Good Math Ed Problems

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Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

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Always on the lookout for good math ed problems, and after years and years of looking, I can announce, openly, that I am riddled with good (and other) math ed problems. Yes, I am, here, making a distinction between good *math problems* and good *math ed problems*.

Based on my perpetual pursuit, I have found that good math problems, for the most part, are good math ed problems (e.g., the Monty Hall problem). However, and the impetus for this article, many good math ed problems are just not always good math problems. Resultantly, good math ed problems don't always get the same level of attention as good math problems.

Let's shed some light, then, on what I consider a very good math ed problem. I ask that you indulge me and, please, take a moment and answer the following problem:

> *A cone and ice cream together cost $3.50. If the ice cream costs $3.00 more than the cone, how much did the cone cost?*

With your answer in tow, I again ask for your indulgence and, only if you wish, of course, please take a moment to contemplate on what might make the cost of the cone and ice cream problem a good math ed problem.

Just so that we're all on the same page, the correct answer to the above problem is 25 cents. Presented in a bit more detail, should the cone cost $0.25 then the ice cream, which costs $3 more than the cone, would cost $3.25, and the cone ($0.25) and the ice cream ($3.25) together ($0.25 + $3.25) would cost $3.50. A simple solution to a seemingly simple problem.

The kicker, however, is that many people think, at least initially, that the correct answer is 50 cents. The problem, now widely known with different numbers and different objects as the "bat and ball problem" (see, for example, Kahneman's book, *Thinking, Fast and Slow*), has had a life of its own in the field of psychology for many years. That our intuition (yet again) leads us astray, to a mathematically incorrect answer, that is, is arguably at the root of how the bat and ball problem has gained such prominence in the field of psychology. I contend, this famous psychology "problem" is a good math ed problem.

As one argument for obtaining the incorrect answer goes, the ice cream costing $3.00 more than the cone gets interpreted as (or substituted for) the cost of the ice cream; and, as such, if the ice cream costs $3.00 and the cone and the ice cream cost $3.50 then the cone would, simply, cost $0.50. Alternatively stated, if the cone were $0.50 and the ice cream were $3.00 then, yes, the cone and the ice cream would cost $3.50 and everything would be fine – except that is not the case. In the words of Red Auerbach, "It's not what you say, it's what they hear.". Even with the words of Red ringing in my ears, I know that the cone and ice cream problem, a just-ok math problem, is a good math ed problem.

Beyond knowing, I would even argue that the problem is a very good math ed problem. I know this because I give the cone and ice cream problem to future teachers of elementary school mathematics each semester. In class, after attempting to answer the cone and ice cream problem on their own at first, students are placed in groups of three and tasked with explaining, to me, epitomizing someone that "just doesn't get it", how the answer to the problem is not 50 cents?! Walking around the room, watching and listening to students fumbling with explaining and understanding the problem and the solution, opens a weird window into the teaching and learning of mathematics, which, for me, is at the very core of a good math ed problem.

Usually after my first pass, I grab the attention of the room to tell everyone that declaring to me the correct answer is 25 cents but doing so in a higher volume and more aggressive tone each time I tell them "I just don't get it…" is not the greatest teaching strategy in the world. I also tell them that while they might be comfortable using the John von Neumann approach (i.e., "In mathematics you don't understand things, you just get used to them.") in their future math class, it is not an acceptable teaching method for explaining, to me, that the answer to the cone and ice cream problem is not fifty cents. "Let's try this again…", I declare.

My second pass around the room is akin to watching tigers find their stripes. The we-love-algebra table quickly emerges, for example. Proudly calling me over to show that they have defined a variable and "Let x" (not c) be the cost, in dollars, of the cone is nice, I tell them, but it's still, I also tell them, just a different version of declaring the right answer, albeit more formally (for them) because algebra is now involved. Also emerging, the not-50-cents table. Pointing to some scrappy calculations, I am told to follow along as the group explains that fifty cents, when combined with three dollars and fifty cents makes four dollars. Looking for some sort of reaction from me at this point, I let the group know that it is always a good idea to be thinking "What do you think they thought?". What I am really looking for during my second pass, however, are those brave individuals, those who will admit that they still don't understand the solution, which is key to my third and final pass around the room.

Once again getting the attention of the entire room, I make note that we, as a class, seem to be in a good spot. Some groups have, convincingly, shown me that the correct answer is 25 cents. Other groups have, also convincingly, shown me that the answer is not 50 cents. The only problem, I tell the room, is that even though many of them are now brimming with confidence, there are still some individuals that, admittedly, do not yet see what they see. As a result, I ask each group to devise a plan, a strategy, an approach, a method (whatever they want to call it) to help understand how someone got the incorrect answer; and, then, help that same someone get from their incorrect answer (of 50 cents or some other answer) to the correct answer (of 25 cents).

Every once and a while, my faith in the teaching and learning of mathematics gets restored. Having a room full of future math teachers, all devising a plan to help each and every last one of them understand the solution to the cone and ice cream problem is one of those moments. That they are helping and care, however, is not where my faith gets restored. It's how they do it.

Called the "Goldilocks and the 'Five' Examples" approach one year, groups start to make neat and tidy tables showing that the cone could not cost, for example, a penny. If so, the cone (a penny) and ice cream (three bucks and a penny) would together cost \$3.02. Next, they demonstrate that the cone could not cost a dime because the cone (a dime) and ice cream (three bucks and a dime) together would cost \$3.20. Leaving a spot blank in the middle of the table (for a quarter), groups go on to demonstrate how the cone costing 35 cents, 50 cents or one dollar would make the final cost, when taking the cone and ice cream together, \$3.70, \$4.00 and \$5.00, respectively. I've asked. For some, seeing the conspicuous blank line in the middle of the table was key for them to getting it, especially seeing \$0.35. For others, seeing the column of \$3.02, \$3.20, \$3.70, \$4.00 and \$5.00, especially when presented as \$3.02 (\$0.01 + \$3.01), \$3.20 (\$0.10 + \$3.10), \$3.70 (\$0.35 + \$3.35), \$4.00 (\$0.50 + \$3.50), \$5.00 (\$1.00 + \$4.00) is key to getting it. For me, I see future math teachers, interestingly, relying on an intuitive approach to the teaching and learning of mathematics to, early on in their career path, some success.

It would be hard to convince me that the 5 to 10 minutes it takes to get all future teachers of elementary school mathematics on board with the correct solution to the cone and ice cream problem is not worth it. Everyone, arguably, benefits from the time spent on the problem. In addition to the benefits for the students, as I mentioned, it restores my faith in the teaching and learning of mathematics, but not because the students show they care. The students are future teachers – of course they care. Rather, my faith gets restored because associated with a math problem that leads our intuition astray, mathematically, there might just be another intuition at play, a teaching and learning of mathematics intuition, that helps us to combat our mathematically incorrect answer, which stems from our intuition (once again) leading us astray. Intuition versus intuition?!

Ultimately, then, good math ed problems are, yes, about mathematics, but they, at their core, easily open a window into the world of teaching and learning of mathematics. If you have a good math ed problem, we'd love to hear about it, here, in the Education Notes section of CMS Notes. While I am littered with good math ed problems, Kseniya and I would like nothing more than the pages of Education Notes to also be littered with good math ed problems. A little window dressing, if you will.
