For $A + B \rightleftharpoons C + D$

$$K_c = \frac{[C][D]}{[A][B]}$$ and $\text{Rate}_{\text{forwards}} = k[A][B]$

$\text{Rate}_{\text{backwards}} = k'[C][D]$

From RHS

$$\frac{\text{Rate}_f}{k} = [A][B] \quad -(1)$$

$$\frac{\text{Rate}_b}{k'} = [C][D] \quad -(2)$$

$$K_c = \frac{[C][D]}{[A][B]} \quad -(3)$$

Subbing (1) and (2) into (3)

$$K_c = \frac{\frac{\text{Rate}_b}{k'}}{\frac{\text{Rate}_f}{k}}$$

at dynamic equilibrium Ratef = Rateb

$$\therefore K_c = \frac{\frac{\text{Rate}_b}{k'}}{\frac{\text{Rate}_f}{k}} = \frac{k}{k'}$$ ← neither k nor k' is affected by pressure or concentration

← In addition, a catalyst increases Ratef and Rateb by the same factor hence K_c is unchanged if a catalyst is present

$$\therefore K_c = \frac{(a)k}{(a)k'} = \frac{k}{k'}$$