greek mathematics in one direction and Medieval, Renaissance, and early modern European mathematics in the other. And just recently, Mahdi Abdeljaouad and I have published the book *Al-Hawārī's Essential Commentary: Arabic Arithmetic in the Fourteenth Century*, in which we present an edition, translation, and commentary of a fourteenth-century Arabic arithmetic textbook. This is the latest installment of Edition Open Sources, a collaborative venture between the Max Planck Institute for the History of Science in Berlin and the University of Oklahoma. As with all EOS books, ours is free to read online or download [1]. It is this book that I now describe.

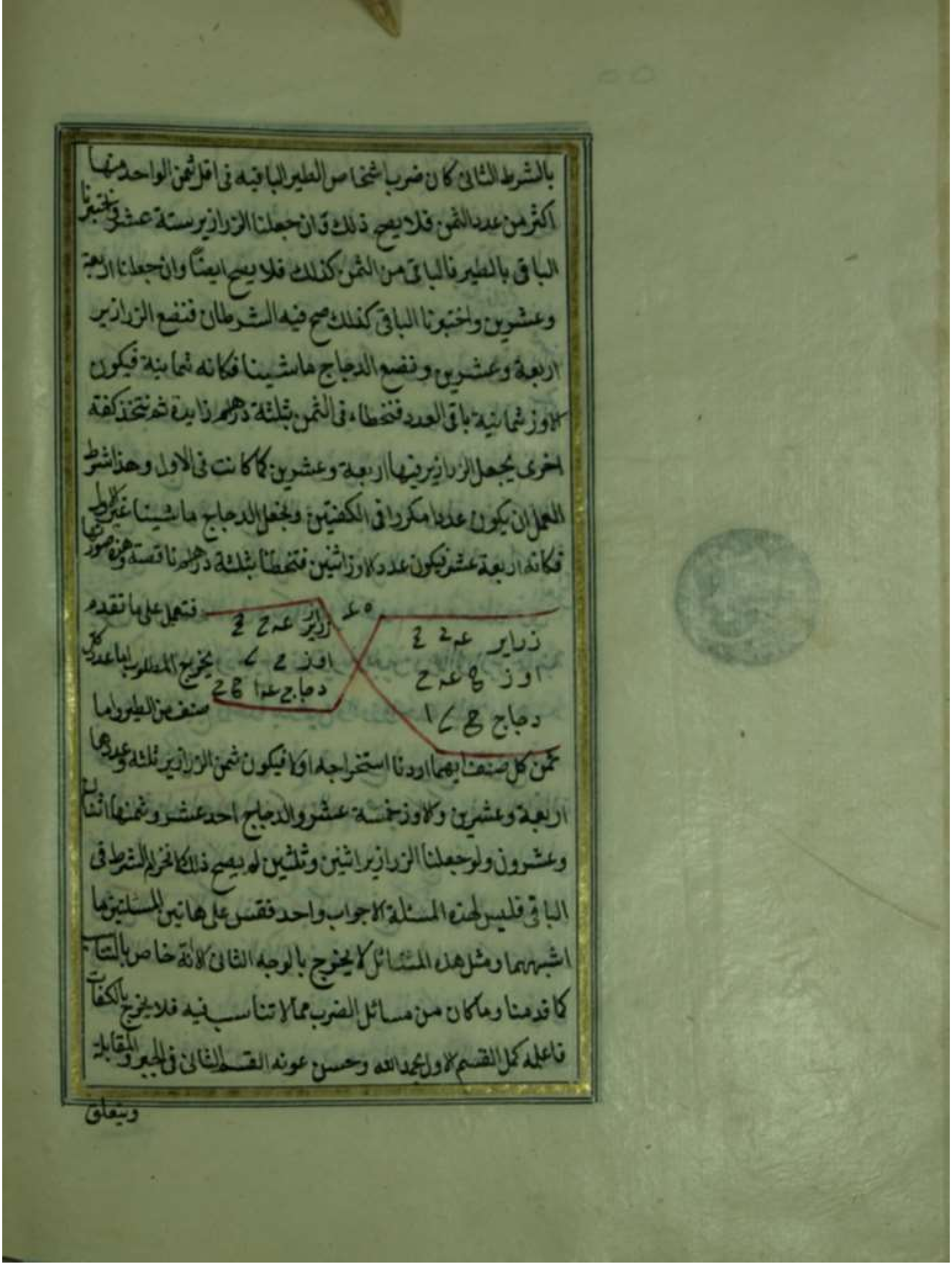
Abd al- ʻAzī z al-Hawārī was a student of the polymath Ibn al-Bannāʻ in Marrakesh in the early years of the fourteenth century CE. Ibn al-Bannāʻ was lecturing on his *Condensed [Book] on the Operations of Arithmetic* (*Talkhī s a ʻmāl al-hisāb*), a book *so* condensed that it included not one numerical example to illustrate the rules. While still attending lectures, al-Hawārī began writing a commentary on his teacher's book with the main goal of providing those numerical examples. He completed his *Essential Commentary on the Condensed [Book] on the Operations of Arithmetic* (*al-Lubāb fī sharh Talkhī s a ʻmāl al-hisāb*) in 1305 CE.

Al-Hawārī followed Ibn al-Bannā "s book chapter by chapter, beginning with a description of different kinds of numbers and how to write them with Indian numerals. (We call them "Arabic numerals" because Europeans learned them from Arabic sources.) He continued with operations on whole numbers, operations on fractions, square root calculations, problem-solving via proportion, double false position, and algebra, and finally a short section on finding secret numbers. Like many medieval Arabic textbooks, this one is a splendid hybrid, combining the rules of Indian calculation with techniques of Middle Eastern finger-reckoning, and mixed with Greek number theory. To add to this complexity, Ibn al-Bannāʻ had copied many passages word-for-word from earlier textbooks, and al-Hawārī included many passages from his teacher's own commentary, titled *Lifting the Veil from the Faces of the Operations of Arithmetic* (*Rafʻ al-hijāb ʻan wujūh a ʻmāl al-hisāb*) [4].

Our book begins with a 28-page general introduction to Arabic arithmetic, something that has been lacking in the literature. There we explain the collisions between the arithmetics of earlier cultures brought together in Arabic texts as well as the roles played by algebra and other arithmetical problem-solving methods. A literal English translation of al-Hawārī's book comes next, followed by a comprehensive mathematical, conceptual, linguistic, and historical commentary that is longer than the translation. Because our book is meant to be read on a screen, we include links throughout, so that, for instance, one can click between the translation and the commentary. Following the commentary we give appendices, including a conspectus of problems, translations of some worked-out problems from other books, a chronological list of mathematicians and other scholars, and a glossary of Arabic terms with links back to the translation and the commentary. This is followed by a bibliography and an index of people that is also linked back to the text. The Arabic edition of al-Hawārī's book comes at the end, with its own introduction. If the book were printed in physical form, the Arabic portion would come first for Arabic readers, since Arabic is written right-to-left.

Al-Hawārī was not a brilliant mathematician with innovative ideas and theoretical insights. His aim was simply to provide students with a clear and comprehensive guide to the practical arithmetic of his time. He offers no proofs or philosophical discussions, just explanations and examples of the rules. The book must have enjoyed a modest success, since at least fourteen manuscripts are extant.

There is much to be gained from reading textbooks like this one, beginning with the window they provide into contemporary forms of mathematical practice. Not only are we treated to different techniques for performing calculations and solving problems, but the way the procedures are presented testifies to the different role that books and writing played in a predominantly oral culture. Ibn al-Bannāʾ was following standard practice by reciting his book aloud to his students, and those students would have been expected to memorize its contents. Calculations were worked out in notation on a dust-board or other ephemeral surface, and if one wished to include a calculation in a book, then a rhetorical version was composed. The notation used in working out the problems is shown in books only as figures illustrating what should be written on the board. It is because some modern writers have been unaware of dust-board calculations that they believe that al-Khwārazmī and other algebraists worked out their problems verbally!



بالشرط الثاني كان ضرب اشخاص الطير الباقية في اقل ثمن الواحد منها
اكثر من عدد الثمن فلا يصح ذلك وان جعلنا الزرازير ستة عشر واخبرنا
الباقي بالطير فالباقي من الثمن كذلك فلا يصح ايضا وان جعلنا الاربعة
وعشرين واخبرنا الباقي كذلك صح فيه الشرطان فنضع الزرازير
اربعة وعشرين ونضع الدجاج ما شئنا فكانه ثمانية فيكون
الاوز ثمانية باقي العدد فتحط ما في الثمن بثلثة دراهم زايدة ثم نتخذ كفة
اخرى يجعل الزرازير فيها اربعة وعشرين كما كانت في الاول وهذا شرط
العمل ان يكون عددا مكررا في الكفتين ونجعل الدجاج ما شئنا غير الاول
فكانه اربعة عشر فيكون عدد الاوز اثنين فنحط بثلثة دراهم ناقصة وهذه صورة
فنعمل على ما تقدم يخرج المطلوب لما عدد كل صنف من الطير واما
ثمن كل صنف ايهما اردنا استخراجه اولا فيكون ثمن الزرازير ثلثة وعددها
اربعة وعشرين والاوز خمسة عشر والدجاج احد عشر وثمنها اثنان
وعشرون ولو جعلنا الزرازير اثنين وثلثين لم يصح ذلك لانخرام الشرط في
الباقي فليس لهذه المسئلة الا جواب واحد فقس على هاتين المسئلتين ما
اشبههما ومثل هذه المسائل لا يخرج بالوجه الثاني لانه خاص بالتناسب
كما قدمنا وما كان من مسائل الضرب مما لا تناسب فيه فلا يخرج بالكفات
ناعليه كمل القسم الاول بحمد الله وحسن عونه القسم الثاني في الجبر والمقابلة
ويتعلق

Figure 1. A sample page from the Medina manuscript of al-Hawārī's book, copied in 1345 CE.

But there is more to learn from elementary textbooks than merely "how they did things then". For the case of medieval Arabic arithmetic, these books can be particularly valuable for what their explanations, wording, and methods reveal about how differently people conceived of numbers and algebraic expressions, which in turn explains their seemingly curious procedures. Numbers today are regarded as existing independently of whatever units they may count or measure, but this was not the case in premodern arithmetic. Numbers in al-Hawārī and other Arabic authors on practical arithmetic are always numbers of some divisible unit, whether material or intelligible. Examples of material units include horses, bricks, parasangs (a unit of length), dirhams (a denomination of coin), hours, and mithqals (a unit of weight), while intelligible units were measured in generic "units", which were often labeled "in number", or again, as "dirhams". Numbers could be any positive amount, including fractions and irrational roots, which makes them incompatible with the multitudes composed of indivisible, intelligible units, as in Book VII of Euclid's *Elements*. Even the foundation of the numbers of Arabic practitioners is different from both Euclid's and our own: they are validated through practice, not via philosophical definitions or axioms. It is on this practical foundation that Arabic algebraic expressions were conceived, which are again radically different from their modern counterparts. Very briefly, Arabic polynomials are not built from arithmetical operations like ours, but were simply aggregations of the powers of the unknown.

This background on the nature of premodern numbers and expressions should then be taken into account in evaluating the contributions of more original authors like al-Karajī, al-Samaw ʻal, al-Khayyam, al-Fārisī, and al-Kāshī, who all developed their ideas on arithmetic and algebra from the practical tradition. Medieval Europeans, too, learned their computational methods directly from this tradition, so that the foundation one gains from studying the Arabic books applies equally to the underlying concepts at play in earlier figures such as Fibonacci, Jean de Murs, and Luca Pacioli, as well as later authors writing throughout the sixteenth century. To understand why Michael Stifel (1544) avoided irrational coefficients in algebra, such as multiplying-in our notation- $\sqrt{6}$ by x to get $\sqrt{6x^2}$, or why even François Viète (d. 1603) would not write multiple roots, insisting on forms like $\sqrt{3888}$ and $\sqrt{\frac{45}{64}}$ instead of $36\sqrt{3}$ and $\frac{3}{8}\sqrt{5}$, one should return to the Arabic sources.

Al-Hawārī reproduces the whole of Ibn al-Bannā"s book. He cites a passage, then gives his examples and comments, and then repeats the pattern with the next passage. We put Ibn al-Bannā"s passages in bold font in both the edition and translation. Al-Hawārī also quoted from Ibn al-Bannā"s own commentary, and we distinguish these passages by rendering them in an all-caps font in the translation. We also indicate in footnotes which passages Ibn al-Bannā' borrowed from other authors.

Our commentary follows the medieval paradigm. We remark on al-Hawārī's book passage-by-passage, with occasional digressions to expand on broader aspects of the mathematics. In addition to explaining the conceptual issues mentioned above, we illustrate in detail how the different rules of calculation function, sometimes bringing in examples from other books. We also identify the origins of various definitions, techniques, and ideas, whether from Euclid, Nicomachus, the finger-reckoning tradition, another Arabic author, or some other source. With all that is going on in al-Hawārī's seemingly mundane textbook, we hope that others will find it as fascinating as we have.

References

[1] Abdeljaouad, Mahdi, and Jeffrey Oaks (2021) *Al-Hawārī's Essential Commentary: Arabic Arithmetic in the Fourteenth Century.* Berlin: Max Planck Institute for the History of Science. **https://edition-open-sources.org/sources/14/**.

[2] Al-Khwārazmī. (1831) *The Algebra of Mohammed ben Musa*. Translated and edited by Frederic Rosen. London: Nachdruck der Ausgabe.

[3] Hughes, Barnabas, ed. (1986) Gerard of Cremona's Translation of al-Khwārizmī's *Al-Jabr*: A Critical Edition. *Mediaeval Studies* 48, 211-263.

[4] Ibn al-Bannā'55. (1994) *Raf' al-hijāb'an wujūh a'māl al-hisāb*. Edited by Muhammad Aballagh. Fās: Jāmi'at Sīdī Muhammad ibn 'Abd Allāh.

[5] Oaks, Jeffrey (2006) Bibliography of the Mathematical Sciences in the Medieval Islamic World (by Topic). University of Indianapolis. **https://uindy.edu/cas/mathematics/oaks/biblio/**. By now this bibliography is very out of date.

Teaching with Computer-Based Proof Assistants: Perspectives from Instructors of Mathematics

Education Notes June 2022 (Vol. 54, No. 3)

Gila Hanna (OISE, University of Toronto)

Xiaoheng Yan (OISE, University of Toronto)

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

John McLoughlin, *University of New Brunswick (johngm@unb.ca)*
Kseniya Garaschuk, *University of Fraser Valley (kseniya.garaschuk@ufv.ca)*

The authors are engaged in a project to investigate the potential benefits of teaching proof to undergraduate students with the help of a computer-based proof assistant. A point of departure was the observation that such proof assistants do not play a role in the undergraduate mathematics curriculum in North America that is in any way commensurate with their increasingly important role in mathematical practice. Our hypothesis is that a proof assistant could be a useful additional tool in teaching undergraduate mathematics, with the potential to foster mathematical understanding and in particular students' competence in proving (Avigad, 2019; Buzzard, 2020; Hanna & Yan, 2021; Thoma & Iannone, 2021). In preparation for our study, we had looked at a number of proof assistants, such as Coq, HOL, and Lean, recently acknowledged as having an important role in mathematics research (Castelvecchi, 2021).[1]

Here we report that there are a number of educators who have already chosen to include the use of a proof assistant in their mathematics courses. We contacted nine of them, to ask for any insights and suggestions stemming from their teaching experience (see Table 1). In what follows, we set out a summary of their responses to our five questions about teaching with the help of a computer-based proof assistant. The specific proof assistant these teachers had all been using is Lean, an interactive theorem prover (ITP) that provides instantaneous feedback, and allows exploration. Lean has a very active and friendly community of users who focus on mathematics rather than on computer science and software engineering. (See *Notes* following the references for further insight and relevant resources concerning Lean.)

Table 1 Participants

Country	Number of Participants
France	1
The Netherlands	1
United Kingdom	1
United States	6
Total	9

Instructors' Responses

1. Was the course you taught using an interactive theorem prover (ITP) an introduction to mathematical reasoning? To logic in general? To proof? Other?

Interactive theorem provers (ITPs) have been used in the following mathematics courses:

- Introduction to Logic for 1st-year students of computer science
- Undergraduate Introduction to Mathematical Reasoning
- Undergraduate Introduction to Proofs
- Undergraduate Real Analysis
- Undergraduate Computer-assisted Logic and Proofs (50 students enrolled in the first year, all pursuing a double major in mathematics and computer science)
- Graduate Introduction to Logic

Most instructors reported that they used the ITP, Lean, in introductory courses oriented towards logic and mathematical reasoning. It is important to note that Lean includes resources for several other topics in mathematics, such as Analysis, Linear algebra, and Geometry.[2]

2. How did you incorporate an ITP into your teaching?

Interactive theorem provers were incorporated into teaching mainly in three ways:

- *to demonstrate proofs in* a lecture (5 out of 9 instructors)
- *for in-class problem-solving activities* (all instructors)
- *for* homework/assignment (4 out of 9 instructors)

The mathematics instructors reported devoting a portion of their class time to demonstrating how Lean could be used to formalize proofs of theorems that had been proven less formally in their lectures. The amount of time spent on Lean depended on the length of the course as a whole.

For example, one instructor dedicated 5 out of 35 class periods to the Lean interactive theorem prover. In those 5 class periods, about one-half of the time was devoted to Lean demonstrations, in which new concepts and worked examples were introduced. The other half of the time was spent on group problem-solving in Lean. In addition, there were 5 homework assignments involving Lean following up on the lectures. Lean, however, was not included in any of the exams.

Similarly, another instructor spent about half an hour in class on Lean demonstrations. Occasionally, she would post Lean formalizations of theorems from the course on her course website. She would also include in each homework assignment one Lean problem for extra credit points. The points were allocated such that the Lean problem could replace any other problem on the homework. To assist students in using Lean, she provided links to online tutorials and support during office hours, but she did not teach the students to use the software in any other way.

The instructor who taught the undergraduate Computer-assisted Logic and Proofs course intentionally provided lecture notes using the Lean format and gave students access to a Lean program he had designed for his class. At the end of the course, students were examined on Lean proofs as well as on paper proofs.

3. Could you describe the learning curve of the ITP for you as an instructor and for your students?

Instructors who had already been using various proof assistants reported the learning curve as relatively flat. Top students and those who had previous programming experience seemed to learn Lean effortlessly. In fact, for many students, learning Lean was very helpful; for others, Lean did not "click" even by the end of the course. For both instructors and students without prior programming experience, the learning curve was steep in the beginning.

One instructor pointed out that it is not easy to assess the Lean learning curve, because it is hard to distinguish difficulties that stem from Lean from those which flow from the mathematical content. The main barriers, according to the instructor, seemed to be issues installing Lean and misunderstandings in reading the Lean user interface. Some instructors suggested that, with more careful attention and support in the first few weeks of the course, it would be possible to increase the number of students who enjoy the Lean part of the course.

4. *From your observation, what worked well when using an ITP in the classroom? What were the major challenges?*

For some students, using Lean made the material less "dry", and motivated them. One instructor described the outcome of his course using the ITP as follows: Good students really learn a lot, and some average students are pleased too, but weak ones don't get anything out of it, even when they get decent grades. As other instructors put it, an ITP works best with mathematically mature students.

The most obvious benefit of using an ITP is that students get a much better idea of what a proof really is, because setting out a proof in Lean forces them to organize their thoughts. Another potential benefit of an ITP is in performance assessment. One instructor mentioned that if all calculus homework were turned in to the instructor in Lean format, then the instances of search-copy-paste solutions to assignments would approach zero.

However, students face some obvious challenges in learning and using the ITP:

- The necessarily rigid ITP syntax can be frustrating
- Learning an additional language (i.e., Lean syntax) is a hurdle for some students
- *Compared to a handwritten proof, completing a proof in Lean may require multiple attempts*

5. *In your view, which group of students at the post-secondary level would benefit most from using LEAN in the classroom?*

The short answer to this question is "all of them!", to quote the very phrasing used by many of the instructors. The instructors were of one mind on this, however they may have expressed it. Their answers to this question may seem incompatible with the ones they gave when asked about the major challenges they faced (Question 4). One respondent did say that weak students do not seem to "get anything out of it", and another that it "works best with mathematically mature students". But all agreed that undergraduate students stand to benefit from using Lean, albeit to varying degrees. The two groups mentioned as most likely to make effective use of Lean were students of the philosophy of mathematics and students who double-major in mathematics and computer science.

Final Thoughts

Exploring the perspectives of instructors who used Lean in their teaching provided some indication of both benefits and challenges. We also noted that all the instructors plan to continue using Lean in their courses, despite the challenges it presents. We intend to carry out a systematic evaluation of the degree to which the ITP Lean could be a useful additional tool in teaching undergraduate mathematics. Such an evaluation could help in the design of new and effective teaching strategies that can make ITPs valuable in an educational setting.

References

Avigad, J. (2019) Learning logic and proof with an interactive theorem prover. In Hanna, G., Reid, D. & de Villiers, M. (Eds.), *Proof technology in mathematics research and teaching* (pp. 277-290). Springer.

Buzzard, K. (2020). Proving theorems with computers. *Notices of the American Mathematical Society, 67*(11), 1791-1799.

Castelvecchi, D. (2021). Mathematicians welcome computer-assisted proof in 'grand unification' theory. *Nature, 559*, 18-19. https://www.nature.com/articles/d41586-021-01627-2

Hanna, G., & Yan, X. (2021). Top of Form

Bottom of Form

Opening a discussion on teaching proof with automated theorem provers. *For the Learning of Mathematics*, *41*(3), 42-46.

Thoma, A., & Iannone, P. (2021). Learning about proof with the theorem prover LEAN: the

Abundant Numbers Task. *International Journal of Research in Undergraduate Mathematics Education*. https://doi.org/10.1007/s40753-021-00140-1

Notes:

The Lean theorem prover is a proof assistant developed principally by Leonardo de Moura at Microsoft Research.

The Lean mathematical library, *mathlib*, is a community-driven effort to build a unified library of mathematics formalized in the Lean proof assistant. The library also contains definitions useful for programming. This project is very active, with many regular contributors and daily activity.

The contents, design, and community organization of mathlib are described in the paper The Lean mathematical library, which appeared at CPP 2020. You can get a bird's eye view of what is in the library by reading the library overview. You can also have a look at our repository statistics to see how it grows and who contributes to it.

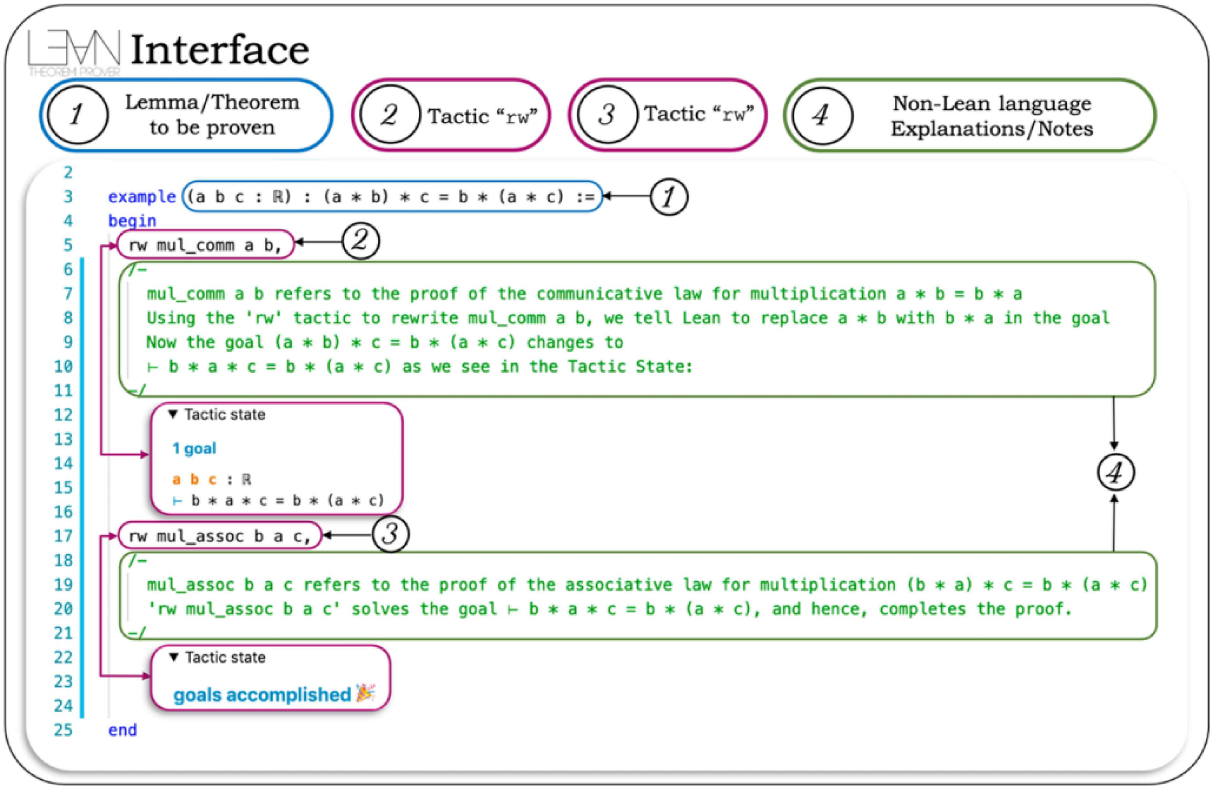
Example 1: A proof of if n is even then so is m * n:

From: https://leanprover-community.github.io/mathematics_in_lean/01_Introduction.html#overview

```
example : ∀ m n : nat, even n → even (m * n) :=
begin
  -- say m and n are natural numbers, and assume n=2*k
  rintros m n ⟨k, hk⟩,
  -- We need to prove m*n is twice a natural. Let's show it's twice m*k.
  use m * k,
  -- substitute in for n
  rw hk,
  -- and now it's obvious
  ring
end
```

As you enter each line of such a proof in VS Code, Lean displays the *proof state* in a separate window, telling you what facts you have already established and what tasks remain to prove your theorem. You can replay the proof by stepping through the lines, since Lean will continue to show you the state of the proof at the point where the cursor is. In this example, you will then see that the first line of the proof introduces m and n (we could have renamed them at that point, if we wanted to), and also decomposes the hypothesis even n to a k and the assumption that n = 2 * k. The second line, use m * k, declares that we are going to show that m * n is even by showing m * n = 2 * (m * k). The next line uses the rewrite tactic to replace n by 2 * k in the goal, and the *ring* tactic solves the resulting goal m * (2 * k) = 2 * (m * k).

Example 2: A proof of (*a* x *b*) x *c* = *b* x (*a* x *c*)



The diagram above brings together what's happening in the Lean interface when one works on a simple proof such as (*a* x *b*) x *c* = *b* x (*a* x *c*), marked as 1. To prove it, using the "rw" tactic with the commutative law and associative law for multiplication closes the goal. The commands in Lean language are marked as 1, 2, and 3. Commands 2 and 3 lead to changes in the tactic state where Lean shows the current goal(s) of the proof. The green text, marked as 4, denotes informal math language that could be inserted between the tactics to communicate with the reader about the proof. This space is particularly useful to instructors who want to add explanations or share notes with students.

Canadian Mathematical Society
Société mathématique du Canada

CMS Competitions This Fall

Canada's most **prestigious** competition
Canadian Open Mathematics Challenge (COMC)
October 27
COMC.MATH.CA

Compete to be part of Math Team Canada!

Fall competition for younger students!
Canada Jay Mathematical Competition (CJMC)
November 17
CJMC.MATH.CA

AGES
CJMC: grade 5-8 curriculum

COMC: students must be under 19 years of age as of June 30, 2022

June 2022 (Vol. 54, No. 3)

Nomination Information

Cathleen Synge Morawetz Prize

> Nominations are currently welcomed for the 2023 Cathleen Synge Morawetz Prize.
> Deadline: September 30

The Cathleen Synge Morawetz Prize is for an author(s) of an outstanding research publication. A series of closely related publications can be considered if they are clearly connected and focused on the same topic. At least one author of any nominated paper should be part of the Canadian Mathematical community.

The Cathleen Synge Morawetz Prize will be awarded according to the following 6-year rotation of subject areas:

1. Geometry and Topology *(2021, and every six years thereafter)*,
2. Combinatorics, Discrete mathematics, Logic and foundations, and Mathematical Aspects of Computer Science *(2022 , and every six years thereafter)*,
3. Applied mathematics, including but not limited to Numerical Analysis and Scientific Computing, Control Theory and Optimization, and Applications of Mathematics in Science and Technology *(2023, and every six years thereafter)*,
4. Probability and Mathematical Physics *(2024, and every six years thereafter)*,
5. Algebra, Number theory, Algebraic geometry *(2025, and every six years thereafter)*,
6. Analysis and Dynamical systems *(2026, and every six years thereafter)*.

All of the above fields will be understood most broadly, to ensure that any outstanding publication can be considered under at least one of the categories. A paper (or a series of papers) which has significantly impacted more than one of the listed fields can be nominated more than once in the six-year rotation. The nomination must focus on a single topic, rather than a broad body of work by the nominee.

This call for nominations is for an author(s) of a publication or a series of closely related publications in the field of Applied mathematics, including but not limited to Numerical Analysis and Scientific Computing, Control Theory and Optimization, and Applications of Mathematics in Science and Technology.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation.

The nomination letter should highlight the research paper(s) being nominated, providing evidence of its impact and significance. The nomination letter should list the chosen referees, and should include a recent curriculum vitae of the nominee(s), if available. Up to three reference letters in support of the nomination should be sent directly to the CMS.

All documents should be submitted electronically, preferably in PDF format and **no later than the deadline date above**, to **csmprize@cms.math.ca**.

Nomination Information

Coxeter-James Prize

The Coxeter-James Prize recognizes young mathematicians who have made outstanding contributions to mathematical research. It is awarded on an annual basis. The selected candidate will deliver the prize lecture at the Winter Meeting.

Nominations are solicited the prior year, typically from early March to the end of September.

> Nominations are currently welcomed for the 2023 Coxeter-James Prize.
> Deadline: September 30

The recipient shall be a member of the Canadian mathematical community. Nominations may be made up to ten years from the candidate's Ph.D: researchers having their Ph.D degrees conferred within the past ten years (e.g.: degree in 2009 or later would be eligible for nomination in 2019 for the 2020 Prize). Where eligible leaves of absence may warrant, nominations may be made more than ten years from the candidate's Ph.D. Such exceptions should be clearly addressed by the nominators. A nomination can be updated and will remain active for a second year unless the original nomination is made in the tenth year from the candidate's Ph.D.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation. A candidate can be nominated for more than one research prize in the applicable categories; several candidates from the same institution can be nominated for the same research prize.

All CMS research prizes (the Coxeter-James Prize, the Jeffery-Williams Prize and the Doctoral Prize) are gender-neutral, except for the Krieger-Nelson Prize, which is awarded to women and female-identifying mathematicians only. Nominations of eligible women and female-identifying mathematicians for general research prizes in addition to the Krieger-Nelson Prize are strongly encouraged.

The Research Committee of the CMS reserves the right to consider a nomination for one of the three research prizes for any other, applicable prize.

Nominations and reference letters should be submitted electronically, preferably in PDF format, by the appropriate deadline, to cjprize@cms.math.ca.

Nominators should ask at least three referees to submit letters directly to the CMS (same email address) by the deadline. Some arms length referees are strongly encouraged. Nomination letters should list the chosen referees, and should include a recent curriculum vitae for the nominee, if available.

Call for Nominations : 2023 Jeffery-Williams Prize

June 2022 (Vol. 54, No. 3)

Nomination Information

Jeffery-Williams Prize

The Jeffery-Williams Prize recognizes mathematicians who have made outstanding and sustained contributions to mathematical research.

> Nominations are currently welcomed for the 2023 Jeffery-Williams Prize.
> Deadline: September 30

The prize lecture will be delivered at the Summer Meeting. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for three years.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation. A candidate can be nominated for more than one research prize in the applicable categories; several candidates from the same institution can be nominated for the same research prize.

CMS research prizes are gender-neutral, except for the Krieger-Nelson prize, which is awarded to women and female-identifying mathematicians only. Nominations of eligible women and female-identifying mathematicians for general research prizes in addition to the Krieger-Nelson Prize are strongly encouraged.

The Research Committee of the CMS reserves the right to consider a nomination for one of the three research prizes for any other, applicable prize.

The deadline for nominations is shown above. Nominations and reference letters should be submitted electronically, preferably in PDF format, by the appropriate deadline, to jwprize@cms.math.ca.

Nominators should ask at least three referees to submit letters directly to the CMS (jwprize@cms.math.ca) by the deadline above. Some arms-length referees are strongly encouraged. Nomination letters should list the chosen referees, and should include a recent curriculum vitae for the nominee, if available.

Nomination Information

Krieger-Nelson Prize

The Krieger-Nelson Prize recognizes outstanding research by a woman or a female-identifying mathematician.

> Nominations are currently welcomed for the 2023 Krieger-Nelson Prize.
> Deadline: September 30

The prize lecture will be delivered at the Summer Meeting. The recipient shall be a member of the Canadian mathematical community. A nomination can be updated and will remain active for two years.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation. A candidate can be nominated for more than one research prize in the applicable categories; several candidates from the same institution can be nominated for the same research prize.

CMS research prizes are gender-neutral, except for the Krieger-Nelson Prize, which is awarded to women and female-identifying mathematicians only. Nominations of eligible women and female-identifying mathematicians for general research prizes in addition to the Krieger-Nelson Prize are strongly encouraged.

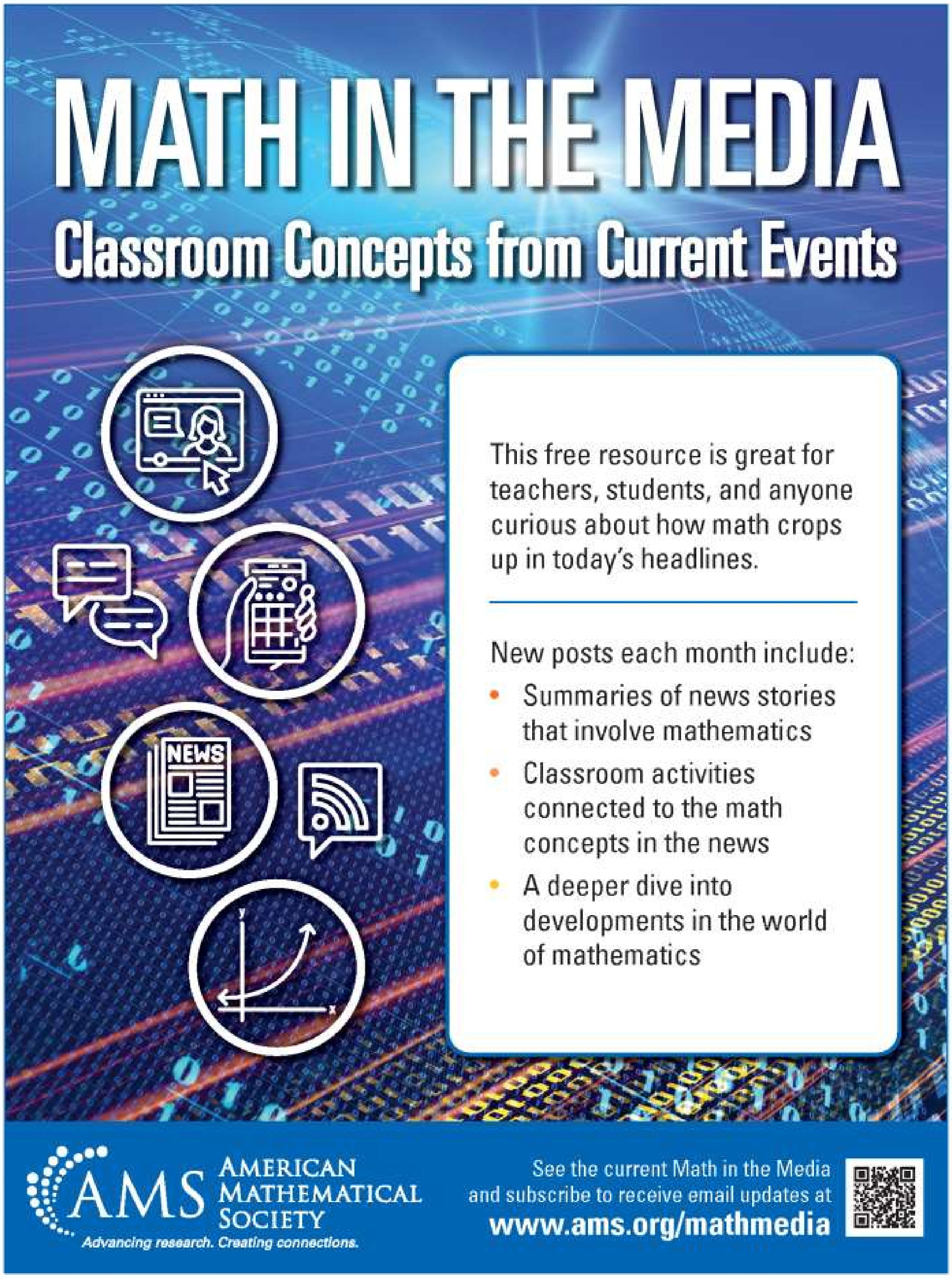
The Research Committee of the CMS reserves the right to consider a nomination for one of the three research prizes for any other, applicable prize.

The deadline for nominations is indicated above. Nominations and reference letters should be submitted electronically, preferably in PDF format, by the appropriate deadline, to knprize@cms.math.ca.

Nominators should ask at least three referees to submit letters directly to the CMS (knprize@cms.math.ca) by the deadline. Some arm's length referees are strongly encouraged. Nomination letters should list the chosen referees, and should include a recent curriculum vitae for the nominee, if available.

American Mathematical Society (AMS)

American Mathematical Society (AMS)



CMS NOTES

Call for Sessions: 2022 CMS Winter Meeting

2022 CMS Winter Meeting

December 2-5, The Chelsea Hotel, Toronto, Ontario

The Canadian Mathematical Society (CMS) welcomes and invites session proposals and mini-course proposals for the 2022 CMS Winter Meeting in Toronto from December 2-5, 2022.

PLEASE NOTE: Due to the uncertainty surrounding the COVID-19 pandemic, the 2022 Winter Meeting may be switched to an online platform. If alternative arrangements must be made, the CMS will advise all registrants of the changes as soon as possible.

CALL FOR SESSIONS:

In accordance with the CMS mandate to *propose conferences which are accessible and welcoming to all groups*, diversity amongst organizers and speakers is strongly encouraged. Diversity includes topics of interest, career stages, geographic location, and demographics.

Proposals should include:

(1) Names, affiliations, and contact information for all session co-organizers. Early career researchers are encouraged to propose sessions.

(2) A title and brief description of the topic and purpose of the session. This can include an overview of the subject.

(3) The total number of expected talks, with a list of possible speakers and/or papers in the theme. It is expected that all sessions will make efforts to include speakers from designated underrepresented groups. These groups include, but are not limited to, women, Indigenous Peoples, persons with disabilities, members of visible minority/racialized groups, and members of LGBTQ2+ communities. Intention to include new PhD's and to make the session accessible to graduate students is also encouraged.

We kindly ask that you refrain from inviting speakers until after your session has been approved and until after the open call for talk abstracts has concluded.

Open Call for Abstracts: The CMS will continue the **open abstract submission process** that was recently introduced to support session organizers in their important work and in their efforts towards inclusivity and diversity.After the open call has concluded, session organizers will be able to select and invite speakers from amongst the abstracts received and/or from their list of potential speakers as indicated in their proposal.

The scientific sessions will take place from **December 3-5**, 2022.

CALL FOR MINI-COURSES:

The CMS is offering three-hour mini courses to add more value to meetings and make them more attractive to students and researchers.

The mini courses will be held on Friday, December 2, before the public lecture, and include topics suitable for graduate students, postdocs and other interested parties.

Proposals should include names, affiliations, and contact information for all the mini-course co-organizers, as well as a title and brief description of the focus of the mini-course.

Deadline: Proposals should be submitted by **Monday, September 12, 2022** to the Scientific Directors and the CMS Office should be cc'ed. Their contact information is as follows:

Ada Chan: ssachan@yorku.ca

Gregory Smith: gregory.george.smith@gmail.com

Jessica Horobetz: meetings@cms.math.ca

2022 CMS Winter Meeting : Toronto, Ontario

CMS Meetings
June 2022 (Vol. 54, No. 3)

December 2-5, 2022 at the Chelsea Hotel

The Canadian Mathematical Society (CMS) is excited to host it's 2022 Winter Meeting in Toronto, Ontario from December 2-5, 2022.

The meeting website will be available in July 2022. Please see the Call for Sessions included in this issue of CMS Notes. Otherwise, keep an eye on the CMS website and social media for updates on this exciting event!

In Memoriam: Daniel Ashlock (1961-2022)

Obituaries June 2022 (Vol. 54, No. 3)

Rajesh Pereira & Allan Willms on behalf of the Department of Mathematics & Statistics, University of Guelph

April 29, 2022

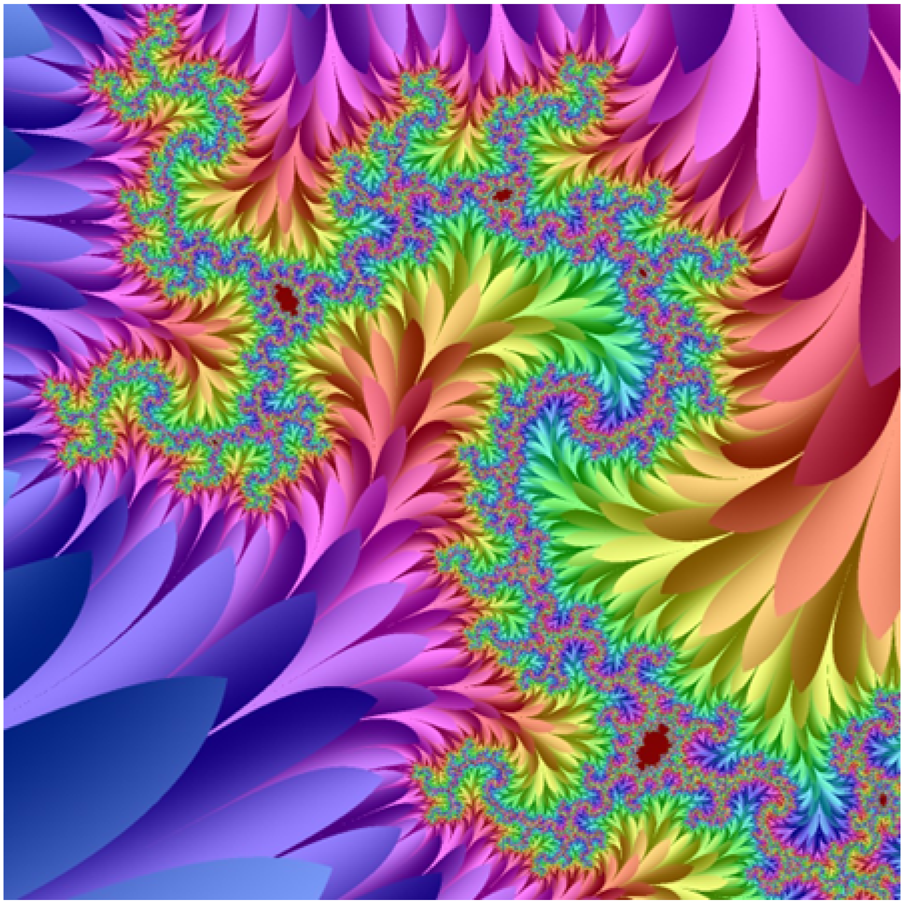
The Department of Mathematics and Statistics of the University of Guelph mourns the passing of our Chair, Daniel Ashlock, after a brief battle with cancer. Dan was an exemplary member of the department and an energetic presence in the university community. Dan completed his Ph.D. in 1990 at the California Institute of Technology under the direction of the combinatorialist Richard M. Wilson. He was a Professor of Mathematics at Iowa State University for fourteen years before joining the Department of Mathematics and Statistics at the University of Guelph in 2004. Dan's contributions to research, teaching, student supervision, outreach, and university administration have been extraordinary.

Dan was a prolific researcher in many fields, most notably Evolutionary Computation, Bioinformatics, Mathe- matical Biology, Games, and Graph Theory. From using evolutionary computation to optimize cooking stove design in Central America to using artificial intelligence to improve vaccine distribution, Dan was a master at using ad- vanced computational techniques to solve real-world problems. An author of more than 300 scholarly articles, Dan's writing and influence have been substantial. He is author or co-author of seven published textbooks including "Evo- lutionary Computation for Modeling and Optimization" (Springer, 2006), "A Course in Evolutionary Computation" (with Wendy Ashlock, independently published, 2021), and five introductory level texts: "Mathematical Problem Factories: Almost Endless Problem Generation" (with Andrew McEachern, Morgan-Claypool, 2021), "An Intro- duction to Proofs with Set Theory" (with Colin Lee, Morgan-Claypool, 2020), and three "Fast Start … Calculus" texts (Morgan-Claypool, 2019). Dan was also the editor for a book series on computational intelligence in games for Morgan and Claypool for the past four years, and served as an associate editor for many years for the journals BioSys- tems, the IEEE Transactions on Evolutionary Computation, the IEEE Transactions on Games, and the IEEE/ACM Transactions on Computational Biology and Bioinformatics. In the area of evolutionary computation, one of Dan's seminal contributions was the idea that representation of the problem has a large, and often determining, effect on the success of an optimizing algorithm. Dan's research led to improved knowledge about fitness landscapes and issues surrounding coevolution.

Dan played a pivotal role in the interdisciplinary bioinformatics programs at both Iowa State and the University of Guelph, being a founding member of the program at Iowa State, and serving on the Bioinformatics Steering Committee at Guelph. Dan was instrumental in the launch and success of Guelph's Ph.D. program in this area. In Iowa, he was involved with the Research Experience for Undergraduates (REU) that attracted students from all over the U.S. to come to Iowa State for a summer of bioinformatics research. At Guelph, Dan taught courses in the program at various points over the years, and served as supervisor or advisory committee member for many graduate students. He developed a particularly close collaboration with Dr. Steffen Graether of the Department of Molecular & Cellular Biology, and their research has resulted in novel clustering methods that can be applied to DNA sequence and other data, and new approaches for studying the evolutionary history and classification of proteins.

Dan was highly involved in the IEEE, being a senior member since 2005. He was on the Chair of the Task Force on Industrial Relations (Games), 2017–2018, and Chair of the Games Technical Committee, IEEE Computational Intelligence Society, 2017–2018. He served on the IEEE Education Committee from 2018 until his death, and was particularly heavily involved for decades with the Bioinformatics and Bioengineering Technical Committee and its associated IEEE Computational Intelligence in Bioinformatics and Computational Biology (CIBCB) conferences, as well as the IEEE Conference on Games (COG). Dan was awarded the IEEE Lifetime Achievement Award in 2015–2016.

Dan was a passionate promoter of Mathematics Education and Outreach as evidenced by his blog Occupy Math, whose byline was "Math is the right of all free people." He posted articles covering a wide array of topics including math anxiety, needing more women in mathematics, using mathematics to know when politicians are lying to you, and enjoying the wonderful structure of mathematics through art. As service to the department, Dan was involved in outreach, high school liaison, and supporting the undergraduate Mathematics & Statistics Club. Dan developed a relationship with a chemistry high school teacher at Centennial CVI in Guelph, who sent him bright students with whom Dan met regularly to mentor. Dan would frequently hold math challenge problem contests for undergraduate students, he designed many educational games, and he held a fractal art competition. Fractals were one of Dan's loves. He, along with his graduate students, would write evolutionary algorithms to find "interesting" portions of the Mandelbrot set, or other fractals. An example of one of his fractals is below; many others can be found on his Occupy Math blog.



One wall of the Mathematics & Statistics Club room at the University of Guelph is a mural of a fractal created by Dan. Dan felt that everyone should have the opportunity to learn mathematics, but he was not convinced that calculus is the best topic at the first year level for many students, especially those outside Physics and Engineering. He has a TEDx talk on this subject, which can be viewed here. Many other of his opinions about teaching and learning mathematics can be found on his Occupy Math blog.

Dan was also a driving force behind curricular renewal. The Physics department had often complained that the order in which topics were taught in our calculus courses did not suitably align with when these topics were needed in the Physics courses. Dan formed a committee to look into this, making sure that the committee had ample student representation. Following the recommendations of the committee, Dan helped design two combined introductory Physics and Calculus courses that would be team taught by both a Physics and a Mathematics instructor. Dan then co-taught that course for many years, often receiving standing ovations from students at the end of the semester. Dan was instrumental in re-designing the department's mathematics for business students course and most recently helped design our department's first Combinatorics and Graph Theory course. While at Iowa State, Dan developed a Math for Biologists course and an Introduction to Evolutionary Computation course.

Dan was mentor and advisor to many undergraduate and graduate students in the Mathematics and Bioinformat- ics programs at the University of Guelph and Iowa State. While at Iowa State, he supervised three Bioinformatics doctorates and two Mathematics doctorates, as well as 11 masters students. At Guelph he advised or co-advised nine doctoral and 15 masters students in Mathematics, one doctoral student in Statistics, one doctoral student in Computer Science, one masters student in Biophysics, six doctorates in Bioinformatics, two Masters of Science in Bioinformatics, and dozens of Masters of Bioinformatics projects, as well as serving on many advisory committees. Many of Dan's students have expressed thanks for his support, and for conveying his enthusiasm for mathematics to them. Dan cared deeply for others and was an advocate for the vulnerable.

Conversing with Dan on the research level was often a mental workout, not because what he said was obtuse, but simply because his thought processes were far faster than most. One often felt like Dan was at the summit surveying the problems on the other side while you were still trying to get your head around the obstacle half way up the hill. At a more general level, Dan always had something interesting to talk about, whether it be the most recent scientific gadget, the fascinating research of some colleague, the latest environmental news, or his own opinion on the woeful state of politics. And he was nearly always ready with a reference to some science fiction story that was directly applicable. An elevator ride with Dan was never a dull moment.

He will be greatly missed.

June 2022 (Vol. 54, No. 3)

The editors welcome the submission of articles, letters and announcements. Indicate the section chosen for your article, and send it to CMS Notes at the appropriate email address indicated above. No responsibility for the views expressed by authors is assumed by the CMS Notes, the editors or the CMS.