Welcome to the December Issue of the CMS Notes

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2022: The End of an Era

Cover Article

December 2022 (Vol. 54, No. 6)

Termeh Kousha (Canadian Mathematical Society)
Executive Director

As I sit and reflect on the past year, I feel grateful for my office in the Canadian House of Mathematics from where I write this article. Despite the many ups and downs that the CMS has faced over the past year (or several), I am pleased with the progress that we have made and continue to make with our endeavours. This was an exciting year, with the resumption of many in-person activities, which allowed young mathematicians to experience the exciting atmosphere of Mathematical Olympiads and to attend a variety of Math Camps across Canada. What's more, professional mathematicians from all domains were able to reunite and collaborate face to face (offline).

It is such a pleasure to be a part of the Canadian Mathematical Society, a group of diverse mathematicians that is always ready to stand with their colleagues in the face of injustice. The Board of Directors has been working hard to create new committees and sub-committees to address the needs of our members and mathematicians alike. This year, two new committees were established: *Human Rights in Mathematics Committee* and *International Prize Committee.*

We had the first full year of in-person CMS Meetings since 2019, where we were able to reconnect with old friends and colleagues to discuss important issues pertaining to the field and those within it. At the Summer Meeting, we felt the residual effects of the pandemic, with our shipment of meeting supplies being stuck at a warehouse in New Brunswick and arriving after the meeting had already concluded. No badges? No problem! Despite this minor hiccough, everyone was pleased to be able to ***finally*** attend an in-person mathematics conference. Whether the Executive Office Staff learned from this experience or was paranoid due to this experience, I cannot say; we loaded up our car and drove to Toronto for the Winter Meeting. Badges or no badges, mathematicians sure know how to enjoy a conference! It was great to see everyone back after a successful semester and to congratulate our Award Winners and our new class of CMS Fellows.

Among those who benefitted from the excitement of attending a mathematical event were the students on **Math Team Canada** who traveled to the International Mathematical Olympiad and the European Girls' Mathematical Olympiad. I'd love to take this opportunity to boast about how wonderful our team performed and encourage you to look at the official competition scores[1]. Of course, none of this would be possible without Dorette Pronk, Robert Woodrow, and the rest of the team that put in endless hours behind the scenes to train the Olympians, coordinate their travel, and provide them with a positive experience.

This year has been a busy year for competitions, with the Executive Office running a campaign to increase school participation in the COMC and the CJMC. 14,792 invitations were sent to Canadian schools inviting them to participate in our competitions this fall. Our competition network expanded this year thanks to new and existing partnerships both in Canada and internationally[2]. Thanks to these efforts, CJMC participation increased by nearly 50%. We are wrapping up a very busy competition season and are looking forward to analyzing the results of this direct marketing campaign in the new year.

Summer 2022 brought with it the resumption of many Math Camp programs across the country and the creation of several new programs. We have received great feedback from students and organizers regarding the camps and are certain that 2023 will bring additional exciting opportunities for youth in mathematics.

Here at the CMS Executive Office, we are no stranger to happy reunions. After over 2 years of working remotely, we have returned to the office. It is worth noting that this office is not just any office, but the newly christened **Canadian House of Mathematics**: a permanent home for the CMS and Mathematics in Canada.

This reunion has been bittersweet, because one of our team members is no longer with us. On September 30th, resident expert on all things CMS, our friend and mentor, Alan Kelm passed away. He was a pillar of the Canadian Mathematical Society for almost 30 years, and he is dearly missed by our entire team.

Speaking of our team, I cannot stress enough how resilient the CMS Executive Office Staff has been over the past year. Through staffing shortages, technological glitches, illness, and everything else that life has thrown their way, everyone at the Executive Office has remained committed to the Society and the role that they play within it. The hard work of Kaileigh, Jessica, Sarah, Steve, and Xinxin did not go unnoticed. I am so lucky to have them as colleagues and wish to express my heartfelt thanks for all their effort this year. Last, but far from least, I would like to thank Yvette for all that she has done for myself and the CMS this year. Her hard work and dedicate have ensured the survival of the CMS during difficult times. I couldn't have asked for a better team.

Finally, I would like to thank Dr. David Pike, Dr. Javad Mashreghi, Dr. Monica Nevins, the Board of Directors, and the members of this wonderful Society for their support and guidance this year. I greatly appreciate their mentorship and friendship as I learn and grow in my role at the CMS.

From my position of privilege, I must acknowledge the sorrow and rage that I experience as an Iranian Canadian Woman when I think of the injustices that my fellow Iranian Women are facing (and have faced) under the current regime. The bravery of those who are and have been peacefully protesting for their freedoms makes me proud to be an Iranian Woman.[3]

At the same time, the Canadian Mathematical Society and the dedication of its wonderful members to the Mathematical Community make me proud to be a Canadian mathematician.

Wishing all of you a wonderful period of rest, and a very happy 2023!

For Women, Life, Freedom

برای زن، زندگی، آزادی

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[1] CMO/CJMO: https://cms.math.ca/news-item/2022-cmo-cjmo-results/

EGMO: https://cms.math.ca/news-item/egmo-results-2022/

IMO: https://cms.math.ca/news-item/math-team-canada-imo-2022/

[2] A list of International Competition Partners can be viewed here: https://cms.math.ca/competitions/international-competition-partners/

[3] More information on current events in Iran can be found here: https://www.cbc.ca/news/topic/Tag/Iran

Over The Transom

Editorial December 2022 (Vol. 54, No. 6)

Robert Dawson (Saint Mary's University)
Editor-in-Chief, CMS Notes

No doubt I'm not the only member of the CMS to receive a steady stream of emails in the same genre as the following:

Dear Professor:

I am just about to graduate with a master's degree in computer science from Utopia National University, where I also obtained my B.Sc. I achieved an average GPA of 3.8 in my undergraduate courses. I have high hopes to work with you on a Ph.D. at your university.

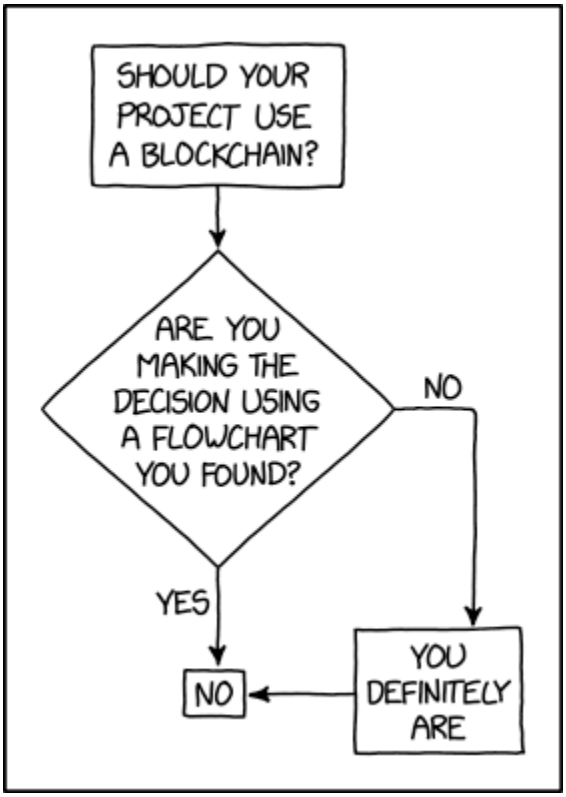
...

My master's thesis was on "Cloud Computing Paradigms for Blockchain Technology in the Design of Smart Cat Toys." I have had summer jobs with a variety of companies. I feel that my software skills will be an excellent fit for your research team. I would very much appreciate the opportunity to work under your supervision to obtain my doctorate.

I attach a resume to this email, summarizing my educational accomplishments and teaching experience. Please feel free to contact me if you have any questions or would like to read my thesis. I am available at any time for an interview.

Thank you in advance,
(etc)

Before you think you've found the perfect holiday present for Fluffy, I must confess that smart cat toys are as far from my field of expertise as cloud computing. As for blockchain, I think Randall Munroe's xkcd comic probably has it right in most cases:



https://imgs.xkcd.com/comics/blockchain.png

—especially the mouseover text. In short, I'm about as far from a potential supervisor for this young person as you could get without going to (say) the classics department. (My lack of research funding would be the cherry on the sundae, were there a sundae.)

But suppose that my correspondent had written a master's thesis not impossibly far from one of the obscure topics in geometry, number theory, or category theory with which I occupy myself between classes; and suppose that I had enough funding to support a student (rarely the case even when I did have a research grant). Would this over-the-transom application persuade me?

Probably not: even with a chance hit, it still bears the marks of a shotgun application. It's hard to make a persuasive case that your work is a good fit to somebody else's project while not mentioning the project! I'd advise potential graduate students searching for a supervisor to carefully research the people doing good work in their area of interest, and send personalized emails making their case in detail for working with that particular supervisor. Nobody likes a spammer.

CMS NOTES 75 1945-2020

Recurring decimals, proof, and ice floes

Education Notes December 2022 (Vol. 54, No. 6)

Brian McMaster (Queen's University, Belfast)

Aisling McCluskey (University of Galway)

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

John Grant McLoughlin, *University of New Brunswick (johngm@unb.ca)*

Kseniya Garaschuk, *University of the Fraser Valley (kseniya.garaschuk@ufv.ca)*

Or: Why do we teach students how to prove things we all know already, such as 0.9999··· = 1?

Partly, of course, so that they develop thinking skills to use on questions whose truth-status they won't know in advance. Another part, however, concerns the dialogue nature of proof: a proof must be not only correct, but also persuasive: and persuasiveness is not objective and absolute, it's a two-body problem. Not only to tango does one need two.

The statements —

(1) *ice floats on water,*

(2) *ice is less dense than water*

— are widely acknowledged as facts and, usually, as interchangeable facts. But although rooted in everyday experience, they are not that experience. We have firstly represented stuffs of experience by sounds English speakers use to stand for them, then represented these sounds by word-processor symbols that, by common agreement, stand for them. Two steps away from reality already! This is what humans do: we invent symbols for perceived realities and, eventually, evolve procedures for manipulating them in ways that mirror how their real-world origins behave. Virtually no communication between two persons, and possibly not much internal dialogue within one mind, can proceed without this. Man is a symbol-using animal.

Statement (1) counts as fact because folk living in cooler climates have directly observed it throughout history (and because conflicting evidence is lacking). Statement (2) is factual in a significantly different sense, arising by further abstraction from (1) and from a million similar experiential observations: partly to explain (1) and its many cousins, we have conceived ideas like mass, volume, ratio of mass to volume, and explored for generations towards the conclusion that mass-to-volume works out the same for similar materials under similar conditions, and that the comparison of mass-to-volume ratios predicts which materials will float upon others.

Statement (3): 19 is a prime number. In what sense is this a fact? Its roots are deep in direct experience: the hunter-gatherer wishing to share nineteen apples equally with his two brothers or his three sons or his five children must have discovered that he couldn't, without extending his circle of acquaintance so far that each got only one, long before he had a name for what we call 'nineteen'. But (3) is many steps away from the experience where it is grounded. It involves conceptualisation of numerical measurements of sets one encounters, and millennia of thought to acquire symbols for these and codify procedures for manipulating them in ways that mirror how reality functions. We've done this so successfully that it's easy to forget how far from the tangibles of experience they stand.

Statement (4): $\sqrt{2}$ is not exactly the ratio of two whole numbers. Most first-year mathematics students know this. But by this stage of abstraction, separating its fact-ness from its demonstration is impossible: the property of being exactly a fraction is not detectable by physical experience. It is a property of how we abstracted and systematised the numbers that proved useful in modelling

reality, not of our hands-on experience of reality. The reason we regard $\sqrt{2}$'s irrationality as factual is precisely because we can give a demonstration within an accepted logical framework.

What then about recurring decimals? For persuasive argument, we must first ascertain the distance from reality at which the question arises: not, in this case, the rarified atmosphere of undergraduate mathematics but the primary school classroom. Once a child has learned rituals for dividing whole numbers and the convenience of decimal notation, she will try to divide, say, 2 by 3 and will hit a problem: the decimal representation of the answer does not cease to spew out digits of lesser and lesser significance no matter how long she keeps turning the handle.

What should we reply when she asks whether zero point infinitely many 6's is or is not two thirds, or even — as a thoughtful child should — whether zero point infinitely many 6's is a legitimate symbol at all?

The answer must be tailored to the questioner's needs, but the natural way forward — though it took us centuries to make it logically watertight — is the nineteenth-century definition of sum of an infinite series. For the primary school kid it may suffice to say that, by writing down enough 6's, we'd get as close to 2/3 as we'd need for any practical purpose. For differential calculus we'd need something better, and for model-theoretic discourse involving infinitesimals something better again. Yet the underpinning mathematics for equalities like $0.6666\cdots = 2/3$ *where the question arises* is the nineteenth-century one. Its fact-ness therefore resembles that of ice being less dense than water, of 19 being prime or of $\sqrt{2}$ being irrational: it can be demonstrated within a logical framework that systematises our observations of real-world experiences. So it is a fact not about reality but about the models we build to explain reality. Demonstration is the only tool available for establishing its truth.

Mathematics without proof is not like an omelette without salt and pepper; it is like an omelette without egg.

About the Authors

Brian McMaster has served his alma mater, Queen's University Belfast (QUB), in multiple capacities including Adviser of Studies, Head of Research and Associate Director of Education. His publication profile covers over sixty refereed journal articles and his teaching interests focus around analysis and analytic topology. Formally retired since 2011, he continues to deliver ad hoc undergraduate teaching on an honorary basis. In 2018 he received the accolade for Special Recognition for Inspirational Teaching awarded by QUB Students' Union.

Aisling McCluskey is a Personal Professor in Mathematics at the University of Galway, Ireland. She maintains an active research profile whilst holding the teaching and learning of mathematics central to her academic endeavour. She has received several institutional and national awards for excellence in teaching, most recently in 2020 and 2021. She is currently a Governor at the University of Galway and Head of the School of Mathematical and Statistical Sciences there.

Aisling McCluskey and Brian McMaster have co-authored several textbooks in the areas of topology, analysis and complex numbers.

Quaternions at Twilight: Remembering Mary Somerville 150 years after her death

CSHPM Notes December 2022 (Vol. 54, No. 6)

Brigitte Stenhouse (University of Toronto)

CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *independent scholar (aackerbe@verizon.net)*

Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

November 29, 2022, marks 150 years since the death of Mary Somerville (1780–1872), one of the most iconic mathematicians of 19th-century Britain. Somerville built her reputation in the drawing rooms of Edinburgh, London, and Paris, in which she demonstrated her expert knowledge of the analytical mathematics recently developed in France. This mathematics was widely believed to be the answer to a long-running decline in British science, owing to the fruitful applications of methods from the differential calculus and the calculus of variations. Somerville's earliest publications were solutions to puzzles printed in *The New Series of the Mathematical Repository*, through which she was one of the earliest adopters of the differential notation in Britain. The work for which she is most famous is undoubtedly her 1831 *Mechanism of the Heavens*, a translation of Laplace's formative *Traité de Mécanique Céleste*, in which he gave an analytical, algebraic treatment of physical astronomy.

Mary Somerville's mathematical works were the focus of my doctoral thesis, which sought to understand what it meant for Somerville, as a woman in 19th-century Europe, to be a mathematician [8]. In early 2022, I was delighted to again have access to the Somerville papers at the Bodleian Library, which I had been unable to consult for the final 18 months of my PhD owing to the Covid-19 pandemic. I especially wanted to resolve the footnotes in my thesis that said, "I know relevant material exists, I just haven't been able to look at it yet!" According to the online catalogue, one folder held a "notebook containing notes and comments in Mary's hand on The Mechanism of the Heavens, n.d. (c.1831)". Much to my surprise, this notebook was accompanied by 70 loose sheets in the shaky handwriting typical of Somerville much later in her life and full of algebraic calculations. One page in particular took my breath away: a draft page from her autobiography, again featuring algebraic formulas down the side, in which she acknowledged that the end of her life was approaching but noted that she was perfectly content in the care and company of her beloved daughters [4]. I was immediately hooked, and could clearly picture Somerville at her writing table, expressing her gratitude that her "intellect [was] still unimpaired", before turning to mathematics, a subject that she had enjoyed for nearly eighty years. I needed to know more about how and why Somerville returned to serious mathematical study when she was nearly ninety.

Figure 1. Self-Portrait of Mary Somerville, undated. Reproduced with the kind permission of the Principal and Fellows of Somerville College, Oxford.

By 1870, when this tale begins, Somerville had been living a peripatetic life on the Italian peninsula for around thirty years. Moving to Italy had been decided upon for health reasons, to seek a better climate than the UK offered, and also in hope of living at a lower cost than in London. During this lengthy sojourn, Somerville published multiple highly successful books—giving surveys of recent scientific developments in the physical sciences and physical geography—yet she still felt on the outside of the scientific community. Individuals such as Giovanni Plana, Professor of Astronomy at the University in Turin, offered her access to their personal libraries, but this was nonetheless a sharp contrast to the lively, sociable scientific community she had been a part of in London. Somerville described how she felt at "a great disadvantage being so entirely deprived of scientific society and of the means of hearing of recent discoveries and new publications" [3]. Both the Royal Institution and the Royal Astronomical Society decreed that Somerville was to be sent copies of their *Proceedings* and the *Greenwich Observations*, respectively, but these did not always materialize and instead she often relied on the goodwill of contemporaries—for example, mathematician Augustus De Morgan—to send her books and papers from Britain.

Beyond communication with her scientific acquaintances via letter, Somerville often hosted visitors in whichever city she was then living. One such visitor was Benjamin Peirce, Professor of Mathematics and Astronomy at Harvard University, USA, who came to Europe in 1870 to view an eclipse. Whilst in Naples he paid Somerville a visit and was so enamoured with his host that, on his return home, he sent her a privately-printed copy of his *Linear Associative Algebra*. In testament to her ongoing reputation, Peirce inscribed the work with: "To the brightest glory of her sex, Mrs Mary Somerville, with the sincere admiration and the profound respect of the Author" [2].

The mathematics in Peirce's *Linear Associative Algebra* would have been entirely new to Somerville. Building on the work of William Rowan Hamilton on quaternions (to which we return later), Peirce considered hyper-complex numbers and presented 162 different algebras in this volume [1, p. 127].Hamilton's quaternions were notable in not satisfying commutativity of multiplication, and Peirce went further in allowing systems in which associativity did not hold, and even those in which division was not well defined.

Apparently struggling to understand these new ideas, which marked a distinctive conceptual shift towards perceiving algebra as the study of structures, Somerville soon began soliciting books to aid her reading of Peirce. In April 1871 she wrote to her publisher, John Murray, telling him of Peirce's book and asking him to send her a copy of Hamilton's work on quaternions. A few months later Hamilton's 1853 *Lectures on Quaternions* were sent to her by the Reverend Whitwell Elwin, a close acquaintance of Murray who was not previously known to Somerville.

It is somewhat unfortunate that Somerville came to the study of quaternions only in 1870, five years after Hamilton's death. The two had met in Cambridge 38 years earlier when Somerville was received at Trinity College, and Hamilton subsequently oversaw her election in 1834 as an honorary member of the Royal Irish Academy. At this time Hamilton was still at the very beginning of his work on quaternions. He was interested in studying complex numbers as 'algebraic couples', or ordered pairs of real numbers on which he defined operations of multiplication and addition. He then began searching for an analogous system of triplets of real numbers, but he struggled to define such a system in which the properties of commutativity, associativity, and distributivity held, and where division (the inverse of multiplication) was well-defined. Eventually, in 1843, Hamilton developed his system of quaternions, namely numbers of the form $a + bi + cj + dk$, where i, j, k are unit vectors such that $i^2 = j^2 = k^2 = -1$. These hypercomplex numbers satisfied all of the properties Hamilton desired, except for commutativity of multiplication; the abandonment of commutativity was revolutionary at the time. In 1848 Hamilton gave a series of four lectures on quaternions at Trinity College Dublin, and these were subsequently expanded into his 1853 book that was sent to Somerville [1, pp. 28–35].

Hamilton's *Lectures* were apparently not sufficient, as Somerville soon reached out to William Spottiswoode, then President of the London Mathematical Society, for further assistance. Like Peirce, Spottiswoode had made Somerville's acquaintance while visiting Naples a few years earlier. In August 1871 Spottiswoode wrote to Somerville:

> I was glad to hear of you again, & especially so as you are still pursuing your studies. As you do not mention the exact subject of the American book, I am not quite sure as to the work best leading to it. But I send you three which I think must cover the ground, & with which I feel sure that you will in any case be interested. . . . The third is [Peter Guthrie] Tait's work on Quaternions. I have sent this rather than Sir William Hamilton's works as the latter are intolerably diffuse, & Tait has carried out the applications of the subject much further than anyone else [7].

During the winter of 1858–59 Tait and Hamilton had shared an intense exchange of letters discussing ideas around quaternions. After Hamilton's death, Tait became one of the leading advocates of quaternions, developing them into a tool with applications in the physical sciences.

A copy of Tait's 1867 *An Elementary Treatise on Quaternions* was part of Somerville's personal scientific library at the time of her death [9]. These books were donated by her daughters as a single collection to Girton College, Cambridge, and the copy of Tait still contains a sheet of handwritten notes by Somerville placed between pages 294 and 295 [10]. This page of formulas hints at the volume of time Somerville spent studying Tait, her last mathematical project.

In the final three years of her life, 1870–72, Somerville was working on numerous publication projects. She revised two mathematical manuscripts that she had first written in the 1830s, prepared new editions of her scientific survey books, and authored her autobiographical *Personal Recollections*. All of these works were left to be published after Somerville's death, for then her government pension would cease, and she intended the royalties from book sales to provide a vital income for her two unmarried daughters.

That Somerville was studying quaternions alongside these authorial projects is clear from manuscript drafts of her *Personal Recollections*. Interspersed with a draft from circa 1872 are sheets on which Somerville has made notes on Peirce's *Linear Associative Algebra*, and some of the draft sheets themselves feature brief jottings of calculations and diagrams; see, for example, Figure 2 [5]. Moreover, in a draft of a letter to Murray from November 1872 Somerville described her morning routine which involved "solv[ing] problems by the higher algebra or add[ing] to the narrative of [her] life" [4].

As can be seen in Figure 3, the draft of this letter was used by Somerville for jotting down calculations and ideas about quaternions. This folio is held with the aforementioned 70 loose sheets, which are a mixture of scrap paper used for rough workings-out and neat pages of notes. Tait provided exercises with no solutions at the end of each chapter of his *Quaternions*, and we can here witness Somerville preparing her own solutions to these problems. Other pages contain summaries of key ideas, and cross-references to results contained in works by Hamilton. One particularly interesting sheet contains an attempt at a proof of a result mentioned but not demonstrated by Tait.

It is highly likely that Somerville was producing these notes for an imagined reader other than herself. As can be seen in the bottom right-hand corner of the sheet in Figure 2, she felt that the quaternion system heralded a new, more powerful age in the mathematical study of the physical sciences. Thus it was a natural continuation of her previous mathematical work advocating for the adoption of analytical methods by those studying the natural world, for instance in her translation of Laplace. In addition, she may have felt that with the recent death of Hamilton, and Tait's declaration that he had moved on from his studies of quaternions, there was a lucrative gap in the book market that she could profit from and aid her daughters. The archival materials provide

further textual evidence that Somerville was in the early stages of preparing a book, possibly a companion to Tait's own. She wrote up multiple copies of the same sheets, returned to others to make edits, and at the beginning of her notes on Chapter 3 she declared that "the whole of this chapter is difficult and requires explanation which I have attempted" [4]. This justification for the material that she has produced and the neatly copied-up pages of notes would be unnecessary, were Somerville writing only as part of her own mathematical learning practice.

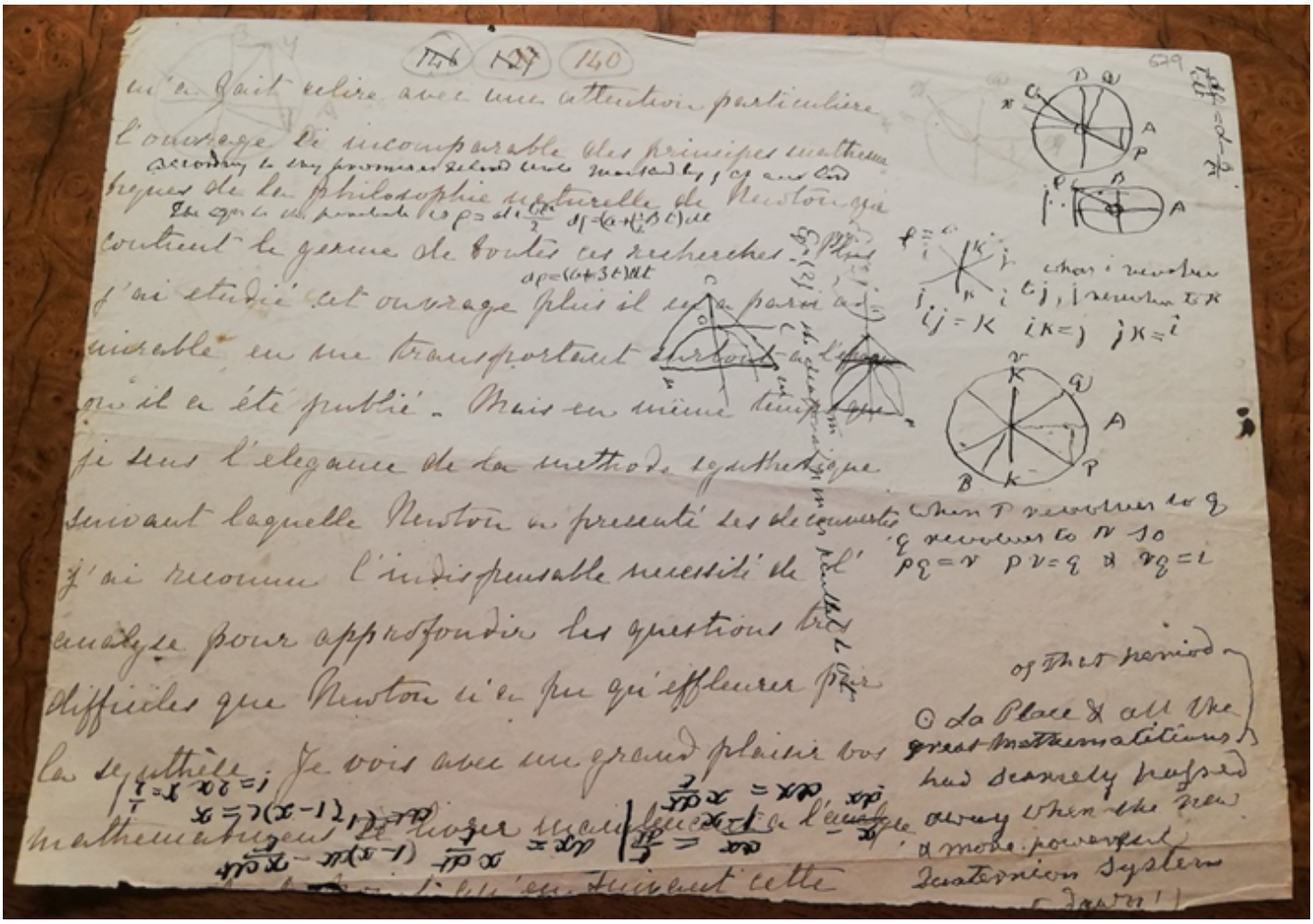


Figure 2. Image of a sheet held with a draft of Somerville's Personal Recollections. This page features a letter from Laplace copied up in Somerville's hand, which was included in the final publication [6, p.181]. Reproduced with the kind permission of the Principal and Fellows of Somerville College, Oxford.

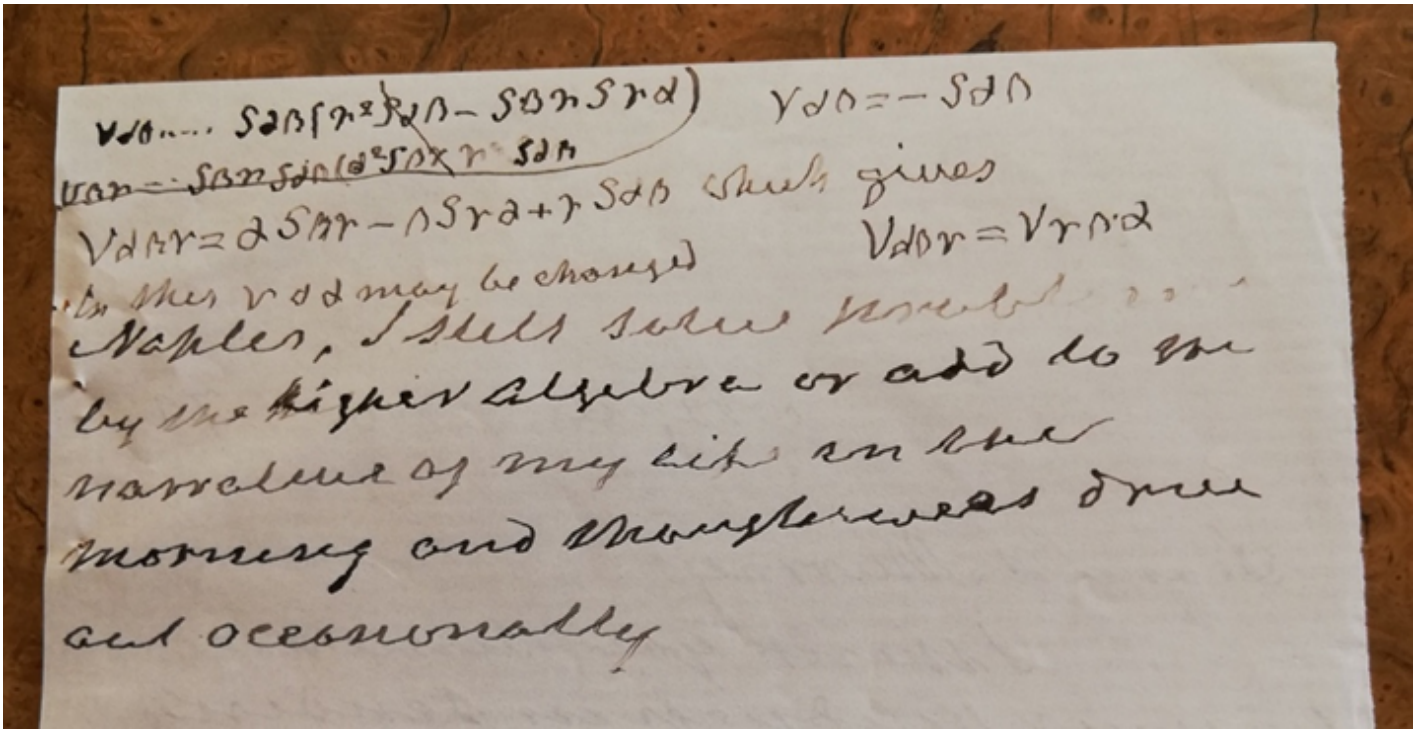


Figure 3. Draft of letter written to publisher John Murray, dated November 1872. Reproduced with the kind permission of the Principal and Fellows of Somerville College, Oxford.

Unfortunately, Somerville's work on the quaternions never came to fruition. She continued studying Tait's book until the day she died, November 29, 1872, at which point she had only written notes up to Chapter 3 [6, p. 376]. Her two other mathematical manuscripts were also left unpublished, but her autobiography was well received and two of her scientific survey books were re-issued after her death, providing some income for her daughters.

These loose sheets full of letter drafts, notes, and calculations raise questions about opportunities for publishing mathematical books in nineteenth-century Britain. Throughout her life Somerville was described as a mathematician, and she described mathematics as the subject she found most congenial. Yet her career as an author was focused on books which detailed results in the physical sciences, without delving into the mathematics used to reach them. It is impossible to say for certain that Somerville was preparing a work on quaternions for publication, but it is clear that at the end of her life she again hoped to reconcile her need for financial stability and her own intellectual gratification through mathematical writing.

Somerville's exposure to emerging ideas in algebra was contingent on the respect and recognition she commanded in scientific circles, which led men such as Peirce and Spottiswoode not only to visit her, but to provide her with recent mathematical works. When she faced difficulties, she did what she had done for over sixty years and turned to her acquaintances to ask for help. It is a testament to her tenacity that rather than being dismayed and discouraged by her difficulties in understanding the works of Peirce, Tait, and Hamilton, Somerville instead enjoyed the opportunity to study an entirely new branch of mathematics. When writing about solving the exercises in Tait, she admitted:

> Sometimes I find them difficult, but my old obstinacy remains, for if I do not succeed to-day, I attack them again on the morrow [6, p. 364].

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[3] Somerville, Mary. (1859, Jan. 5) Letter to Henry Holland. Somerville College Library, University of Oxford.

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[5] Somerville, Mary. (1872) Early draft of Somerville's Personal Recollections. Dep. c. 355, MSAU–2. Mary Somerville Collection. Bodleian Library, University of Oxford.

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[8] Stenhouse, Brigitte. (2021) Mary Somerville: Being and Becoming a Mathematician. PhD thesis. The Open University. https://oro.open. ac.uk/78959/.

[9] Tait, Peter Guthrie. (1867) *Elementary Treatise on Quarternions*. Oxford: Clarendon Press.

[10] Thanks to Jenny Blackhurst, Librarian and Fellow at Girton College, Cambridge, for consulting this material on my behalf.

Dr Brigitte Stenhouse is currently a postdoctoral researcher at the IHPST at the University of Toronto, and in January 2023 will take up the post of Lecturer in the History of Mathematics at The Open University, UK. Her research looks at the role of marriage and domesticity in mathematics, and 19th-century mathematics in Britain more generally.

The Best Writing on Mathematics, 2021

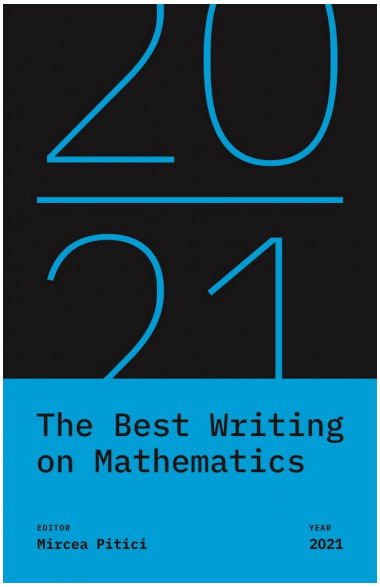
Book Reviews

December 2022 (Vol. 54, No. 6)

Karl Dilcher (Dalhousie University)

Book Reviews bring interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

Karl Dilcher, *Dalhousie University (notes-reviews@cms.math.ca)*



The Best Writing on Mathematics, 2021
Edited by Mircea Pitici
Princeton University Press, 2022
ISBN: 978-0-691-22570-8
Paperback: US $25, CAN $32

Reviewed by Karl Dilcher

Reviews of past issues of *The Best Writing on Mathematics* can be found in the 2019, 2020, and 2022 volumes of CMS Notes.

"So much to read, so little time!" is a common complaint that is known in several variations. Different solutions have been suggested, including speed reading. I tried learning a technique once when I was younger; but in the end, I had to agree with Woody Allen who reportedly said in one of his early comedy routines, "I read War and Peace in 20 minutes—It's about Russia."

A much better solution is to get advice from a trusted friend or colleague on what to read, and then read it slowly and in depth. When it comes to the large number of shorter pieces of writing in any given field, including mathematics, such a friend or colleague can also be a trusted and knowledgeable editor and/or anthologist.

This is the solution I have now happily relied on for several years, with the help of the series *The Best Writing on Mathematics*, edited by Mircea Pitici and published by Princeton University Press since 2010. The book under review is the twelfth volume in this remarkable series of annual anthologies. In the brief review of *The Best Writing on Mathematics, 2018*, I addressed some general features shared by all volumes. I will not repeat these remarks here; the interested reader will find them in the September 2019, issue. Instead, I quote from the Introduction to the current volume, where the Editor recalls that the series brings together "diverse perspectives on mathematics, its application, and their interpretation—as well as on their social, historical, philosophical, educational, and interdisciplinary contexts. The volume should be seen as a continuation of the previous volumes."

A bit later in the Introduction, Pitici writes, "The pieces offered this time originally appeared during 2020 in professional publications and/or in online sources. The content of the volume is the result of a subjective selection process that started with many more candidate articles. […] Once again, this anthology contains an eclectic mix of writings on mathematics, with a few even alluding to the events that just changed our lives in major ways."

I will now quote from the overview of this current volume and add the titles of the 26 individual pieces of writing, as I did in the two previous years. The average length of the pieces is almost exactly 10 pages.

"To start, Viktor Blåsjö takes a cue from our present circumstances and reviews historical episodes of remarkable mathematical work done in confinement, mostly during wars and imprisonment. [*Lockdown Mathematics: A Historical Perspective*].

"Andrew Lewis-Pye explains the basic algorithmic rules and computational procedures underlying cryptocurrencies and other blockchain applications, then discusses possible future developments that can make these instruments widely accepted. [*Cryptocurrencies: Protocols for Consensus*].

"Michael Duddy points out that the ascendancy of computational design in architecture leads to an inevitable clash between logic, intellect, and truth on one side—and intuition, feeling, and beauty on the other side. He explains that this trend pushes the decisions traditionally made by the human architect out of the resolutions demanded by the inherent geometry of architecture. [*Logical Accidents and the Problem of the Inside Corner*].

"Steve Pomerantz combines elements of basic complex function mapping to reproduce marble mosaic patterns built during the Roman Renaissance of the twelfth and thirteenth centuries. [*Cosmatesque Design and Complex Analysis*].

"Ben Logsdon, Anya Michaelsen, and Ralph Morrison construct equations in two variables that represent, in algebraic form, geometric renderings of alphabet letters—thus making it possible to generate word-like figures, successions of words, and even full sentences through algebraic equations. [*Nullstellenfont*].

"Maria Trnkova elaborates on crocheting as a medium for building models in hyperbolic geometry and uses it to find results of mathematical interest. [*Hyperbolic Flowers*].

"Yelda Nasifoglu decodes the political substrates of an anonymous seventeenth century play allegorically performed by geometric shapes. [*Embodied Geometry in Early Modern Theatre*].

"In the next piece, Stephen K. Lucas, Evelyn Sander, and Laura Taalman present two methods for generating three-dimensional objects, show how these methods can be used to print models useful in teaching multivariable calculus, and sketch new directions pointing toward applications to dynamical systems. [*Modeling Dynamical Systems for 3D Printing*].

"Joshua Sokol tells the story of a quest to classify geological shapes mathematically—and how the long-lasting collaboration of a mathematician with a geologist led to the persuasive argument that, statistically, the most common shape encountered in the structure of the (under)ground is cube-like. [*Scientists Uncover the Universal Geometry of Geology*].

"Don Monroe describes the perfect similarity between foundational algorithms in quantum computing and an experimental method for approximating the constant p, then asks whether it is indicative of a deeper connection between phenomena in physics and mathematics or it is a mere (yet striking) coincidence. [*Bouncing Balls and Quantum Computing*].

"Kevin Hartnett relates recent developments in computer science and their unforeseen consequences for physics and mathematics. He explains that the equivalence of two classes of problems that arise in computation, recently proved, answers in the negative two long-standing conjectures: one in physics, on the causality of distant-particle entanglement, the other in mathematics, on the limit approximation of matrices of infinite dimension with finite-dimension matrices. [*Landmark Computer Science Proof Cascades through Physics and Math*].

"David Hand reviews the risks, distortions, and misinterpretations caused by missing data, by ignoring existing accurate information, or by falling for deliberately altered information and/or data. [*Dark Data*].

"In the same vein, Michael Wallace discusses the insidious perils introduced in experimental and statistical analyses by measurement errors and argues that the assumption of accuracy of data collected from observations must be recognized and questioned. [*Analysis in an Imperfect World*].

"In the midst of our book—like a big jolt on a slightly bumpy road—John Conway, Mike Paterson, and their fictive co-author Moscow, bring inimitable playfulness, multiple puns, and nonexistent self-references to bear on an easy game of numbers that (dis)proves to be trickier than it seems! [*A Headache-Causing Problem*].

"Next, Sanjoy Mahajan explains (and illustrates with examples) why some mathematical formulas and some physical phenomena change expression at certain singular points. [*A Zeroth Power is Often a Logarithm Yearning to Be Free*].

"Stan Wagon describes the counterintuitive movement of a bicycle pedal relatively to the ground, also known as the "bicycle paradox", and uses basic trigonometry to elucidate the mathematics underlying the puzzle. [*The Bicycle Paradox*].

"Jacob Siehler combines modular arithmetic and the theory of linear systems to solve a pyramid-coloring challenge. [*Tricolor Pyramids*].

"Natalie Wolchover untangles threads that connect foundational aspects of numbers with logic, information, and physical laws. [*Does Time Really Flow? New Clues Come from a Century-Old Approach to Math*].

"The late Harold Edwards pleads for a reading of the classics of mathematics on their own terms, not in the altered "Whig" interpretation given to them by the historians of mathematics. [*The Role of History in the Study of Mathematics*].

"Michael Barany uncovers archival materials surrounding the birth circumstances, the growing pains, and the political dilemmas of the Notices of the American Mathematical Society—a publication initially meant to facilitate internal communication among the members of the world's foremost mathematical society. ["*All of These Political Questions*": *Anticommunism, Racism, and the Origin of the Notices of the American Mathematical Society*].

"Mike Askew pleads for raising reasoning in mathematics education at least to the same importance give to procedural competence—and describes the various kinds of reasoning involve in the teaching and learning of mathematics. [*Reasoning as a Mathematical Habit of Mind*].

"Roger Howe compares the professional opportunities for improvement and the career structure of mathematics teachers in China and in the United States—and finds that in many respects the Chinese ways are superior to the American practices. [*Knowing and Teaching Elementary Mathematics—How are We Doing?*].

“Stephen Ramon Garcia draws on his work experience with senior undergraduate students engaged in year-end projects to distill two dozen points of advice for instructors who supervise mathematics research done by undergraduates. [*Tips for Undergraduate Research Supervisors*].

“Adam Glesser, Bogdan Suceavă, and Mihaela B. Vâjiac read (and copiously quote) Sophie Germain’s French *Essays* (not yet translated into English) to unveil a mind not only brilliant in original mathematical contributions that stand through time, but also insightful in humanistic vision. [“*The Infinite Is the Chasm in Which Our Thoughts Are Lost”: Reflections on Sophie Germain’s Essays*].

“Melvyn Nathanson raises the puzzling issues of authorship, copyright, and secrecy in mathematics research, together with many related ethical and practical questions; he comes down uncompromisingly on the side of maximum openness in sharing ideas. [*Who Owns the Theorem?*]

“In the end piece of the volume, Terence Tao candidly recalls selected adventures and misadventures of growing into one of the world’s foremost mathematicians.” [*A Close Call: How a Near Failure Propelled Me to Succeed*].

Returning to the Introduction, the Editor mentions the difficulties of compiling this volume, the main work of which was done during the height of the COVID crisis. This meant that many print-only resources were not available to him. Still, in addition to the fascinating 26 pieces of writing published here, the volume ends with a chapter on *Notable Writings*, containing a sizeable list of *Notable Journal Articles* and a list of close to 30 *Notable Journal Issues* which are “fully or partly dedicated to the specified topics—or contain symposia on the respective theme”.

The tone of the Introduction to this volume is more subdued than that of earlier volumes, and the Editor also mentions that it is shorter than in the past, and that, due to the COVID crisis, some additional material (e.g., a book list) is lacking. More concerning, the Editor indicates that the series faces an uncertain future. I sincerely hope that any difficulties facing further publication have been, or will be, overcome, and that Princeton University Press will make it possible for Mircea Pitici to continue providing this wonderful service to the community.

Call for Submissions: CMS Notes Mathematics, Outreach, Society, Accessibility and Inclusiveness Column (MOSAIC)

MOSAIC December 2022 (Vol. 54, No. 6)

The Canadian Mathematical Society (CMS) invites you to submit articles to be featured in the MOSAIC column of the CMS Notes.

MOSAIC (Mathematics, Outreach, Society, Accessibility, and Inclusiveness Column) is directed by the CMS Equity, Diversity, and Inclusion (EDI) committee.

The column offers a space of expression for you to ask, listen, learn, share experience, and propose solutions to build a more diverse, just, and stronger mathematical community. For instance, you are welcome to submit an article sharing challenges and successes in enacting EDI initiatives within your university, with competitions, outreach activities, or other events.

Your email submission should include your article in both Word and PDF formats. Please submit your article to the EDI Committee at edic@cms.math.ca.

CMS NOTES
December 2022 (Vol. 54, No. 6)

Call for Sessions: 2023 CMS Summer Meeting

December 2022 (Vol. 54, No. 6)

Call for Scientific Sessions

The Canadian Mathematical Society (CMS) welcomes and invites session proposals for the 2023 CMS Summer Meeting in Ottawa from June 2 to 5, 2023.

- Sessions are scheduled in 2.5-hour blocks and take place **June 3-5**, **2023**. Typical scientific sessions have between 10 and 20 talks of 30 minutes each but 1-hour talks are possible.
- In accordance with the CMS mandate to *propose conferences which are accessible and welcoming to all groups*, diversity amongst organizers and speakers is strongly encouraged. To support organizers in their important work and in their efforts towards inclusivity and diversity, the CMS will host an **open call for abstracts** for all sessions, and asks organizers to consider all eligible abstract submissions for their session.
- Diversity includes topics of interest, career stages, geographic location, and demographics; designated underrepresented groups include, but are not limited to, women, Indigenous Peoples, persons with disabilities, members of visible minority/racialized groups, and members of LGBTQ2+ communities.
- Note that there will be a separate follow-up call for **Education Sessions**.

Proposals should include:

1. Names, affiliations, and contact information for all session co-organizers. Early career researchers are encouraged to propose sessions.
2. A title and brief description of the topic and purpose of the session. This can include a brief overview of the subject. Include a two to three sentence summary that will be posted on the website for potential speakers.
3. The number of 2.5 hour blocks expected, with a list of possible speakers.

Proposals will be selected by the Scientific Organizing Committee, limited by available classroom space, with priority for sessions that show intention to include a mix of senior and junior researchers, to make parts of their session accessible to graduate students, and to include speakers from designated underrepresented groups.

Deadlines:

There will be two rounds of submissions. Proposals submitted by **Wednesday, December 14, 2022** to the Scientific Directors with the CMS Office in cc will be considered in the first round, with responses early in January. The deadline for the second round will be February 28, 2023.

Monica Nevins mnevins@uottawa.ca

Aaron Tikuisis Aaron.Tikuisis@uottawa.ca

CMS Office: meetings@cms.math.ca

Call for Education Sessions: 2023 Summer Meeting

CMS Meetings December 2022 (Vol. 54, No. 6)

The Canadian Mathematical Society (CMS) welcomes and invites **education session** proposals for the 2023 CMS Summer Meeting in Ottawa from June 2th to 5th, 2023.

This year, the education session proposals will be selected by the CMS Meeting Education Session Committee, which will also schedule the accepted sessions, in communication with their co-organizers.

Each proposal should follow the guidelines indicated in the call for Scientific Sessions. In addition, organizers are asked to specify the structure of their session (e.g., 20-minute talk followed by 5 minute Q&A and 5 minute transition; or a panel, or interactive session/workshop, etc.).

In parallel to the Scientific Session Proposals, there will be two rounds of submissions. Proposals submitted by **Wednesday, December 14th, 2022** will be considered in the first round, with responses early in January. The deadline for the second round will be February 28th, 2023.

Email education sessions proposals (and any questions) to:

Andie Burazin a.burazin@utoronto.ca

With Monica Nevins mnevins@uottawa.ca, Aaron Tikuisis Aaron.Tikuisis@uottawa.ca, and the CMS Office meetings@cms.math.ca in cc.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Call for Nominations: 2023 CMS Blair Spearman Doctoral Prize

Calls for Nominations December 2022 (Vol. 54, No. 6)

The CMS Blair Spearman Doctoral Prize recognizes outstanding performance by a doctoral student.

Nominations are currently welcomed for the 2023 CMS Blair Spearman Doctoral Prize.Deadline: January 31

The prize is awarded to one recipient of a Ph.D. from a Canadian university whose overall performance in graduate school is judged to be the most outstanding. Although the dissertation will be the most important criterion (the impact of the results, the creativity of the work, the quality of exposition, etc.) it will not be the only one. Other publications, activities in support of students and other accomplishments will also be considered.

Nominations that were not successful in the first competition, will be kept active for a further year (with no possibility of updating the file) and will be considered by the Doctoral Prize Selection Committee in the following year's competition.

Nominations

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding candidates for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation.

Candidates must be nominated by their university and the nominator is responsible for preparing the documentation described below, and submitting the nomination to the address below. The deadline for the receipt of nominations is indicated above.

The documentation shall consist of:

- A curriculum vitae prepared by the student.
- A resumé of the student's work written by the student and which must not exceed ten pages. The resumé should include a brief description of the thesis and why it is important, as well as of any other contributions made by the student while a doctoral student.
- Three letters of recommendation of which one should be from the thesis advisor and one from an external reviewer. A copy of the external examiner's report may be substituted for the latter. More than three letters of recommendation are not accepted.

All documentation, including letters of recommendation, should be submitted electronically, preferably in PDF format, **by the deadline date above**, to **docprize@cms.math.ca**.