October 2023

Canadian Mathematical Society
Société mathématique du Canada

(Vol. 55, No. 5)

CMS NOTES de la SMC

IN THIS ISSUE

PG

October 2023

Canadian Mathematical Society
Société mathématique du Canada

(Vol. 55, No. 5)

CMS NOTES de la SMC

PG

IN THIS ISSUE

Down the CMS memory lane

Cover Article October 2023 (Vol. 55, No. 5)

Alina Stancu (Concordia University)
CMS Director VP - Quebec

As I began my mandate as vice-president (Quebec) this past June, I took a moment to reflect on the many contributions that the Canadian Mathematical Society (CMS) makes to our community in Canada. If you are reading this, you are probably already aware of the society's activities and its multi-sided initiatives. Yet there are also various ways in which the Canadian Mathematical Society directly impacts each of its members, and others who may not even be CMS members. From this perspective, I pinpointed three moments in my own career when the Canadian Mathematical Society played a memorable role for me.

I have completed my PhD at University of Rochester under the supervision of Michael Gage who is best known for his pioneering work on the curve shortening flow. Shortly after graduation, I had the occasion to attend a CMS Winter Meeting, my first, held at Queen's University. For what I know, this chance might have come about because my advisor could not attend and suggested that I go in his place. As one of my first academic trips, I remember it very well. I flew into Montreal's Mirabel airport, which is no longer in use for commercial flights holding a touch of historical significance for the Montreal resident that I am now, and I drove to Kingston, Ontario along St. Lawrence River, and Thousand Islands. Beyond the lectures and presentations which, honestly, I do not remember well, what stands out in my memory is the excitement of meeting authors of papers in my field and engaging in conversations with them. At this meeting, I have made several connections that would shape my career and I, unknowingly, encountered some of the colleagues I now have in Montreal. While my story may not completely capture it, I experienced a sense of integration in the broader mathematical community or, at least, it undeniably felt that way back then.

In those days, before the prevalence of online collaboration tools, personal interactions were particularly important. Even today, I believe that face-to-face interactions hold great value, especially for young researchers for which we may underestimate the impact of in-presence professional meetings on their careers. It is worth pointing out that the CMS meetings have evolved since my first participation to offer a larger array of activities: mini-courses, career advice workshops, panels of discussion on various academic topics, sessions for students, and more. So, we should encourage even more our current and former students to attend the Canadian Mathematical Society's meetings.

The other two CMS memorable instances in my career occurred somewhere between six and seven years ago. Alongside an amazingly dedicated instructor, Ildiko Pelczer, I was involved in creating a weekly enriching math program for elementary and high school students in Montreal. At an early stage of the process, the Canadian Mathematical Society provided support through one of its Endowment Grants (for which a call just went out recently!) and this contributed to the creation of the *Montreal Math Circle*. Although not a substantial grant, the CMS Endowment Grant provided essential support and, implicitly, the recognition that helped secure further funding for this activity, paving its way to survive the pandemic and to continue to this day.

Lastly, during my tenure as scientific director of Quebec's *Institut des sciences mathématiques* (ISM), the institute's funding faced potential cuts. The ISM plays a vital role in coordinating and supporting financially all graduate programs in mathematical sciences in Quebec. It also serves as equal partner of the *Centre de recherches mathématiques* (CRM) in their postdoctoral program, and coordinates several activities (colloquia, student conferences, summer schools, outreach) at all levels. Close to 500 mathematicians signed then a letter addressed to the provincial government in support of the institute and its mission. Among signatories, the CMS president at the time, Lia Bronsard, lent her support. Meanwhile, I am happy to report that ISM did have his funding renewed and, eventually, increased.

While my experiences provide a modest glimpse into the meaningful impact of the Canadian Mathematical Society, I offer them as an invitation to readers to embark on their own journey of reflection. It is through the collective personal experiences that we become aware of the role that CMS plays in our lives, how important is to count on its presence, and why it deserves our wholehearted support.

Furthermore, allow me to conclude by donning my hat as one of the Scientific Directors of the upcoming CMS Winter Meeting in Montreal and extending, or perhaps renewing, a warm invitation to you, your students, and postdocs to attend the meeting this December. I hope that this Winter meeting will create some memorable moments that, one day, one of you will find worth writing about too. No, strike that! Looking at how the program shapes out, I know that it will create some notable memories that one of you could write about one day.

My lords and ladies: do any of you remember the late Lord Buckley?

Editorial October 2023 (Vol. 55, No. 5)

Professor Robert Dawson (Saint Mary's University)
Editor-in-Chief, CMS Notes

That dude was laying it down in the middle of the Two-Zero century, in an accent as real as a three-pound note and a vocabulary he'd borrowed somewhere in Harlem and never got around to taking back. And he was the hippest, swingingest, funniest cat you could hope to hear in a lifetime. You don't know what I'm talking about, you just put this editorial down and go off and straighten yourself on the Internet. I got time.

Back when Joan Baez was just a young chick, she wrote about him on an album cover; and, cats and kitties, let me tell you, there ain't much higher honour than a shout-out on a Baez album cover. And the Spiderman, that would be Canada's own Spider Robinson, he's been saying the same to anybody ready to listen since the Nazz was a carpenter.

So you're probably asking, have we flipped our editorial wig? Why (apart from pure green envy of the cat who could *really* pull the act off) pastiche the late Lord Buckley here? Because it's the beginning of a new term, and I'm lecturing to a new class, and I'm finding out all over again the joy — and the power — of standing up there at the front of the room and hamming it up. Not because the material (abstract algebra) is boring: nothing could be further from the truth. But I deeply believe that a lecture delivered with a little seasoning — and your recipe will differ from mine — will be remembered better than "the facts, ma'am. Just the facts."

You don't have to juggle the whiteboard markers or make up limericks about the folks who proved the theorems in your lecture. But think about the last conference you were at. Which talks do you remember? Maybe one of them was a dry-as-dust presentation, delivered nervously by a young colleague who's going to win a Fields Medal for that work in two years' time. But the chances are that many of the memorable talks combined interesting material with a little entertainment.

It's not just what you say. It's how you say it. So…my lords and ladies… go out there and knock their socks off.

Kseniya Garaschuk (University of the Fraser Valley)

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Kseniya Garaschuk, *University of the Fraser Valley (kseniya.garaschuk@ufv.ca)*
John Grant McLoughlin, *University of New Brunswick (johngm@unb.ca)*

As I was going through my office reorganizing it for my sabbatical, I stumbled upon my notes from the education sessions of the 2023 CMS Summer meeting. So many great ideas, all in one place. Good thing I'm forced into occasional organizing.

One particular theme stood out to me at the 2023 CMS Summer meeting (and selfishly the one theme that I never get tired of discussing). So in this piece, I highlight a few presentations with a general common theme of assessments, specifically the different ways we can think about, approach and implement assessments. This article also has a second, more subtle goal: to give readers a taste of education sessions at CMS meetings. As such, below I summarize my thoughts from a few talks, making for a mixture of presented materials and my take on it.

*****

Assessment aftermath or notes around Darja Barr's talk

As a math instructor, you have surely thought of test anxiety before. Maybe you experienced it yourself, as a student or as an educator – I myself nearly had a nervous breakdown before my PhD defense and to this day I get nervous when my students write a test. Of course tests are stressful by nature. Research shows that the higher the person's anxiety, the lower their performance on the test. However, in contrast to the actual research, we often hear the argument that higher anxiety is caused by lower preparation, so the anxiety is allegedly the middle man in this lower-preparation-lower-result game.

Not all tests are created equal. Maybe we can affect the stress levels by varying the types of assessment we give our students, for example in terms of the grade weight – high stakes versus low stakes. Sadly, each format has their pros and cons. Specifically, while low stakes tests are, in fact, low stakes and hence less stressful, they also produce a 'delayed consequences' phenomenon. This occurs when students don't consider it a big deal to fail a small test since it isn't worth much and hope to do better on the next one. Unfortunately, despite the fact that students optimistically believe that future-them is always better at math than present-them, hoping to do better is not the same as planning to do better.

While I can see many flaws with traditional testing, the assessments' transitory fleeting nature is the most significant. As an instructor, I see any mid-semester assessment as a learning opportunity; in cooking terms, it's a chance to taste the dish during the cooking process and adjust spices/temperature/consistency as necessary. However, without explicit prompting, many students do not use the test results to assess the situation, to reflect on their mathematical mistakes and adjust their approach to studying for the course. This very issue was one of the big motivators in my own research on collaborative exams. I have also used a variety of test-aftermath assignments, where students are asked to not only identify and correct their mistakes, but also go through the test and for each problem find at least 3 examples from the available course materials (lecture notes, homeworks, practice problems, quizzes, past tests) that target the same concept. So we have established that they have access to the necessary materials, but how do I know if they are using them? To allow for honest self-assessment and to prompt students to look inwards at their study habits, Darja uses a "What did I do?" survey to give students a tangible way to estimate their engagement in the course. Here is one version of it:

Post-Test Behaviour Self-Evaluation

Give yourself points for each action item:

1. *Attendance*

   – *Attended every lecture* *(2 points)*

   – *Missed a single lecture* *(1 points)*

– *Missed more than one lecture (0 points)*

2. *Practice*

– *Complete the assigned problems from each section in the text (2 points)*

– *Attempt most of the problems (1 points)*

– *Attempt some or none of the problems (0 points)*

3. *Assignments*

– *Completed every assignment (grade not important) (2 points)*

– *Missed completing one assignment (1 points)*

– *Missed completing more than one assignment (0 points)*

4. *Tutorials*

– *Attended every tutorial and completed the work (2 points)*

– *Missed a single tutorial (1 points)*

– *Missed more than one tutorial (0 points)*

5. *Seeking Help*

– *Emailed your prof with a MATH question at least twice (2 points)*

– *Emailed your prof with a MATH question once (1 points)*

– *Did not email your prof with any math questions (0 points)*

As the prof, you can decide on your minimum cut-off, include more categories for the point-worthy behaviour or exclude some that are there. But this type of survey gives a quick and easy tool for students to assess their approach to the class and think about how they position themselves for success. Furthermore, embedded within this survey are clear suggestions as to how a student could course-correct since categories provide proactive ways to engage with the class.

*****

Communicating math or notes around Fabian Parsch's talk

Any mathematics course is a communications course. A quote attributed to Einstein eloquently summarizes the idea: "You don't really understand something unless you can explain it to your grandmother". The true study of the subject is about understanding and applying various mathematical concepts, which is achieved and strengthened by students' learning to communicate these concepts to themselves and others. In my practice, I have tried a variety of things, including for example learning journals.

Fabian's context was an engineering calculus course where the instructional team focused on developing students' communication skills with motivation that, as future engineers, the students will need to communicate technical aspects to their non-technical clients. Best practice for any kind of assignments includes a draft stage, which can provide an opportunity for the student to get feedback on their work so far; once again cooking-wise, this is akin to someone else tasting your dish during preparation and making suggestions for possible adjustments. Ability to get feedback and act on it is especially important for writing assignments in technical courses – a skill that students likely have limited (if any) experience with, so building in the draft feedback cycle is a helpful step in the process. Next up is marking: grading writing assignments can be an intimidating and daunting task, so a rubric to guide both students and markers is a must and so is the process of sharing and discussing such rubric with students well before the drafts are due. This is Fabian's rubric:

	2 points	1 point	0 points
Audience	The answer could be understood by a student of this course without the need to ask questions of clarification, even if they never saw this assignment before.	The answer could mostly be understood by a student of this course, but issues persist. For example, questions of clarification would be necessary.	The answer could only be understood by someone with extensive knowledge of the subject (e.g. a TA or an instructor).
Style	The answer uses full sentences that guide the reader through the steps.	Full sentences are being used, but to limited extent and/or in isolation.	The answer is mostly a "wall of formulas" with no explanation.
Vocabulary	Accurate usage of terminology, used appropriately in context	Terminology is generally used, but is either lacking or used excessively	Vague, weak or no usage of appropriate terminology

There are many ways of developing a rubric. For instance, instead of directly providing one, you can provide examples of sample submissions and build a rubric together with students based on the discussions of what they found good or bad, helpful or frustrating, useful or obstructive in the writing samples.

There are many ways to design writing assignments as well. With the ever-improving quality of AI-generated text, we need to get a bit more creative ourselves. Here are some ideas:

- a memo on the results of a technical paper or report
- an executive summary of a technical presentation
- an explanation or critique of a piece of math found in the media this week/month

*****

Not all tests need to be written or notes around Diana Skrzydlo's talk

One of the biggest noticeable impacts of the pandemic on education was our collective re-thinking of assessment practices. With invigilated exams no longer an option, we were forced to question everything about the state of the exams, from their goals down to the format. Veselin Jungic and I wrote about our view of assessments and academic integrity in the times of COVID, but I'm sad to say that I see more and more people going "back to normal" in their teaching and not capitalizing on the lessons learned in early days of the lockdown. In particular, I am on the lookout more than ever before for more meaningful formative and summative assessment formats that underline more authentic and purposeful teaching (and, if possible, also lighten the academic integrity concerns). Enter oral exams.

Oral exam is a dialogue. It is an opportunity for students to explain conceptual ideas in words, while allowing instructors to ask for clarification. Just like any other assessment though, it needs to align with the course goals and in-term opportunities to practice oral communication. It is essential to create clear guidelines for what will be asked of the student and how the examiner will assess it, since few (if any) students come in with previous experience in this format and hence find it very intimidating.

Diana's format is as follows: 15 minutes, 5 questions each of a different type and drawing on a different section or topic of a course. The 5 questions types are:

- definition,
- advantages/disadvantages,
- compare/contrast,
- describe a process,
- predict an impact.

Here are some specific examples from her statistics course:

- Compare and contrast: Binomial vs Poisson distribution, Type I vs Type II error.
- Describe a procedure: joint transformation, Bayesian estimation, generating random walk/Brownian Motion.

For practice, students are encouraged to think of at least one of each type of question for each section of the material. Note that with this approach, the questions target different levels of Bloom's taxonomy: from remembering and applying in the definitions and describing procedures to evaluating and creating in compare/contrast and predicting impact categories.

*****

Some of the most inspirational are my notes and thoughts from the session run by Peter Taylor and Chris Suurtamm that involved a panel with high school teachers. While other sessions largely focused on specific teaching tools and techniques, this session focused on overall approach and philosophy of teaching. I often think about similarities and differences in teaching math and English in a standard first year university class. Both are service courses meaning the audience consists of mostly non-majors in the corresponding subject, both are mandatory classes that students often do not wish to be enrolled in, both appear to students as a necessary evil that isn't relevant to their future careers or current interests, both carry the reputation of "I was never good at...", both a mathematical solution and a well-organized essay are just a series of logical steps presented within the specified context, both aim to teach communication and general critical

thinking skills rather than being heavy in the discipline-specific jargon. However, take a look at the course outlines. English courses give outcomes, not context: they mention types of analysis or writing students will do (rhetoric analysis, close reading, response and reflection writing), but not what specific works they will use in the course (Shakespeare vs Tolstoy). Math courses give content, not outcomes: students will be able to apply differentiation rules, row-reduce a matrix, calculate probability distributions and so on with no mention of neither communicating their findings nor developing higher-level problem solving skills, analytical and computational thinking that demonstrate the power and relevance of mathematics to mathematicians and non-mathematicians alike. Of course the two subjects of math and English differ by nature, but contrasting the two helps me focus on the ultimate goal stated by someone in the session: we need to be teaching more than the alphabet of math.

*****

What you see above is a snapshot of just a small portion of the talks. I took many more notes about many other topics. If you've read this far, there are two "morals" I would like to get across. First of all, go to the education sessions at the CMS meetings. If teaching is a part of your everyday life, you will find many inspirational and very practical ideas in these education sessions to bring into your own classroom. Secondly, it was August, two months since the conference and this was the first time I looked through my notes from the session. I am generally a good note taker, so the notes were easy to follow as I re-discovered some ideas that I thought were great back in June. Clearly, I need to become more systematic about reviewing notes I take for future development of my teaching practice. But also in general, as a community, we need to do better with sharing and archiving this information. For people who couldn't make it to the conference or couldn't make it to a particular talk, how can CMS collect and make available the materials and ideas that are being developed by educators across Canada that we can all benefit from? My first attempt was writing this piece; if you have other ideas, please contact me – I'm happy to talk more.

Annette Rouleau (VP of Education and Engagement at the Julia Robinson Mathematics Festival (jrmf.org))

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

Kseniya Garaschuk, *University of the Fraser Valley (kseniya.garaschuk@ufv.ca)*
John Grant McLoughlin, *University of New Brunswick (johngm@unb.ca)*

The tables are missing! Not exactly what you want to hear when you have invited over 500 students to have some fun exploring math with you. But there were a few benches and a gym with a lot of floor space, so I announced that this was going to be JRMF's first ever Floor festival. And it was wonderful! Sitting (or laying!) all across the gym, the students were genuinely engaged in JRMF's math activities.

Figure 1. Cup Stacking

JRMF, or the Julia Robinson Mathematics Festival (jrmf.org), is a nonprofit organization that connects K-12 students with math experiences that foster a positive mathematical identity. We achieve this by designing hands-on math activities that require problem-solving and collaboration, and by training teachers, parents, and community members on how to lead these engaging, yet challenging activities. We also host (or help others host) Math Festivals, which are an opportunity for children to gather and explore mathematics through our activities. So far in 2023, we have had almost 200 Math Festivals across North America that have reached almost 30,000 children, of whom one-third attend underserved and underrepresented schools.

As the majority of math events for children are usually held outside regular school hours, holding math festivals during the school day is something we have been encouraging others to consider. It feels like an inclusive way to bring engaging, challenging mathematics to children who might not otherwise be able to attend a math event. During a JRMF in-school math festival, the math activities are typically led by two older classes of students, with one class volunteering for the morning and the other volunteering for the afternoon. It is truly wonderful to hear a grade 6 student gently encouraging a grade 2 student to "Tell me more about your thinking" or bravely asking a grade 9 student if they want to try the next challenge.

Why a Math Festival?

It is not a secret that many students experience math anxiety and even more profess to dislike math entirely. Unfortunately, this leads to math avoidance, which closes the door to many opportunities as they become adults (Namkung et al., 2019). We want to change that, so, at a JRMF Math Festival, we offer the kind of meaningful math students seldom encounter, yet is so

necessary for developing a love of mathematics and cultivating an interest in pursuing it further. In doing so, we focus on four areas:

Promoting inclusion.

We know that extracurricular math programming has a long history of targeting high-achieving students. This results in disproportionately high levels of participation by males who thrive on competition and by those who have the support required to register and attend after school math events.

Often overlooked are females and students of color — the same children who have also traditionally been excluded in the math classroom. Successful inclusion of these particular groups of children means allowing them the voice and agency to freely express ideas and grapple openly with math tasks (Schettino, 2016) .

Building confidence.

Math is often presented with a one-size-fits-all approach, which quickly devolves into the artificial binary of being a math person or not. At a Math Festival, we offer a different approach. We design open-ended activities with multiple entry points that allow students to gain confidence from their initial success and then build on that as they tackle more challenging mathematics (Lambdin, 2002). They have the opportunity to see themselves as mathematical beings, often for the first time.

Inspiring joy.

Many children (and adults) find math tedious, yet joy has everything to do with wanting to learn and engage in math. Skills such as persistence, determination, and willingness to problem solve lay at its very foundation. So, at a Math Festival, students learn through play which helps learners of all ages better assimilate new ideas and leads to deeper conceptual and procedural mathematical understandings (De Holton et al., 2001).

Creating community.

Finally, we encourage collaboration. Mathematics learning is a social endeavor, not individual (Burton, 2002). Students need opportunities to empower themselves and each other in a productive struggle with challenging problems.

There is a burning need for joyful math as school districts deal with the aftermath of successive years of learning interruptions. And JRMF brings that joyful math, and it is contagious. If you ever have, or have had, the chance to attend a JRMF Math Festival, you'll understand why. To see a whole gym full of children eagerly engaging in collaborative problem solving is powerful and makes one want to see it happen more. This is true in the schools we visit — once a school community has participated in a festival hosted by us, they immediately realize its value. And, more importantly, in the manner of 'teach someone to fish', they also recognize that this is something that they could do on their own. And that's really what we want — entire communities of people connecting children with joyful math, not just us. We like to imagine a world where math is so celebrated that we have made JRMF redundant!

If you are interested in hosting a festival and are wondering how to get started, please reach out to annette.rouleau@jrmf.org . We work closely with K-12 schools, but we also support faculty at post-secondary institutions who are interested in math events that connect local children with their campus. Additionally, all of our resources are freely available on our website (https://jrmf.org/puzzle/) and are an amazing collection of math activities suitable to a wide range of interests and abilities.

Figure 2. Pentominoes

Figure 3. Star Battle

References

Burton, L. (2002). Recognising commonalities and reconciling differences in mathematics education. *Educational Studies in Mathematics*, 50(2), 157–175.

De Holton, D., Ahmed, A., Williams, H., & Hill, C. (2001). On the importance of mathematical play. *International Journal of Mathematical Education in Science and Technology*, 32(3), 401–415.

Lambdin, Diana. (2002) Benefits of teaching through problem solving. In F. Lester (Ed.), *Teaching mathematics through problem solving* (pp. 3–14). Reston, Virginia: NCTM.

Namkung, J., Peng, P., & Lin, X. (2019). The relation between mathematics anxiety and mathematics performance among school-aged students: A meta-analysis. Review of Educational Research, 89(3). 459-496. https://doi.org/10.3102%2F0034654319843494

Schettino, C. (2016). A framework for problem-based learning: Teaching mathematics with a relational problem-based pedagogy. *Interdisciplinary Journal of Problem-Based Learning*, 10(2), 12.

Annette Rouleau is VP of Education and Engagement at the Julia Robinson Mathematics Festival (jrmf.org) and her passion is to share the joy and wonder that is possible in the teaching and learning of mathematics, yet so seldom experienced.

A former elementary teacher, she studied for her doctorate at Simon Fraser University under Peter Liljedahl and has taught mathematics pedagogy courses to both preservice and inservice teachers. She also conducts professional development workshops on Building Thinking Classrooms.

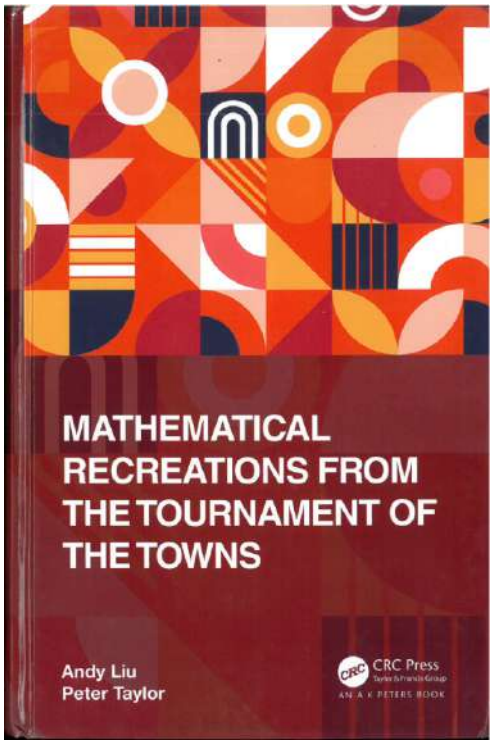
Professor Robert Dawson (Saint Mary's University)
Editor-in-Chief, CMS Notes

The Tournament of the Towns was first held in Moscow, Leningrad, and Riga in 1980. Since then it has spread to other towns and other countries. Until recently, several Canadian cities participated: I write this review in the hope that the Russian government will soon return to sanity, and such friendly competition may again be possible.

The Tournament was founded by the late Nikolay N. Konstantinov, as a less-elitist alternative to the International Math Olympiad. The papers are written locally, allowing as many students as wish to to participate. Junior and senior students write different papers. Within each age group, there are fall and spring sessions, each consisting of an easy ("O-level") paper and a harder ("A-level") paper two weeks later. There are thus eight five-question papers a year, of which one student can write up to four. While the same problem may appear as a hard question on the junior paper and a more lightly-weighted easy question on the senior paper, the contest generates about thirty problems per year. This book contains all the problems from the Tournament of the Towns from Fall 2007 to Spring 2021, and their solutions.

As a useful bonus, as well as the complete chronological listing, the book contains a number of suggested "highlight" sets. In Part I of the book are eighteen themed sets with a particularly recreational flavor; in part II are selections focusing on arithmetic, geometry, and combinatorics.

Tournament problems can be very difficult: the Senior A-level problems are often comparable with those on the IMO. They tend, however, to have more of the flavor of recreational mathematics than Olympiad problems do. Problems often involve wizards, knight, and dragons; and some problems involving a particularly counterintuitive result are hinted at by the introduction of that notorious teller of tall tales, Baron Münchausen! The solver may expect to make more use of parity and the Pigeonhole Principle than of Jensen's inequality or obscure triangle geometry.



By Andy Liu and Peter Taylor
CRC Press, Boca Raton, 2023
978-1-032-35292-3

There are occasional places where the wording of a problem is not entirely clear. For instance, in problem S.O.3 for 2010 we find:

Is it possible to cover the surface of a regular octahedron with several regular hexagons, without gaps or overlaps?

It is, but the hexagons must be interpreted as flexible, which is nowhere stated. (This is presumably not the fault of the editors, but how the question appeared in the English-language version of the Tournament.) Arguably it's a better problem this way, requiring a little lateral thinking!

On the other hand, problem C3-4 on page 45 seems to have an actual error:

Alice and Betty are sixteen, Carla is fifteen, Debra is fourteen, and Ellen is thirteen. They want to cross the river in a boat. No girl may be in the boat alone, and no two girls whose ages differ by more than 2 may be in the boat at the same time. Is this task possible?

As asked, the solution's almost trivial: Alice, Betty, Carla, and Debra cross, then Carla and Debra go back for Ellen. The given answer (requiring nine crossings) suggests that the question should have read "by two or more."

This book will be of interest to any moderately advanced math puzzler, and useful to anybody training for a math contest at the high school or university level. It should definitely be in every university and high school library, and on many private bookshelves. The online price is under C$75, so price should not be too much of an obstacle. (I've seen it discounted well below this price, though it doesn't seem to be so at the time of reviewing.)

CMS NOTES 75 1945-2020

Teaching Calculus Through History's Lens

CSHPM Notes October 2023 (Vol. 55, No. 5)

Eugene Boman (Pennsylvania State University Harrisburg)

Robert Rogers (State University of New York at Fredonia)

CSHPM Notes bring scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

Amy Ackerberg-Hastings, *Independent Scholar (aackerbe@verizon.net)*
Hardy Grant, *York University [retired] (hardygrant@yahoo.com)*

Try this. Go to your library and, for each decade after 1950, take down a typical Calculus textbook first published during those ten years. You will find that, by and large, most of the textbooks used to teach Differential Calculus in college classrooms for the past 70 or 80 years have presented the topics in essentially the following order. (The dates below indicate when each concept was introduced. Obviously, these should be taken with a grain of salt. New concepts tend to emerge slowly rather than via a single invention.)

- Limits (1823)
- Continuity (1817)
- The Derivative (1797)
- Differentiation Rules (1684) and Tangent Lines
- Implicit Differentiation (1684)
- Related Rates (1665)
- Rolle's Theorem (1691), the Mean Value Theorem (1797), and Extreme Value Theorem (1874)
- The First Derivative Test and Curve Sketching
- The Second Derivative Test and Curve Sketching
- Applied Optimization
- Antiderivatives

Logically, this makes perfect sense. We start with the theoretical foundation and build from there. But the differentials on which Leibniz founded his Calculus were used for nearly 200 years before limits were invented to replace them. Notice that differentials don't appear in our list. They sometimes make a small cameo appearance in a section on approximations, but they are otherwise absent from standard teaching practices.

Note also that, as we read this list from top to bottom, we move backwards in time. This is common in mathematics teaching. In order to present a "clean" finished product to students we start with the foundational underpinnings and then move on to the motivating applications.

Think about your own research. When wrestling with a research problem are the foundational questions top-of-mind, or do you keep those on the back burner until you've actually made progress? Learning new mathematics is a creative process, whether one is doing original research or learning in a classroom. But when we teach in reverse-historical order, we are illustrating neither how mathematics is best created nor how it is best learned. We are illustrating how it is best presented. We (the authors) believe that preserving the historical order helps students see and appreciate the creative, problem-solving aspect of our discipline better than the reverse-historical order that has been the norm for some decades.

Replacing limits with differentials as the basis for Calculus is entirely within reach of first-year students if we are willing to proceed intuitively. Indeed, Leibniz based his "Calculus Differentialis" (Calculus of Differentials) on the notion of a differential because it is both intuitive and visually suggestive. He knew it was not a strong logical foundation, but his goals were to construct tangents and to solve optimization problems in a systematic way—his Calculus was a means to those ends. This is manifest from the title of his first publication on the topic:

> A New Method for Maxima and Minima as Well as Tangents, Which is Impeded Neither by Fractional Nor by Irrational Quantities, and a Remarkable Type of Calculus for This [1].

On the other hand, the limit concept was suggested by Newton, refined by Cauchy (and others), and finalized by Weierstrass in 1874, nearly 200 years after Leibniz. Like all of mathematics, the limit concept was invented to address very specific questions—in this case, how can we be sure that the computational procedures described by Leibniz (and Newton) really work? Perhaps more important, when do they fail? When we begin a Calculus course with the non-intuitive subtlety of the limit concept, we are asking students to understand the answers to those questions without actually asking the questions first, or even indicating why they are important. Although this may be good mathematics, it seems to us to be poor pedagogy. It is like teaching a student to solve a crossword puzzle by giving them the grid, the answers, and their locations, and telling them to fill in the grid. Certainly, something will be learned this way. And, we can build from there. But how many students would stick with us until it gets interesting? Would you?

Rather than starting at the logical beginning, suppose we start at the chronological beginning, with Leibniz' Calculus of Differentials. The differentiation rules are easy to explain intuitively using differentials and diagrams. As Leibniz stated in his paper, "The Demonstration of all this [the differentiation rules] will be easy to one who is experienced in these matters [infinitesimals]" [1]. A beginning Calculus student might balk at calling them "easy," but once differentials are accepted, the differentiation rules are straightforward. In fact, the Constant Rule $(d(constant) = 0)$

and the Sum Rule $(d(x+y) = dx + dy)$ are directly analogous to their finite counterparts. The subtleties of differentials will force a later switch to limits, but having students accept these objects in the beginning is akin to having beginning geometry students accept that there is a geometric object called a point.

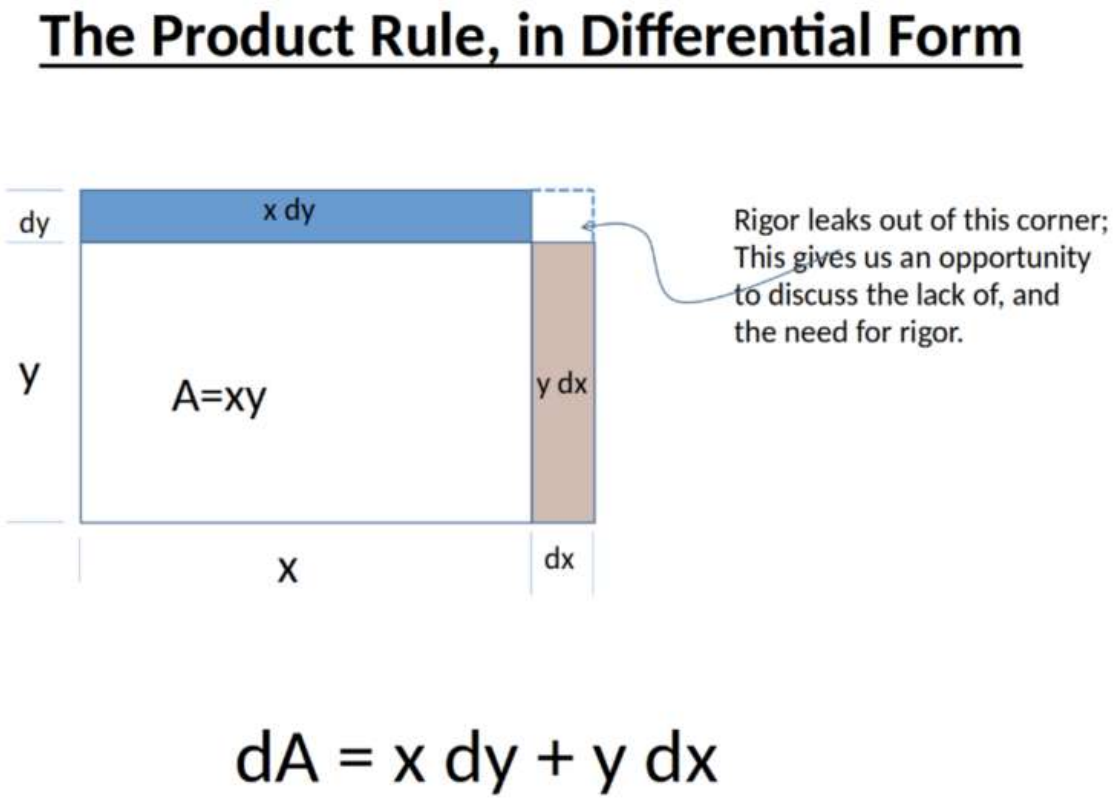


Figure 1. Diagrammatic proof of the Product Rule. Diagram created by the authors.

Figure 1 shows a simple proof of the Product Rule. In order to believe that the Product Rule is valid, a student only needs to accept that the corner indicated can be safely ignored. This is not a rigorous proof. But it is intuitive and therefore believable. Moreover, the lack of rigor can be pointed out in passing, thereby sowing the seeds of curiosity: "I wonder what a rigorous proof would look like?"

We contend that a clear, but intuitive, explanation will give the beginner more faith in a computational tool than any fully rigorous, but abstruse proof. All that is required to use a tool effectively is faith that it is the right tool for the job—and practice. Lots of practice.

Once these first three differentiation rules are known, the Constant Multiple Rule, the Power Rule, and the Quotient Rule all follow without much fuss (assuming that functions involved are differentiable). They can all be developed, and substantial practice given, fairly quickly.

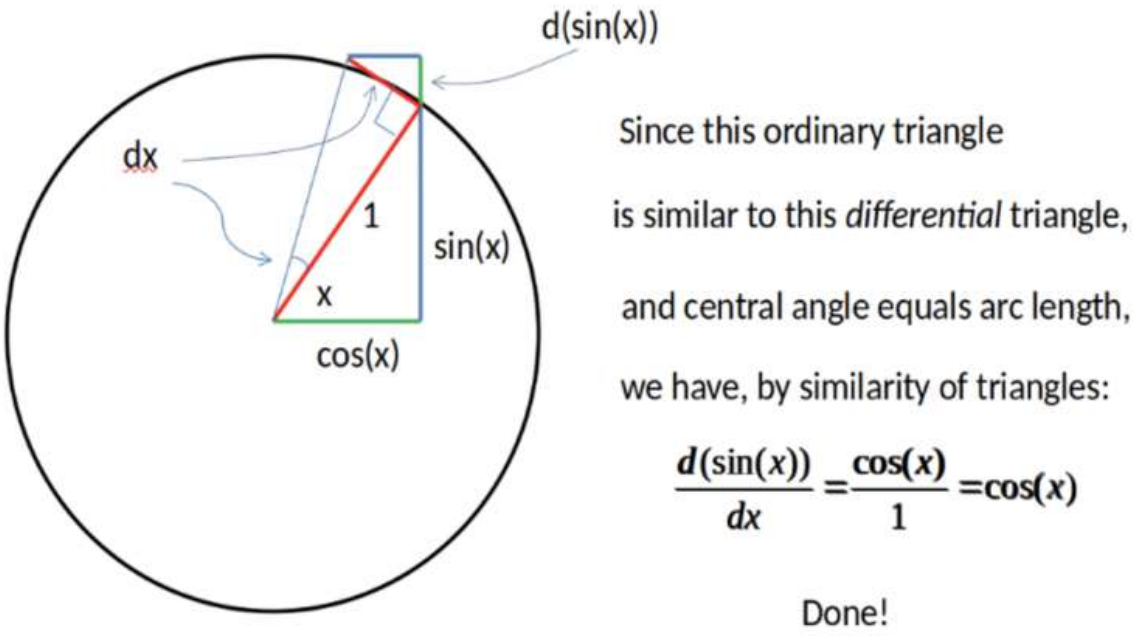


Figure 2. Proof by differentials of $\frac{d(sin(x))}{dx} = cos(x)$. Diagram created by the authors.

Differentiating the trigonometric functions without limits is similarly straightforward. For example, Figure 2 shows a differential-based proof that the derivative of sin *x* is cos *x*. (This proof was devised by Roger Cotes, a colleague of Newton.)

Again, nothing here is meant to be rigorous. We simply provide some *ad hoc*, intuitive arguments that can be used to justify the differentiation rules without limits so that we can quickly move on to using the computational tools of Calculus.

For beginning students, the use of function notation and Lagrange's prime notation—$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$—can make the differentiation of composed functions unnecessarily unwieldy. But using differentials renders this moot. For example, if we let $z = \sin x$ then

$$d(\sin^2 x) = d(z^2) = 2zdz = 2(\sin x)d(\sin x) = 2\sin x \cos x dx.$$

When we use differentials, topics such as implicit differentiation, the chain rule, and related rates do not warrant special consideration. For example, given an equation such as $x^2 + y^2 = 1$, direct application of the differentiation rules yields $2xdx + 2ydy = 0$. If we wish to compute a slope, we divide by dx to obtain $\frac{dy}{dx} = \frac{-x}{y}$. If we wish to see how the rates of change of x and y are related, we divide by dt to obtain $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ or, in Newton's fluxion notation, $2x\dot{x} + 2y\dot{y} = 0$. There is no need to "remember the chain rule" because the chain rule did not exist in either Newton's or Leibniz' version of Calculus.

In fact, the phrase "chain rule" doesn't seem to appear in Calculus texts until the late 19th or early 20th centuries, although earlier arithmetic and algebra books used the term to describe the computations involved in changing units (e.g. $\frac{feet}{second} = \frac{mile}{hour} \cdot \frac{feet}{mile} \cdot \frac{hour}{minute} \cdot \frac{minute}{second}$), which is very similar to the Calculus chain rule expressed in differential form. We conjecture that textbook authors simply co-opted the name of the older rule for use in Calculus.

Computations of this kind are similar to, and are therefore good preparation for, the types of calculations students will be expected to perform later in contexts where differentials are already heavily relied upon. Think of integration by substitution, integration by parts, and line and path integrals.

Furthermore, this approach is not constrained to single-variable functions. For example, if $z = x^2y^3$ then

$$dz = x^2(3y^2dy) + y^3(2xdx) = 2xy^3dx + 3x^2y^2dy = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy.$$

Notice that the partial derivatives emerged quite naturally.

Freedom from the constraint of one-variable Calculus can be a powerful tool. Consider the derivation of Snell's Law of Refraction using Figure 3.

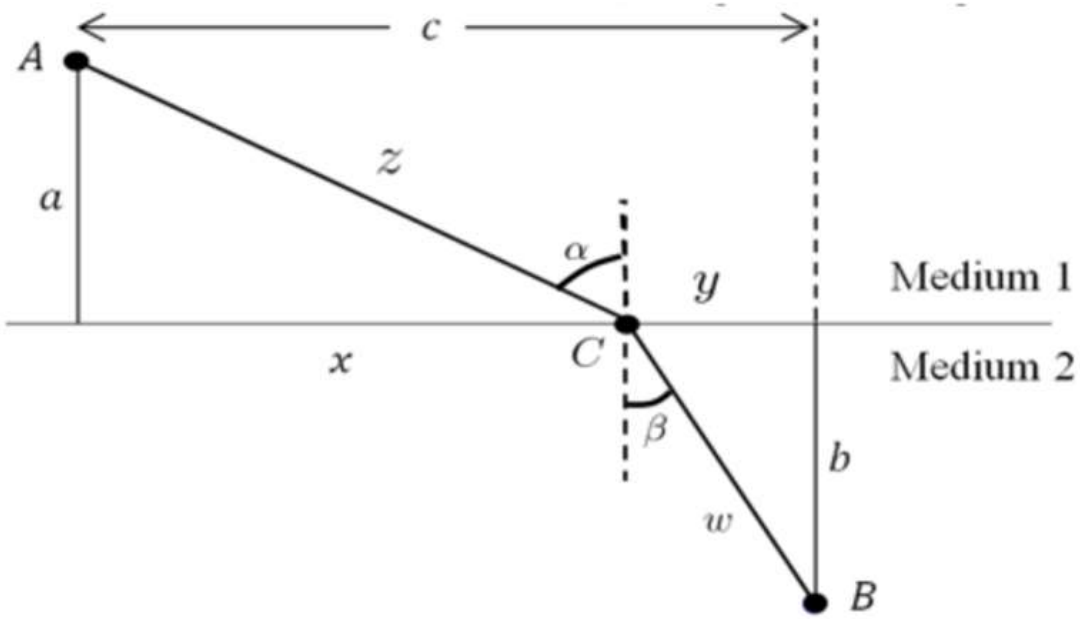


Figure 3. Visualization of the derivation of Snell's Law of Refraction. Diagram created by the authors.

Let v_1 and v_2 represent the velocities of light in medium 1 and medium 2, respectively. The total time for light to travel from point A to point B is given in terms of the single variable x by

$$T(x) = \frac{\sqrt{a^2+x^2}}{v_1} + \frac{\sqrt{b^2+(c-x)^2}}{v_2}.$$

Minimizing $T(x)$ to find the path that light travels is messy, and it only gives us the value of x that minimizes $T(x)$. To derive Snell's Law we still have to show that $\frac{\sin\alpha}{v_1} = \frac{\sin\beta}{v_2}$. If instead we write down our objective function

$$T = \frac{z}{v_1} + \frac{w}{v_2}$$

along with all of our constraints

$$x + y = c, \qquad z^2 = x^2 + a^2, \qquad w^2 = y^2 + b^2$$

in terms of the variables that appear in the diagram and then differentiate (using differentials) we get

$$dx + dy = 0, \qquad 2zdz = 2xdx, \qquad 2wdw = 2ydy$$

so that

$$dT = \frac{dz}{v_1} + \frac{dw}{v_2} = \frac{1}{v_1}\cdot\frac{x}{z}dx + \frac{1}{v_2}\cdot\frac{y}{w}dy = \frac{1}{v_1}\cdot\frac{x}{z}dx - \frac{1}{v_2}\cdot\frac{y}{w}dx.$$

Setting $\frac{dT}{dx} = 0$ gives us $\frac{(\frac{x}{z})}{v_1} = \frac{(\frac{y}{w})}{v_2}$ or $\frac{\sin\alpha}{v_1} = \frac{\sin\beta}{v_2}$, which is Snell's Law.

Treating $\frac{dT}{dx}$ as a ratio of two differentials rather than as a single entity can streamline the use of Calculus as a problem-solving tool in many instances.

You may not yet be fully convinced that the "differentials first" approach is better pedagogy, but if you are still reading you are clearly intrigued and you are probably wondering where you could find a textbook that takes this approach.

We're so glad you asked. We have written such a book, and we have used it ourselves in the classroom. Our text, *Differential Calculus: From Practice to Theory*, is an Open Educational Resource (OER) that you are welcome to download from https://milneopentextbooks.org/differential-calculus-from-practice-to-theory/ and use in your Differential Calculus course at no cost to you or your students. It has a Creative Commons license, which means that you are welcome to use it as is or to alter it to suit your needs. We deliberately designed our textbook so that a student learning from it will end the first semester with at least the same skills and understanding as a student learning from a "limits first" approach.

Part I of our textbook (*From Practice*) presents "Calculus Differentialis" as the intuitive, focused-on-problem-solving "Remarkable Calculus" that Leibniz described in his paper [1]. The differentiation rules are introduced intuitively, leaning heavily on the notion of the differential. We do not hide the questionable nature of the differential; we just don't dwell on it. When the use of differentials becomes problematic we point that out so the student is aware of the issue and, hopefully, becomes curious. But we defer the resolution of this foundational question to Part II (*to Theory*), where we develop the limit concept with full epsilon/delta rigor.

This is a theme throughout our textbook. Substantial questions (e.g., "What is the shape of a hanging chain?") are often introduced before they can be easily answered. The students are then led to partial solutions. Once the necessary theory and techniques have been developed the problem is then re-addressed. The goal is to induce curiosity, and to provoke questions in the student ("If these differentials aren't really a viable foundation, what is?")—and thus reproduce the research experience as much as possible in the context of a Calculus classroom.

Reference

[1] Leibniz, Gottfried Wilhelm. (1684, Oct.) Nova methodus pro maximis et minimis, itemque tangentibus, quae nec fractas, nec irrationales quantitates moratur, et singular pro illis calcului genus. *Acta Eruditorum*, 467–473.

Eugene Boman *is Professor Emeritus of Mathematics at the Harrisburg campus of the Pennsylvania State University. In 2008 he won the Carl B. Allendorfer Award for excellence in expository mathematical writing from the editors of* Mathematics Magazine *for the article "Mom! There's an Astroid in My Closet" (Mathematics Magazine 80 (2007), 247–273).*

Robert Rogers *is a SUNY Distinguished Teaching Professor of Mathematics at SUNY Fredonia where he has been on the faculty since 1987. He has held leadership positions in the Association of Mathematics Teachers of New York State and the Mathematical Association of America Seaway Section, edited the* New York State Mathematics Teachers' Journal, *and received numerous teaching and service awards, including membership in the New York State Mathematics Educators' Hall of Fame.*

P. Mark Kayll (University of Montana, Missoula, MT, USA)

Abstract

We present a short proof of Sylow's famous 'First Theorem.' Stripped to its essentials, the proof—attributed to Wielandt (1959)—relies only on basic concepts (equivalence relations and divisibility). Whereas most textbook proofs invoke plenty of group-theoretic jargon (stabilizers, conjugacy classes, etc.), this one avoids all that, stands alone, and fits on a page.

A (binary) million years ago, Helmut Wielandt published an appealing counting proof [3] of Sylow's First Theorem. Stripped to its bare essentials, it requires surprisingly little group theory and ought to be better known. The version below lends itself to an hour or so in a classroom with second- or third-year university math enthusiasts.

Theorem ('Sylow's First') *If p is a prime, r is an integer coprime to p, and G is a group of order $p^\alpha r$, then G contains a(t least one) subgroup of order p^α.*

Proof. Let $\mathcal{S}$ denote the family of all sub***sets*** of G of size p^α, and write $\mathcal{S} = \{A_1, A_2, \ldots, A_n\}$. We shall argue that at least one member of $\mathcal{S}$ is a group—indeed, is a subgroup of G.

It's convenient to know that p and n are coprime, so we first establish this fact. We have

$$n = \binom{p^\alpha r}{p^\alpha} = \frac{p^\alpha r(p^\alpha r-1)(p^\alpha r-2)\cdots(p^\alpha r-(p^\alpha-1))}{p^\alpha\cdot 1\cdot 2\cdot\cdots\cdot(p^\alpha-1)} = r\prod_{k=1}^{p^\alpha-1}\frac{p^\alpha r-k}{k}.$$

Consider the factors $\frac{p^\alpha r-k}{k}$. If $p \nmid k$, then $p \nmid (p^\alpha r - k)$. On the other hand, if $p \mid k$, then $k = p^\beta s$ for some integer β with $1 \le \beta < \alpha$ and $p \nmid s$. So here, we have

$$\frac{p^\alpha r-k}{k} = \frac{p^\beta(p^{\alpha-\beta}r-s)}{p^\beta s} = \frac{p^{\alpha-\beta}r-s}{s},$$

and $p \nmid p^{\alpha-\beta}r - s$ (for otherwise, $p \mid s$). In either case, p fails to divide each factor $\frac{p^\alpha r-k}{k}$, and thus we see that $p \nmid n$.

Now if $x \in G$ and $A_i \in \mathcal{S}$, then the set $A_i x$ also contains p^α elements (for the map $A_i \to A_i x$ given by $a \mapsto ax$ is injective). Hence, $A_i x = A_j$ for some $j \in \{1, \ldots, n\}$. Let us define a relation $\sim$ on $\mathcal{S}$ by

$$A_i \sim A_j \iff A_i x = A_j \text{ for some } x \in G.$$

Using the group axioms for G, it's easy to see that $\sim$ is an equivalence relation. Since $p \nmid n$, at least one equivalence class $\mathcal{C}$ of $\sim$ contains some q sets A_i with $p \nmid q$; say $\mathcal{C} = \{A_1, A_2, \ldots, A_q\}$ (relabelling the A_i's as necessary).

Let $H = \{x \in G\colon A_1 x = A_1\}$; one easily verifies that H is a subgroup of G—write h for its order. For $x, y \in G$, observe that

$$A_1 x = A_1 y \Leftrightarrow A_1 x y^{-1} = A_1 \Leftrightarrow xy^{-1} \in H \Leftrightarrow Hxy^{-1} = H \Leftrightarrow Hx = Hy;$$

therefore, $A_1x \neq A_1y$ if and only if $Hx \neq Hy$. It follows from this equivalence and the definition of $\mathcal{C}$ that $\mathcal{C}$'s order q coincides with the number of distinct (right) cosets of H in G; i.e., $q = p^\alpha r/h$, or $h = p^\alpha r/q$. But since $p \nmid q$, this implies that $q \mid r$, so that $h \geq p^\alpha$.

To see the reverse inequality, consider an element $a \in A_1$. The definition of H implies that the (left) coset aH is a subset of A_1; whence,

$$h = |H| = |aH| \leq |A_1| = p^\alpha,$$

and it now follows that $h = p^\alpha$. Therefore, the subgroup H of G is indeed a member of $\mathcal{S}$ □
.

Remark Other authors—e.g., [1, 2]—have presented Wielandt's proof; Professor Petrich (see below) distilled it particularly nicely on one chalkboard.

In memoriam

Submitted in honour of **Mario Petrich (1932–2021)**, who showed me (and the rest of the class) this proof at Simon Fraser University in 1986.

References

[1] I.N. Herstein, *Topics in Algebra*, Second edition, Xerox College Publishing, Lexington MA, 1975.

[2] I. Martin Isaacs, *Algebra. A Graduate Course*, Brooks/Cole, Pacific Grove CA, 1994.

[3] Helmut Wielandt, Ein Beweis für die Existenz der Sylowgruppen, *Arch. Math.* (Basel), **10** (1959), 401–402.

AMS (MOS) Subject Classifications: 20D20, 05A10, 00A05, 11-01

Call for Submissions: CMS Notes Mathematics, Outreach, Society, Accessibility and Inclusiveness Column (MOSAIC)

The Canadian Mathematical Society (CMS) invites you to submit articles to be featured in the MOSAIC column of the CMS Notes.

MOSAIC (Mathematics, Outreach, Society, Accessibility, and Inclusiveness Column) is directed by the CMS Equity, Diversity, and Inclusion (EDI) committee.

The column offers a space of expression for you to ask, listen, learn, share experience, and propose solutions to build a more diverse, just, and stronger mathematical community. For instance, you are welcome to submit an article sharing challenges and successes in enacting EDI initiatives within your university, with competitions, outreach activities, or other events.

Your email submission should include your article in both Word and PDF formats. Please submit your article to the EDI Committee at mosaic@cms.math.ca.

Join a CMS Committee!

Call of Interest for CMS Committee Membership

The Canadian Mathematical Society Nominating Committee invites expressions of interest in membership on the following committees. CMS committee members must hold CMS membership, however applicants need not be current members. Terms commence on January 1, 2024 and run for 3 years.

Current and upcoming committee vacancies:

- Education: 1 vacancy
- Endowment Grants: 2 vacancies (Ontario and Quebec representation preferred)
- Equity, Diversity and Inclusion Committee: 1 vacancy
- Finance: 1 vacancy
- Human Rights: 2 vacancies
- International Affairs: 2 vacancies
- International Prize: 2 vacancies
- Invested Funds: 2 vacancies
- Publications: 2 vacancies
- Reconciliation in Mathematics: 3 vacancy (Pacific, West, Atlantic)
- Research: 1 vacancy
- Women In Mathematics: 2 vacancies (Pacific, Ontario)

Terms of Reference for each committee can be found here.

How to express interest

Please send a Letter of Interest to chair-nomc@cms.math.ca with the following information:

1. Your name
2. Your career stage
3. Current university or institutional affiliation
4. Name of committee(s) you wish to join
5. Expression of interest in the particular committee(s): Why you want to be on this committee, or what you would do on this committee (this can be brief if you have a clear vision, or longer if need be.)
6. For International Affairs, Publications and Research, please also indicate your research domain.

Applicants are encouraged to self-identify. The information will be used by the Nominating Committee to ensure committees are diverse in their representation. The information may also be used in aggregate to report on CMS equity, diversity and inclusion initiatives. The information provided will be kept confidential.

Please submit your letter no later than November 17, 2023.

Who should apply

We encourage everyone to consider becoming an engaged member of a CMS committee, however, we particularly welcome people who have not previously served with the CMS, or identify with, are connected to, or have experience with historically excluded groups:

- Racialized, Black, and/or People of Colour ("Visible Minorities")
- People with disabilities (including invisible and episodic disabilities)
- 2SLGBTQIA+ and/or gender and sexually diverse individuals
- "Aboriginal" and/or Indigenous Peoples (First Nation Peoples, Métis Nation, and Inuit)
- Women

If you are excited about participating in CMS activities but you aren't sure if your past experiences align perfectly with a given role/committee, the Nominating Committee encourages you to express your interest.

Determination of membership

Every committee of the CMS operates under its own Terms of Reference (TOR). The Nominating Committee will take into account the current composition of each committee and its TOR when selecting persons for nomination. Once nominated, approval is required by the CMS Executive and Board prior to appointment.

Feedback

Feedback on this new approach and suggestions on how to advance diverse representation on CMS committees are welcomed. Please send comments and suggestions to: chair-nomc@cms.math.ca.

One final push

If you have ever wanted to be more involved with the CMS or you would like to champion a particular cause or activity, please submit a Letter of Interest in being a member of a CMS Committee. Sometimes people are hesitant to put themselves forward, or just need some encouragement or support, whether they are a student, post-doc or established veteran. If you know someone who would be a good fit for a CMS committee, please encourage them to submit a Letter of Interest. **Become involved to build a stronger mathematical community!**

2024 CMS Excellence in Teaching Award

The CMS Excellence in Teaching Award Selection Committee invites nominations for the **2024 Excellence in Teaching Award**.

The **Excellence in Teaching Award** focuses on the recipient's proven excellence as a teacher at the undergraduate level, including at universities, colleges and CÉGEPs, as exemplified by unusual effectiveness in the classroom and/or commitment and dedication to teaching and to students. The dossier should provide evidence of the effectiveness and impact of the nominee's teaching. The prize recognizes sustained and distinguished contributions in teaching at the post-secondary undergraduate level at a Canadian institution. Only full-time teachers or professors who have been at their institution for at least five years will be considered. The nomination will remain active for three years, with a possibility to update.

The CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding colleagues regardless of race, gender, ethnicity or sexual orientation.

A nomination will consist of:

- a signed nominating statement from a present or past colleague, or collaborator (no more than three pages) having direct knowledge of the nominee's contribution;
- a curriculum vitae (maximum five pages);
- three letters of support, at least one from a former student (who has followed a course more than a year ago) and one from the chair of the nominee's unit. The letter of the Chair of the nominee's unit could include a one-page summary on information from student evaluations, or similar information;
- other supporting material (maximum 10 pages).

Nominations and reference letters should be submitted electronically, preferably in PDF format, to: **etaward@cms.math.ca** no later than the deadline of **November 15, 2023**.

2024 CMS Math Competition Grants

The CMS is now accepting applications for the ***2024 CMS Math Competition Grants*** program. The CMS supports activities that promote the learning of mathematics among Canadian youth. In addition to the Society's math competitions, the CMS offers math competition grants for activities at the elementary and secondary school levels.

The deadline for submissions is **November 15, 2023**. Successful applicants will be informed in January 2024 and the grants issued in February 2024.

Further details and guidelines about the math competitions grants and the **application process are available on the CMS website here**.

The Committee on Grants for Provincial Competitions (CGPC) adjudicates proposals for support. Should you have further questions or comments, please contact the Committee by e-mail at **chair-grants-pc@cms.math.ca**

Applications should be submitted electronically using the online application form and additional documents preferably in PDF format, **no later than November 15, 2023** to **mathgrants@cms.math.ca**.

Call for Speakers

2023 CMS Winter Meeting

Call for Speakers and Abstracts

The Canadian Mathematical Society (CMS) invites you to submit an abstract to participate in one of the **planned sessions at the 2023 CMS Winter Meeting** which will take place from December 1-4, 2023.

The Canadian Mathematical Society has created an **open** abstract submission process to support session organizers in their important work and in their efforts towards inclusivity and diversity. We encourage applications from members who identify as part of traditionally under-represented groups, including, but not limited to: Women, Indigenous Peoples, Persons with Disabilities, Members of Visible Minorities and/or Racialized Groups, and members of the LGBTQ+ Community. The CMS also welcomes applications from Graduate Students.

If you are already an invited speaker, please do not submit your abstract through our submission system. Submit your abstract here for invited speakers.

Deadlines:

Candidates are required to submit their abstracts for approval by the session organizers by using our abstract submission form by no later than **November 6, 2023.** Please submit your abstract for only one session.

The CMS encourages organizers to review submitted abstracts on an ongoing basis and to accept all eligible speakers. Successful applicants (invited speakers) must register for the meeting and submit their abstract to the CMS website by **November 13, 2023**.

To Submit Your Abstract:

For Non-Invited Speakers

To submit your abstract to a session, please click here.

The Canadian Mathematical Society (CMS) welcomes and invites session proposals and mini-course proposals for the 2024 CMS Summer Meeting in Saskatoon from May 31 – June 3, 2024. In accordance with the CMS mandate to propose conferences that are accessible and welcoming to all groups, diversity amongst organizers and speakers is strongly encouraged. Diversity includes topics of interest, career stages, geographic location, and demographics.

CALL FOR SESSIONS:

Proposals should include:
(1) Names, affiliations, and contact information for all session co-organizers. Early career researchers are encouraged to propose sessions.
(2) A title and brief description of the topic and purpose of the session. This can include an overview of the subject.
(3) The total number of expected talks, with a list of possible speakers and/or papers in the theme. Sessions should strive to respect the above CMS policy of accessibility and diversity.

Open Call for Abstracts: The CMS will continue the open abstract submission process that was recently introduced to support session organizers in their important work and in their efforts towards inclusivity and diversity.

The CMS kindly asks session organizers to consider all eligible abstract submissions for their session, as up to 30 speakers per session can be accommodated.

The scientific sessions will take place from May 31 – June 3, 2024.

Deadline: Proposals should be submitted by **Wednesday, January 31, 2024** to the Scientific Directors and the CMS Office should be cc'ed. There will be a second deadline of **March 29, 2024**, but earlier submissions will be considered first. Their contact information is as follows:

Elana Kalashnikov: e2kalash@uwaterloo.ca
Steven Rayan: rayan@math.usask.ca
Jacek Szmigielski: szmigiel@math.usask.ca

Sarah Watson: meetings@cms.math.ca

La Société mathématique du Canada (SMC) sollicite des propositions de sessions scientifiques et de mini-cours pour sa Réunion d'été 2024, qui se tiendra à Saskatoon du 31 mai au 3 juin. Conformément à son mandat de proposer des congrès accessibles et accueillants pour tous les groupes, la SMC encourage fortement la diversité parmi les personnes qui organisent ses réunions ou y donnent des conférences. La diversité s'applique aux domaines d'intérêt, à l'étape de la carrière, à l'emplacement géographique et aux caractéristiques démographiques.

APPEL DE SESSIONS :

Les propositions doivent inclure :
1) Les noms, affiliations et coordonnées de tous les co-organisateurs de sessions. On encourage les chercheurs en début de carrière à proposer des sessions.
2) Un titre et une brève description du sujet et de l'objectif de la session; peut aussi comprendre un aperçu du sujet.
3) Le nombre de conférenciers attendus, avec une liste de communications et/ou de conférenciers potentiels pour le thème. Dans la mesure du possible, les sessions devraient respecter la politique d'accessibilité et d'accueil de la SMC.

Appel ouvert de résumés : La SMC met en place un appel ouvert de résumés pour aider les organisateurs de sessions dans leur important travail et dans leurs efforts d'inclusion et de diversité.
La SMC vous prie de considérer les soumissions de tout candidat admissible. Nous jusqu'à 30 conférenciers par session seront accommodés.

Les sessions scientifiques se dérouleront du 31 mai au 3 juin 2024.

La date limite pour présenter une proposition de session ou de mini-cours est le **mercredi 31 janvier 2024.** Une deuxième date limite sera fixée au **29 mars 2024**, mais les demandes antérieures seront examinées en premier lieu. Toute demande doit être envoyée aux Directeurs scientifiques et le bureau de la SMC doit y être copié. Vous trouverez ci-dessous leurs coordonnées :

Elana Kalashnikov : e2kalash@uwaterloo.ca
Steven Rayan : rayan@math.usask.ca
Jacek Szmigielski : szmigiel@math.usask.ca

Sarah Watson: meetings@cms.math.ca

The new **2023 CMS Online Education Meeting** is a new CMS-organized event. Planned to complement the in-person mathematics education sessions at the Summer and Winter CMS meetings, the 2023 CMS Online Education Meeting will feature plenary talks and presentations on various themes in mathematics education and provide ample time for comments and discussion. These meetings will take place over two days, once a year, prior to the in-person CMS Winter meetings.

The CMS will handle the organization and registration for these events. For those who register for the in-person CMS Winter meeting, registration for the preceding online meeting is free.

The inaugural CMS Online Education Meeting is scheduled for **November 25th and 26th, 2023**. Information about the meeting (schedule, themes, etc.) will be posted on the CMS website. A call for presentations will go out soon, with the submission deadline of **October 13th, 2023.**

Call for Proposals

he Canadian Mathematical Society (CMS) welcomes and invites education presentation proposals for its new event, the 2023 CMS Online Education Winter Meeting from November 25-26, 2023.

For this meeting, we will accept proposals on any theme in math education.

Education presentation proposals will be selected by the CMS Meeting Education Session Committee, which will also schedule the accepted sessions, in communication with their proposer(s).

Proposals should include:

1. Names, affiliations, and contact information of the presenter(s). Early career researchers are welcome to propose presentations.
2. A title and a brief abstract of the presentation.

All presentations will be of the standard CMS length: 20 minutes presentation + 5-10 Q&A.

The deadline for the presentation proposals is **October 13th, 2023**. There is a limited number of presentation spots. Preferences will be given to early submissions.

Please send your education session proposals (and questions) to:

- Andie Burazin a.burazin@utoronto.ca
- CMS Office, Sarah Watson, meetings@cms.math.ca in cc.

Call for University Hosts: Winter '25 / Summer '27

The Canadian Mathematical Society (CMS) welcomes and invites host proposals from Canadian Universities for the 2025 CMS Winter Meeting, and the CMS Summer Meeting for 2027.

CMS will provide all logistical support and contract negotiation with local venues. CMS is looking for Canadian Universities that are willing and able to showcase their department and University to students and faculty from across Canada. It is asked that proposals include the following information:

1. Location

- How would people get from the airport to the venue?
- What are the reasons your city may be of interest to Canadian Mathematicians?

2. Site

(For summer meetings) Describe the University where the meeting would be held.

- Which building would the meeting be in and how many rooms are available for meeting sessions and plenaries?
- What technological support is available in session rooms?
- Will these rooms be available during the proposed dates?

3. Lodging

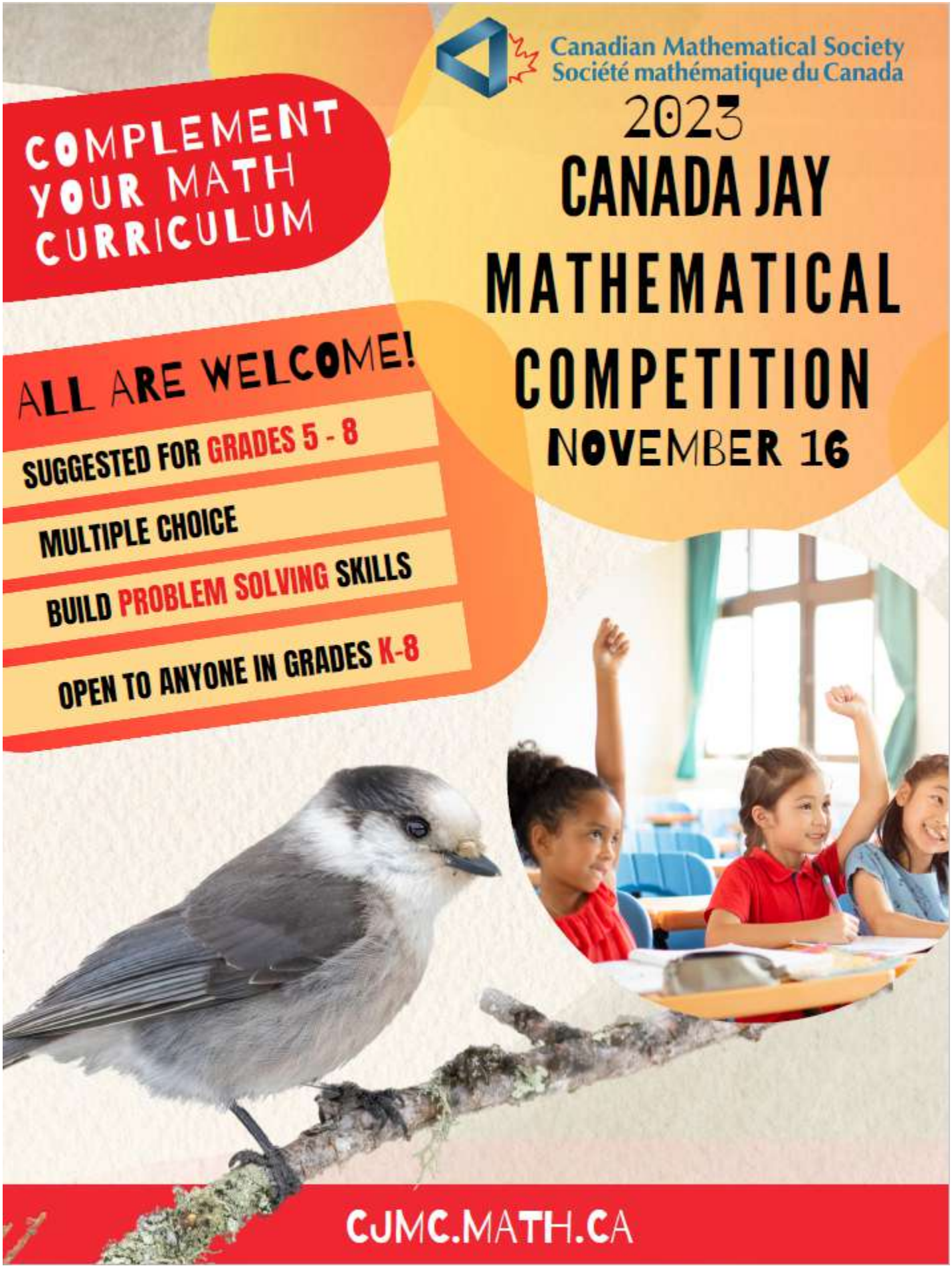
Is your university able to offer any residence lodging during the conference dates? CMS will take care of contracting and negotiating with hotels.

4. Host University

Please describe your institution and department briefly.

- What funding support will the Host University have for the CMS Meeting?
- Is the University available for regular calls and updates on the meeting's progress?
- Can the Host University commit and provide at least one scientific director for the meeting?
- What level of participation do you think there might be from academics at your institution?

The CMS Meetings typically run from Friday to Monday on the first weekend in June and December but we are open to other possibilities. Summer meetings typically have 250-350 registrants and winter meetings are typically 400-600 in larger cities. Please admit your submissions to Sarah Watson (meetings@cms.math.ca).


Canadian Mathematical Society
Société mathématique du Canada
2023
CANADA JAY
MATHEMATICAL
COMPETITION
NOVEMBER 16
COMPLEMENT YOUR MATH CURRICULUM
ALL ARE WELCOME!
SUGGESTED FOR GRADES 5 - 8
MULTIPLE CHOICE
BUILD PROBLEM SOLVING SKILLS
OPEN TO ANYONE IN GRADES K-8
CJMC.MATH.CA

Tenure Track Faculty Position in Fluid Mechanics
Department of Applied Mathematics
University of Waterloo

The Department of Applied Mathematics at the University of Waterloo invites applications for a tenure-track Assistant Professor position in **Fluid Mechanics**. Areas of particular interest include environmental and geophysical fluid dynamics and sustainability related to climate, atmosphere, oceans and lakes. In special cases a position at the rank of Associate or Full Professor may be considered. The successful candidate will be expected to establish an outstanding research program. We are looking for applicants with an enthusiasm for teaching at both the undergraduate and graduate level, and for the supervision of graduate research. In exceptional cases, outstanding applicants in other areas of Applied Mathematics may also be considered.

The Department of Applied Mathematics is one of four departments that, together with the School of Computer Science, comprise the Faculty of Mathematics at the University of Waterloo. The department has 30 regular faculty members, and has leading research programs in Scientific Computing Methods, Mathematical & Quantum Physics, Environmental & Geophysical Fluid Dynamics, Control & Dynamical Systems, and Mathematical Medicine & Biology. With 300 faculty members, 8,000 undergraduate students and more than 1,000 graduate students in mathematics and computer science, Waterloo's Faculty of Mathematics is a global powerhouse in research, education and innovation. Research in the department is enhanced by interdisciplinary and industrial collaborations and links to interdisciplinary institutes including the Waterloo Climate Institute, the Water Institute, the Waterloo Institute for Sustainable Aeronautics, the Waterloo Artificial Intelligence Institute, the Centre for Computational Mathematics, the Institute for Quantum Computing, and the Perimeter Institute for Theoretical Physics. The department has a substantial graduate program with over 100 graduate students pursuing Masters or PhD degrees, and strong undergraduate programs in applied mathematics, scientific computing and mathematical physics. More information about the department can be found at https://uwaterloo.ca/applied-mathematics/.

Candidates interested in this position should have a PhD or equivalent in Applied Mathematics or a related field. The salary range for this position is $110,000-$160,000. Salary will be commensurate with qualifications, experience, and research record. Negotiations beyond this salary range will be considered for exceptionally qualified candidates. The effective date of appointment is July 1, 2024. Interested individuals should apply using **MathJobs** (https://www.mathjobs.org/jobs/list/23161). Applications should include a cover letter, a curriculum vitae, research and teaching statements, teaching evaluation summaries (if available) and up to three reprints/preprints. Applicants are welcome to include a statement on Equity, Diversity and Inclusion. In addition, applicants should arrange to have at least three reference letters submitted on their behalf.

Applications will be reviewed starting **November 1, 2023**, but all complete applications received by **December 1, 2023** will receive full consideration.

The University of Waterloo understands the impact that career interruptions (e.g. parental leave, leave due to illness) can have on a candidate's achievement and encourages potential candidates to explain in their application the impact this may have on their record; this information will be taken into careful consideration during the assessment process.

If you have any questions regarding the position, the application process, assessment process, eligibility, or a request for accommodation during the hiring process, please contact: Prof. Hans De Sterck, Chair, Department of Applied Mathematics, University of Waterloo, Canada (hdesterck@uwaterloo.ca).
The University of Waterloo acknowledges that much of our work takes place on the traditional territory of the Neutral, Anishinaabeg and Haudenosaunee peoples. Our main campus is situated on the Haldimand Tract, the land granted to the Six Nations that includes six miles on each side of the Grand River. Our active work toward reconciliation takes place across our campuses through research, learning, teaching, and community building, and is centralized within our Indigenous Initiatives Office (https://uwaterloo.ca/human-rights-equity-inclusion/indigenousinitiatives).

The University values the diverse and intersectional identities of its students, faculty, and staff. The University regards equity and diversity as an integral part of academic excellence and is committed to accessibility for all employees. The University of Waterloo seeks applicants who embrace our values of equity, anti-racism and inclusion. As such, we encourage applications from candidates who have been historically disadvantaged and marginalized, including people with disabilities and applicants who identify as Indigenous peoples (e.g., First Nations, Métis, Inuit/Inuk), Black, racialized, women and/or 2SLGBTQ+.

The University of Waterloo is committed to accessibility for persons with disabilities. If you have any application, interview or workplace accommodation requests, please contact Maureen Fraser (mcfraser@uwaterloo.ca).

All qualified candidates are encouraged to apply; however, Canadians and permanent residents will be given priority.

Three reasons to apply: https://uwaterloo.ca/faculty-association/why-waterloo.

Tenure Track Faculty Position in Scientific Machine Learning
Department of Applied Mathematics
University of Waterloo

The Department of Applied Mathematics at the University of Waterloo invites applications for a tenure-track Assistant Professor position in the area of **Scientific Machine Learning**. In special cases a position at the rank of Associate or Full Professor may be considered. The successful candidate will be expected to establish an outstanding research program. The department is particularly interested in methods or applications of Scientific Machine Learning that relate to the departmental areas of strength: Scientific Computing Methods, Mathematical & Quantum Physics, Environmental & Geophysical Fluid Dynamics, Control & Dynamical Systems, and Mathematical Medicine & Biology. We are looking for applicants with an enthusiasm for teaching at both the undergraduate and graduate level, and for the supervision of graduate research. In exceptional cases, outstanding applicants in other areas of Applied Mathematics may also be considered.

The Department of Applied Mathematics is one of four departments that, together with the School of Computer Science, comprise the Faculty of Mathematics at the University of Waterloo. With 300 faculty members, 8,000 undergraduate students and more than 1,000 graduate students in mathematics and computer science, Waterloo's Faculty of Mathematics is a global powerhouse in research, education and innovation. The Applied Mathematics department has 30 regular faculty members. Research in the department is enhanced by interdisciplinary and industrial collaborations and links to interdisciplinary institutes including the Waterloo Artificial Intelligence Institute, the Centre for Computational Mathematics, the Waterloo Climate Institute, the Perimeter Institute for Theoretical Physics, and the Institute for Quantum Computing. The department has a substantial graduate program with over 100 graduate students pursuing Masters or PhD degrees, and strong undergraduate programs in applied mathematics, scientific computing and mathematical physics. More information about the department can be found at https://uwaterloo.ca/applied-mathematics/.

Candidates interested in this position should have a PhD or equivalent in Applied Mathematics or a related field. The salary range for this position is $110,000-$160,000. Salary will be commensurate with qualifications, experience, and research record. Negotiations beyond this salary range will be considered for exceptionally qualified candidates. The effective date of appointment is July 1, 2024. Interested individuals should apply using **MathJobs** (https://www.mathjobs.org/jobs/list/23162).

Applications should include a cover letter, a curriculum vitae, research and teaching statements, teaching evaluation summaries (if available) and up to three reprints/preprints. Applicants are welcome to include a statement on Equity, Diversity and Inclusion. In addition, applicants should arrange to have at least three reference letters submitted on their behalf. Applications will be reviewed starting **November 1, 2023**, but all complete applications received by **December 1, 2023** will receive full consideration.

The University of Waterloo understands the impact that career interruptions (e.g. parental leave, leave due to illness) can have on a candidate's achievement and encourages potential candidates to explain in their application the impact this may have on their record; this information will be taken into careful consideration during the assessment process.

If you have any questions regarding the position, the application process, assessment process, eligibility, or a request for accommodation during the hiring process, please contact: Prof. Hans De Sterck, Chair, Department of Applied Mathematics, University of Waterloo, Canada (hdesterck@uwaterloo.ca).

The University of Waterloo acknowledges that much of our work takes place on the traditional territory of the Neutral, Anishinaabeg and Haudenosaunee peoples. Our main campus is situated on the Haldimand Tract, the land granted to the Six Nations that includes six miles on each side of the Grand River. Our active work toward reconciliation takes place across our campuses through research, learning, teaching, and community building, and is centralized within our Indigenous Initiatives Office (https://uwaterloo.ca/human-rights-equity-inclusion/indigenousinitiatives).

The University values the diverse and intersectional identities of its students, faculty, and staff. The University regards equity and diversity as an integral part of academic excellence and is committed to accessibility for all employees. The University of Waterloo seeks applicants who embrace our values of equity, anti-racism and inclusion. As such, we encourage applications from candidates who have been historically disadvantaged and marginalized, including people with disabilities and applicants who identify as Indigenous peoples (e.g., First Nations, Métis, Inuit/Inuk), Black, racialized, women and/or 2SLGBTQ+.

The University of Waterloo is committed to accessibility for persons with disabilities. If you have any application, interview or workplace accommodation requests, please contact Maureen Fraser (mcfraser@uwaterloo.ca).

All qualified candidates are encouraged to apply; however, Canadians and permanent residents will be given priority.

Three reasons to apply: https://uwaterloo.ca/faculty-association/why-waterloo.