The Optimal Macro Tariff*

Oleg Itskhoki
itskhoki@fas.harvard.edu

Dmitry Mukhin
d.mukhin@lse.ac.uk

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Abstract

Can tariffs close a long-run trade deficit? Is the optimal tariff larger for countries running persistent trade deficits? We argue that the answer to these questions depends crucially on a country's external balance sheet which shapes financial valuation effects of trade policy. An import tariff appreciates the real exchange rate, raising the value of the country's foreign liabilities and transferring wealth to the rest of the world. This improves the long-run trade balance, but has a negative welfare effect and reduces the size of the optimal tariff regardless of the *net* asset position or trade deficit. When calibrated to the U.S. economy, its large external dollar liabilities reduce the optimal tariff from 35% to 9% and provide a hedge for its trade partners against a trade war. Furthermore, tariffs induce a retrenchment of external asset positions, partially eroding "exorbitant privilege" for a country that enjoys excess returns on its international portfolio.

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1 Introduction

After decades of trade liberalization, tariffs and trade wars have returned to the center stage of economic policymaking. Despite many rationales for this shift given by the policymakers, including strategic rivalry, supply-chain resilience, and fiscal considerations, none has been as prominently cited as the goal of reducing trade deficits. Indeed, after running trade deficits for more than forty years, the United States announced on April 2, 2025 ("Liberation Day") a new set of import tariffs explicitly calibrated to bilateral trade imbalances, imposing higher duties on countries whose exports to the U.S. exceeded corresponding imports.

This raises two central policy questions. First, from a positive perspective, can trade taxes be used to manage external imbalances? Second, from a normative perspective, is it optimal to set higher tariffs when a country runs a trade deficit? This paper addresses these questions by bridging the gap between two largely separate literatures — trade literature which typically studies tariffs under balanced trade and international macroeconomics which focuses on global imbalances with limited attention to trade policy.

A simple accounting identity provides an important starting point for our analysis as it clarifies the channels through which tariffs can (and, conversely, cannot) change trade imbalances. In particular, a balance of payments requires that $\mathcal{B}_t = \mathcal{R}_t \mathcal{B}_{t-1} + NX_t$, where NX_t is the country's trade balance, $\mathcal{B}_t$ is the net foreign asset position, and $\mathcal{R}_t$ is the realized return on the entire NFA position. Aggregating this condition over time and discounting by a factor $\bar{R} > 1$, we obtain the country's intertemporal budget constraint:[1]

$$\underbrace{-\sum_{t=0}^{\infty} \bar{R}^{-t} NX_t}_{\text{long-run trade deficit}} = \underbrace{\bar{R}\mathcal{B}_{-1}}_{\substack{\text{exogenous}\\\text{initial NFA}}} + \underbrace{(\mathcal{R}_0 - \bar{R})\mathcal{B}_{-1}}_{\substack{\text{on-impact}\\\text{valuation effect}}} + \underbrace{\sum_{t=1}^{\infty} \bar{R}^{-t} \left(\mathcal{R}_t - \bar{R}\right) \mathcal{B}_{t-1}}_{\text{future realized excess returns}}. \tag{1}$$

In words, to sustain a long-run trade deficit on the left-hand side of (1), a country must have a sufficiently positive net foreign asset position (inclusive of the valuation effect in response to a shock or policy change) and/or systematically earn abnormal returns ("exorbitant privilege") on its international portfolio (Gourinchas and Rey 2007a). As an accounting identity, this condition must always hold, in the data as well as in any theory.

The key implication of the budget constraint (1) is that trade policies cannot change the long-run trade deficit unless they generate a valuation effect on the existing net foreign asset position or alter future excess returns on the country's international portfolio. This is trivially true in standard trade models without intertemporal trade, where $\mathcal{B}_t \equiv 0$ and trade is balanced under any policy. The same insight applies more broadly to environments with non-zero

[1] See Appendix A2 for derivation. If a no-bubble condition on net foreign assets (liabilities) is violated, the corresponding term can be combined with the last term in (1). One possible choice for the constant discount factor $\bar{R}$ is the inverse of the unconditional mean of a stochastic discount factor corresponding to the average risk-free interest rate. Note that excess return may reflect both risk premia and/or "convenience yields" of the country's assets and liabilities.

asset positions and exogenous returns. In such cases, the common claim that a country can always close its external imbalance by imposing arbitrarily large import and export tariffs is incorrect. While these policies affect relative prices and allocations, the resulting exchange rate adjustment ensures that the long-run trade (im)balance remains unchanged.[2]

However, it would be equally incorrect to conclude that trade policies generically have no effect on permanent imbalances. Even if, as is often argued, (permanent) tariffs have limited intertemporal effects and do not substantially alter aggregate saving and investment — the variables that determine the current account — they do nonetheless affect returns on the international portfolio. To show this, we use a standard two-country Armington model with cross-border positions in equity, FDI, bonds and/or loans. We show that the external portfolio returns $\mathcal{R}$ depend on the real exchange rate and its appreciation increases the relative value of the country's external liabilities, hence, generating a negative valuation effect (Gourinchas and Rey 2007b). Since tariff shocks move the exchange rate, trade policies induce valuation effects and therefore have implications for the long-run trade balance.

Importantly, an import tariff and an export tax are no longer perfect substitutes in the presence of gross asset positions, breaking Lerner (1936) symmetry. Intuitively, the two instruments have the same effect on the terms of trade — the ratio of import and export prices. Yet, the two trade taxes move the real exchange rate in opposite directions: an export tax raises the foreign price level and depreciates the home exchange rate, whereas an import tariff raises domestic prices and appreciates the real exchange rate. Without cross-border asset positions, the terms of trade are a sufficient statistic for the equilibrium allocation, and the two policies are equivalent. With non-zero gross positions, however, returns on the international portfolio depend on the real exchange rate, so the two instruments are no longer substitutable (see also Farhi, Gopinath, and Itskhoki 2014, Costinot and Werning 2019). As a result, an appropriate combination of import and export taxes can be used to engineer a pure transfer to the rest of the world through the valuation effects in (1), thereby closing a long-run trade deficit. Of course, the optimal policy aims to do the opposite — maximize the valuation transfer from the rest of the world and then optimize the terms of trade.

Motivated by the fact that export taxes are rarely used in practice, we then focus on an import tariff and show that it alone is sufficient to close the long-run trade deficit. Indeed, an import tariff appreciates the real exchange rate, raises the value of home assets held by the foreigners, and therefore generates a negative valuation effect. By the budget constraint (1), the initial trade deficit is then no longer sustainable, and the trade balance must improve. The exchange rate appreciation is the key step in this transmission and has several noteworthy policy

[2]With the long-run trade imbalance unchanged, the dynamic path of period-by-period net exports can still be affected by static or dynamic trade policies. In particular, higher trade frictions reduce the magnitude of transitory trade imbalances (Obstfeld and Rogoff 2001, Reyes-Heroles 2016, Costinot and Werning 2025, and Section 5 below). See also Cuñat and Zymek (2024) on the effects of trade barriers on bilateral trade imbalances in the long run.

implications. First, it runs counter to the standard expenditure-switching logic that a depreciation is needed to improve trade balance. In general equilibrium, this logic fails: improving the long-run trade balance requires a negative valuation effect induced via an appreciation. Second, the required appreciation is pinned down by the external balance sheet of the country irrespective of trade shares and trade elasticities. Finally, the required appreciation can be also achieved with an export subsidy, in place of an import tariff, thus turning Lerner symmetry on its head. From the perspective of closing a trade deficit, an import tariff is similar to an export subsidy, not an export tax, even though the two policies have drastically different implications for trade flows and welfare. In our calibration to the U.S. economy, a permanent 2% trade deficit can be closed with either a 64% import tariff or a 42% export subsidy, both resulting in a 22% appreciation of the dollar and reducing home welfare.

We next show that the tariff-induced valuation effects have first-order implications for the optimal trade policy. We extend the analysis of Johnson (1950) by incorporating cross-border asset positions, formulating the optimal policy problem using a primal approach (Lucas and Stokey 1983, Costinot, Lorenzoni, and Werning 2014), and expressing the optimal tariff in terms of measurable sufficient statistics (Arkolakis, Costinot, and Rodríguez-Clare 2012). The central insight is that the planner must trade off the conventional terms-of-trade motive in the goods market against the valuation effect in the financial market. As valuation effects depend on gross (rather than net) asset positions, a trade deficit per se does not map into a higher or lower optimal tariff. Yet, in the empirically relevant case in which foreigners hold asset positions in the U.S., an appreciation of the dollar increases the value of the U.S. external liabilities. As a result, the optimal import tariff is lower relative to the case of balanced trade. The effect is quantitatively large with the empirical cross-border positions reducing the optimal tariff from 35% to 9% and the associated welfare gains from 1% to 0.1% of permanent consumption. Interestingly, this also implies that countries have an incentive to accumulate assets of their major trade partners, as it both provides an insurance in case of a trade war and reduces the size of the optimal tariff for their trade partners.

Similarly, tariff-induced valuation effects substantially alter the optimal retaliation by other economies. Under balanced trade, foreign welfare falls monotonically as its terms of trade deteriorate with an increase in home's tariffs. However, with cross-border asset positions, a sufficiently high import tariff can raise foreign welfare by generating a positive valuation transfer to the rest of the world. In that case, retaliation is unnecessary. Such high tariffs can arise when the home policymaker attempts to close external imbalances, but then they are necessarily welfare-reducing for the home economy. In the trade-war Nash equilibrium with large external asset positions, the equilibrium tariffs are low for everyone, and largely independent of net positions and trade deficits. In this sense, financial globalization reinforces trade globalization by helping to sustain a low-tariff equilibrium without the need for external cooperation.

Finally, we develop a tractable dynamic version of the model with international portfolio choice where we introduce convenience yield on home assets. This acts as a simple way to capture excess returns enjoyed by the U.S. on its external portfolio, corresponding to the final "exorbitant privilege" term in the budget constraint (1). We first show that the dynamic model aggregates to a single-period long-run representation that we used in our baseline analysis. Furthermore, the effects of tariff on equilibrium prices and quantities in the long-run model are sufficient statistics for comparative dynamics as they proportionately shift the entire path of the dynamic economy. We then revisit our positive and normative results in the presence of convenience yield and exorbitant privilege. We show that trade policies now operate not only via valuation effects, but also via endogenous change in exorbitant privilege. Tariffs reduce trade between the economies and amplify the equilibrium portfolio home bias. The induced retrenchment in cross-border asset positions reduces the wealth transfer from the rest of the world to the U.S. for a given value of the convenience yield on the U.S.-issued assets. Although this mechanism helps narrow the long-run trade deficit, it unambiguously reduces the optimal tariff. If, additionally, a trade war triggers a reduction in the convenience yield on the U.S. assets, it may reverse the exchange rate appreciation and turn it into a dollar depreciation.

Related literature Our work bridges two largely separate strands of literature. The first strand belongs to international trade, relies mostly on static models, and studies the effects of trade policies. This includes classics by Lerner (1936), Baldwin (1948), Johnson (1950, 1953), Jones (1967), Gros (1987) and Bagwell and Staiger (1999), as well as Dixit and Norman (1980) and Helpman and Krugman (1989). Costinot, Donaldson, Vogel, and Werning (2015) and Chari, Nicolini, and Teles (2023) apply the Ramsey approach to optimal trade policy, Lashkaripour and Lugovskyy (2023) discuss the interactions between trade and industrial policies, and Ossa (2016) and Caliendo and Parro (2022) provide surveys of the recent optimal tariff literature. The second strand belongs to international macroeconomics, uses dynamic models, and studies the sources and implications of global imbalances. This includes research on the savings glut (e.g. Caballero, Farhi, and Gourinchas 2008, Mendoza, Quadrini, and Rios-Rull 2009) and exorbitant privilege (e.g. Gourinchas and Rey 2007a, 2014, Atkeson, Heathcote, and Perri 2025).

Our paper contributes to a large wave of studies motivated by the 2025 spike in U.S. tariffs. In particular, Ignatenko, Lashkaripour, Macedoni, and Simonovska (2025), Kalemli-Özcan, Soylu, and Yildirim (2025), Rodriguez-Clare, Ulate, and Vasquez (2025), Caliendo, Kortum, and Parro (2025) provide quantitative analyses of recent tariffs, Ostry, Lloyd, and Corsetti (2025) and Schmitt-Grohé and Uribe (2025) provide empirical evidence, Alessandria, Ding, Khan, and Mix (2025) and Kocherlakota (2025) focus on the fiscal implications of tariffs, Dávila, Rodríguez-Clare, Schaab, and Tan (2025) revisit optimal tariffs in an environment with heterogeneous agents, Hassan, Mertens, Wang, and Zhang (2025) analyze the implications of tariffs for U.S. financial dominance, while Auray, Devereux, and Eyquem (2025), Werning, Lorenzoni, and Guerrieri (2025), Bianchi and Coulibaly (2025), Monacelli (2025), Bergin and Corsetti (2023) characterize the optimal monetary response to tariffs.

Directly related to our analysis, there is a smaller literature that studies the interactions between tariffs and the current account. Razin and Svensson (1983) and Lloyd and Marin (2023) characterize the intertemporal effects of time-varying tariffs, Obstfeld and Rogoff (2001), Reyes-Heroles (2016), Alessandria and Choi (2021), Costinot and Werning (2025) analyze the effects of permanent tariffs and trade costs on the absolute size of imbalances, Pujolas and Rossbach (2024) revisit optimal tariffs in the presence of foreign debt, and Auray, Devereux, and Eyquem (2024) study the interactions between trade policy and monetary regimes in the presence of imbalances. Our analysis of tariff implications for long-run imbalances is most closely related to the earlier contribution of Lorenzoni (2019) and the contemporaneous work of Aguiar, Amador, and Fitzgerald (2025).

2 Baseline Model

This section introduces a two-region exchange economy that we use as a benchmark in our analysis. To keep things simple, we start with a one-period model, although this should not be viewed as a limitation. Section 5 shows that a full-fledged dynamic stochastic model with portfolio choice maps one-for-one into the long-run model studied here. In particular, all prices and quantities in the static model should be interpreted as permanent (i.e., long-run) components of the corresponding variables in a dynamic environment.

2.1 Setup

Consider a two-region world consisting of the home economy (the U.S.) and the rest of the world (ROW, a unified foreign economy indexed by $*$). Home is contemplating imposing tariffs on imports and exports against the rest of the world. For now, we assume that the latter does not retaliate and generalize the analysis to a trade war equilibrium in Section 4.3.

There are two goods — home H and foreign F — and the preferences of households in the two regions over consumption of these goods are given by $u(C_H, C_F)$ and $u^*(C_H^*, C_F^*)$, respectively. For concreteness, we focus on CES utility, even though our results extend to arbitrary separable preferences and the method can be applied more generally. We allow for different elasticities of substitution and home bias parameters to capture asymmetries across the regions. Specifically, the utilities and aggregate consumption bundles C and C^* are defined by:

$$\begin{aligned} u &= C^{\frac{\theta-1}{\theta}} = (1-\gamma)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}, \\ u^* &= C^{*\frac{\eta-1}{\eta}} = \gamma^{*\frac{1}{\eta}} C_H^{*\frac{\eta-1}{\eta}} + (1-\gamma^*)^{\frac{1}{\eta}} C_F^{*\frac{\eta-1}{\eta}} \end{aligned} \tag{2}$$

where $\gamma, \gamma^* \in [0,1)$ are the openness parameters and $\theta, \eta > 1$ are the elasticities of substitution which are greater than one to ensure that optimal tariffs are finite. Trade costs are not modeled explicitly and are implicitly captured by the home bias in preferences such that $\gamma^* < 1 - \gamma$ (see e.g. Obstfeld and Rogoff 2001).

Each region has an exogenous output (endowment) of its own good that can be consumed locally or exported to the other economy, with the resource constraint given by:

$$Y = C_H + C_H^* \qquad \text{and} \qquad Y^* = C_F + C_F^*. \tag{3}$$

Let P_H, P_F denote consumer prices in the home economy and P_H^*, P_F^* denote consumer prices in the foreign economy, all expressed in dollars (the home currency).[3] We assume a competitive equilibrium such that the law of one price holds for the sellers. At the same time, consumer prices may differ across regions due to the ad valorem import tariff τ^I and export tax τ^E:

$$P_F = \tau^I P_F^* \qquad \text{and} \qquad P_H^* = \tau^E P_H, \tag{4}$$

and $\tau^I = \tau^E = 1$ corresponds to the free-trade equilibrium.

International asset market The novelty of our approach to the analysis of tariffs is that we allow for gross international positions in a rich set of assets that includes equity (or FDI) claims issued on the countries' output (Lucas trees), home- and foreign-currency nominal bonds (or deposits), and real bonds denominated in units of home and foreign goods. Similarly, we can handle equity and currency derivatives, such as futures, forwards, options, and swaps. Thus, we account for asset classes that dominate cross-border positions (Gourinchas and Rey 2014). We rule out defaults and expropriations. Furthermore, we assume that the countries' portfolios are predetermined when an unexpected tariff shock materializes, and we discuss the implications of tariffs for the international portfolio choice in Section 5.

The next result establishes that for the tariff analysis, all these assets can be conveniently aggregated into an ex-ante international portfolio position $(-B_0, B_0^*)$, where B_0 corresponds to the home country's short position in home assets and B_0^* to its long position in foreign assets. Furthermore, the value of the country's net foreign assets is given by

$$NFA = P_F^* B_0^* - P_H B_0,$$

where P_H and P_F^* are local-market producer prices such that $\mathcal{Q} \equiv P_F^*/P_H$ is the (producer-price) real exchange rate. The positions B_0 and B_0^* are invariant to the tariffs, while their asset-market valuation effects are fully captured by the response of local producer prices P_H and P_F^*. We denote with $\mathcal{Q}_0 \equiv P_{F0}^*/P_{H0}$ the free-trade (or no-tariff) real exchange rate. For

[3] In the context of this long-run model, we assume that prices are flexible. Given this, we do not need to specify monetary and exchange rate policy, apart from Proposition 1 when we handle nominal bonds. For concreteness, we assume that each region's monetary policy stabilizes producer prices in local currency, while the nominal exchange rate is freely floating, which implies that P_F^* expressed in dollars equals the nominal exchange rate.

concreteness, we make an empirically-realistic assumption that countries hold a long position in foreign assets and do not short home assets, that is, $0 \leq B^* \leq Y^*$ and $0 \leq B \leq Y$.

We prove in Appendix A2 the following characterization result:

Proposition 1 *Assume that monetary policy in each region stabilizes prices of local goods. Then cross-border asset positions in nominal and real bonds, equity and FDI can be mapped into B_0 and B_0^* that are invariant to tariffs, and the allocation-relevant asset valuation effects of tariffs equal:*

$$\Delta(NFA/P_F^*) \;=\; \underbrace{\frac{\Delta \mathcal{Q}}{\mathcal{Q}}}_{\text{real depreciation}} \;\times\; \underbrace{B_0/\mathcal{Q}_0}_{\substack{\text{pre-tariff real value}\\ \text{of external liabilities}}} \;,$$

where $\Delta\mathcal{Q}/\mathcal{Q} \equiv (\mathcal{Q}-\mathcal{Q}_0)/\mathcal{Q}$ is the tariff-induced depreciation of the real exchange rate.

To understand this result, we first write the net foreign asset position in terms of its purchasing power of foreign goods in the foreign market, P_F^*; we have $NFA/P_F^* = B_0^* - B_0/\mathcal{Q}$. The reason to do so is that the one relative price that the home country's tariffs cannot distort is the purchasing power of foreign assets in terms of foreign goods in the foreign market. The real valuation effect of tariffs is then summarized by the resulting percentage change in the real exchange rate times the pre-tariff value of external liabilities, $B_0/\mathcal{Q}_0$, in terms of claims on home's assets such as dollar bonds and home equity. An appreciation of the home currency — i.e., a reduction in $\mathcal{Q}$ — results in a negative valuation effect, that is, a wealth transfer from home to foreign.

It is easiest to see why this is the case for nominal and real bonds. Under monetary policy that stabilizes local-good prices, the real and the nominal appreciation coincide, and so do the valuation effects on real and nominal bonds.[4] Indeed, the purchasing power of home-currency bonds, both real and nominal, increases with the real appreciation. To the extent foreigners hold a net long position in such bonds, as is the case in practice for the U.S., an appreciation results in a transfer from home to foreign residents.

It turns out that the same logic applies for equity-like claims to the country's output which pays the dividend $b_0 Y^*$ and $b_0^* Y$ in proportion to the external positions b_0 and b_0^*, respectively. With such equity claims, the net foreign asset position can be written as $NFA = P_F^* Y^* b_0^* - P_H Y b_0$. As a result, the real valuation effect is again given by $\Delta(B_0/\mathcal{Q})$, where now $B_0 = b_0 Y$ and is invariant to tariffs. We fully spell out this case in a model of international portfolio choice in Section 5. In practice, tariffs may have additional effects on production, investment, cash flows, product and asset prices beyond their effect via the real exchange rate that acts

[4]In practice, producer and consumer real exchange rates comove very closely together, and both comove closely with the nominal exchange rate for countries with price stability (see Itskhoki and Mukhin 2021). Therefore, for simplicity, we assume that real bonds pay out in terms of goods produced by the country rather than in terms of the country's consumption basket.

as a summary statistic in our endowment economy. Nonetheless, Proposition 1 establishes a natural benchmark for the analysis of valuation effects in richer environments.[5]

2.2 Competitive equilibrium

For a given trade policy τ^I and τ^E, the equilibrium prices and allocations are determined as follows. Home households choose consumption to solve:

$$\begin{aligned} \max_{C_H, C_F} \quad & u(C_H, C_F) \\ \text{s.t.} \quad & P_H C_H + P_F C_F = P_H Y + P_F^* B_0^* - P_H B_0 + T, \end{aligned}$$

where a lump-sum transfer from the government reflects the revenue collected from tariffs:

$$T = (\tau^I - 1) P_F^* C_F + (\tau^E - 1) P_H C_H^*.$$

The household optimality condition is given by:

$$\frac{u_F}{u_H} = \frac{P_F}{P_H}, \tag{5}$$

where u_i denotes a partial derivative, i.e., the marginal utility of good $i \in \{H, F\}$.

The problem of foreign households is symmetric and their demand for goods satisfies:

$$\frac{u_F^*}{u_H^*} = \frac{P_F^*}{P_H^*}. \tag{6}$$

Tariffs create deviations from the law of one price across markets according to (4) such that the relative prices differ, $P_F/P_H = \tau \cdot P_F^*/P_H^*$, by a combined wedge $\tau \equiv \tau^E \tau^I$.

The home country's budget constraint follows from the budget constraint of the households, fiscal balance, and market clearing conditions:

$$\underbrace{P_H^* C_H^* - P_F^* C_F}_{=NX} + \underbrace{P_F^* B_0^* - P_H B_0}_{=NFA} = 0, \tag{7}$$

where net exports are expressed in terms of international prices. In sum, given home tariffs (τ^I, τ^E), prices (P_H, P_F, P_H^*, P_F^*) and consumption allocations (C_H, C_F, C_H^*, C_F^*) are determined by equilibrium conditions (3)-(7).[6]

[5]For example, in economies with optimizing investment and price setting choices, the result of Proposition 1 should provide an accurate first-order approximation by envelope theorem. In contrast, situations in which tariffs have additional non-marginal consequences for productivity and asset prices need to be modeled explicitly as we do in Section 5 where we consider the loss of "exorbitant privilege" as a result of the tariff war. See also Amiti, Gomez, Kong, and Weinstein (2021).

[6]Notice that there are seven equations and eight variables; since only relative prices matter, we normalize $P_H = 1$, consistent with the assumption about monetary policy in Proposition 1.

In our static setup, the country budget constraint (7) is the counterpart to the intertemporal budget constraint (1), where the NFA term combines together the initial value of net foreign assets and the on-impact valuation effect from trade policy.[7] According to (7), a country runs an equilibrium long-run trade deficit, $NX < 0$, if and only if it has a positive net foreign asset position, $NFA > 0$.

2.3 Trade possibilities frontier and Lerner symmetry

Our ultimate objective is to characterize the best use of tariffs from the perspectives of different objectives. To do so in a tractable way, we adopt the primal approach of Lucas and Stokey (1983) to reduce the equilibrium system (3)–(7) to a much simpler implementability constraint faced by a home policymaker with import and export tariffs as the only policy tools. We refer to this implementability condition as the *trade possibilities frontier*.

Trade possibilities frontier We derive the implementability condition in two steps. First, we rewrite the equilibrium system (3)–(7) in terms of allocative international relative prices. The foreign consumer optimality (6) relates the *terms of trade*, $\mathcal{S} \equiv P_F^*/P_H^*$, to the consumption allocation in the rest of the world according to:

$$\frac{u_F^*}{u_H^*} = \mathcal{S}. \tag{8}$$

The country budget constraint (7) can be rewritten as:

$$\big(C_H^*/\mathcal{S} - C_F\big) + \big(B_0^* - B_0/\mathcal{Q}\big) = 0, \tag{9}$$

and it features both the terms of trade as a part of net exports and the real exchange rate, $\mathcal{Q} \equiv P_F^*/P_H$, which determines the value of the country's net foreign asset position. Finally, the law of one price conditions (4) imply that the two relative prices are related by:

$$\mathcal{Q} = \tau^E \mathcal{S}, \tag{10}$$

and, in particular, $\mathcal{Q} = \mathcal{S}$ in the absence of the export tax (with $\tau^E = 1$).

With the use of tariffs, the policymaker can choose any allocation (C_H, C_F, C_H^*, C_F^*) and relative prices $(\mathcal{S}, \mathcal{Q})$ that satisfy the three conditions (8)–(10) above along with the resource constraint (3). Indeed, the remaining equilibrium condition for home optimality (5) acts as a side equation which determines the size of the total distortion to free trade, $\tau \equiv \tau^I \tau^E$, from

[7] Formally, the initial pre-tariff NFA position is $NFA_0 = P_{F0}^* B_0^* - P_{H0} B_0$, where P_{F0}^* and P_{H0} are the free-trade equilibrium prices. The change in the real exchange rate, $\Delta \log \mathcal{Q} = \Delta \log(P_F^*/P_H)$, then captures the on-impact valuation effect on the home's external liabilities B_0, as formally stated in Proposition 1, corresponding to the "valuation effect" term in the general accounting budget constraint (1).

$u_F/u_H = \tau\mathcal{S}$, where we have used the law of one price conditions (4). Combining this with (8) to solve out the terms of trade, we obtain a (distorted) contract curve:[8]

$$\frac{u_F(Y - C_H^*, C_F)}{u_H(Y - C_H^*, C_F)} = \tau^I\tau^E \cdot \frac{u_F^*(C_H^*, Y^* - C_F)}{u_H^*(C_H^*, Y^* - C_F)}, \tag{11}$$

where we spelled out the arguments of the marginal utilities and expressed consumption levels of local goods $C_H = Y - C_H^*$ and $C_F = Y^* - C_F$ from the resource constraints (3). Contract curve (11) is the locus of home export and import quantities, C_H^* and C_F, which are consistent with household optimality in the two regions given the combined tariff distortion.

Finally, we use (8) and (10) to solve out relative prices $\mathcal{S}$ and $\mathcal{Q}$ from the country budget constraint (9), which results in the implementability condition on the home policymaker entirely in terms of allocations (specifically, trade quantities). We prove in Appendix A2:

Lemma 1 *The policymaker can choose any combination of import and export quantities, C_F and C_H^*, that satisfy the implementability condition $C_H^* = g(C_F; B_0/\tau^E, B_0^*)$ implicitly defined by*

$$u_F^*(C_H^*, Y^* - C_F) \cdot \big(C_F - B_0^*\big) = u_H^*(C_H^*, Y^* - C_F) \cdot \big(C_H^* - B_0/\tau^E\big) \tag{12}$$

for some export tax $\tau^E \in (0, \infty)$. Under CES preferences, $g(\cdot)$ is strictly increasing and convex in C_F. For any pair of tariffs (τ^I, τ^E), the equilibrium allocation is the solution of (11) and (12).

The implementability condition (12) defines a mapping from export to import quantities, which we write $C_H^* = g(C_F)$ as shorthand, and refer to it as the home's *trade possibilities frontier*.[9] This condition encodes information about both foreign preferences u^* and foreign endowment Y^* (more generally, production structure) and hence reflects both demand and supply sides of the foreign economy, as jointly disciplined by the trade balance (or, more precisely, the country budget constraint). The strict monotonicity and convexity of the frontier follows directly from decreasing marginal utility: the home country needs to export more units of its good for each additional unit of the foreign good imported due to deteriorating terms of trade.

Given Lemma 1, the home's planning problem boils down to maximizing its objective subject to the domestic resource constraint $Y = C_H + C_H^*$ and the implementability condition $C_H^* = g(C_F)$, or simply written as a single constraint $Y - C_H = g(C_F)$. This constraint fully encodes all relevant information about the rest of the world and the international asset positions $(-B_0, B_0^*)$. This characterization applies for any objective of the planner provided that

[8] The undistorted (free-trade) contract curve is simply $u_F/u_H = u_F^*/u_H^*$ corresponding to $\tau = \tau^I\tau^E = 1$.

[9] The earlier literature that followed Johnson (1950) referred to this condition as either "foreign export supply" or "foreign import demand" schedule; in microeconomics, this condition is the foreign "offer curve"; Dixit (1985) called it a "Baldwin envelope". Given strict monotonicity of $g(\cdot)$, its inverse $C_F = g^{-1}(C_H^*)$ summarizes the trade "production function" of foreign goods from home goods (Diamond and Mirrlees 1971).

the policy instruments are tariffs. The required tariff distortion that implements the allocation can then be recovered from the contract curve (11). Conversely, given tariffs (τ^I, τ^E), the contract curve (11) and the trade possibilities frontier (12) pin down the corresponding trade equilibrium.

Lerner symmetry Equilibrium characterization above has an immediate implication for the equivalence of the import tariff and the export tax. Note that only the export tax τ^E features explicitly in the implementability condition (12), while the two tariffs enter symmetrically in the contract curve (11). Furthermore, when $B_0 = 0$, the value of τ^E no longer affects the implementability condition (12), and the two tariffs only appear jointly in (11) determining the overall distortion $\tau = \tau^I \tau^E$. We, therefore, have:

Proposition 2 *Lerner symmetry between the import tariff τ^I and the export tax τ^E holds if and only if the net external position in terms of home's assets is zero, $B_0 = 0$, irrespective of whether there is a long-run trade imbalance $NX \neq 0$.*

When $B_0 = 0$, the international terms of trade $\mathcal{S}$ are a sufficient statistic for the equilibrium allocation, while the value of the real exchange rate $\mathcal{Q}$ is immaterial for the allocation as with $B_0 = 0$ there are no valuation effects (Proposition 1). An import tariff appreciates the real exchange rate, while an export tax depreciates it, yet the two trade taxes have exactly the same effect on the terms of trade, driving a combined wedge τ between home and foreign relative prices, as featured in the distorted contract curve (11). Thus, despite having two trade policy instruments, the policymaker in this case effectively has only one degree of freedom — the terms of trade — when choosing the equilibrium allocation. This delivers Lerner symmetry.

Lerner (1936) symmetry is conventionally derived in standard trade models with balanced trade under financial autarky with $B_0 = B_0^* = 0$. Proposition 2 shows that the argument extends to economies with trade deficits or surpluses as long as all international borrowing and lending is denominated in foreign currency, that is, $NX = -P_F^* B_0^* \neq 0$ according to (7). In this case, the payoff on foreign assets is always proportional to the price of foreign goods P_F^*, irrespective of the home's trade policy. As a result, equilibrium allocations continue to depend exclusively on the terms of trade $\mathcal{S}$, and Lerner symmetry remains valid.

The logic changes fundamentally once countries hold external positions in terms of home assets, $B_0 \neq 0$. In this case, equation (9) makes clear that the terms of trade are no longer a sufficient relative price for trade policy: movements in the real exchange rate $\mathcal{Q}$ now affect equilibrium allocations through asset valuation effects. An export tax drives a wedge between $\mathcal{S}$ and $\mathcal{Q}$ that is absent under an import tariff, as implied by (10), which breaks the equivalence between the two instruments when $B_0 \neq 0$. A home real appreciation induced by an import tariff revalues the home's external liabilities when $B_0 > 0$, resulting in a transfer of purchasing power (wealth) to foreign. A reverse transfer happens under a real depreciation induced by an export tax which is equivalent from the perspective of the terms of trade.[10]

3 Closing Imbalances

Before studying welfare-maximizing tariffs in the presence of trade imbalances, we first ask whether tariff instruments can eliminate such imbalances altogether. This can be viewed as an optimal tariff question under the sole objective of balanced trade.[11]

Consider a country with a non-zero international gross asset position, $B_0, B_0^* > 0$, such that under a free-trade equilibrium its trade is imbalanced, that is, $NX_0 = -NFA_0 \neq 0$, where $NFA_0 = P_{F0}^* B_0^* - P_{H0} B_0$. An immediate observation from the country budget constraint (7) is that a tariff policy can implement a balanced-trade equilibrium, $NX = 0$, if and only if it results in a zero net foreign asset position, $NFA = P_F^* B_0^* - P_H B_0 = 0$. In other words, what matters for the answer to this question is whether tariffs can induce a sufficiently large asset valuation effect to eliminate the entire net foreign asset position resulting in an international transfer of wealth equal to NFA_0 from home to foreign (vice versa if $NFA_0 < 0$). When both tariff instruments are available, this policy outcome is always feasible.

Proposition 3 *A necessary and sufficient condition for tariffs to close the equilibrium trade deficit, $NX = 0$, is that the real exchange rate takes the following value:*

$$\mathcal{Q} \equiv \frac{P_F^*}{P_H} = \frac{B_0}{B_0^*}. \tag{13}$$

When both import and export tariffs are available, the policymaker can unilaterally implement any balanced-trade equilibrium, including trade autarky.

The first part of this proposition follows directly from the country budget constraint (7) and Proposition 1 which shows that the real exchange rate summarizes all asset valuation effects induced by tariffs. Even when the trade balance is the policymaker's ultimate goal, the budget constraint brings the country's external balance sheet into the spotlight, shifting the emphasis away from the terms of trade and towards the real exchange rate. Condition (13) defines the unique value of the equilibrium real exchange rate that ensures $NFA = 0$ and,

[10] See related analyses of valuation effects in Blanchard (2009), Farhi, Gopinath, and Itskhoki (2014), Barbiero, Farhi, Gopinath, and Itskhoki (2019), Costinot and Werning (2019) and Itskhoki and Mukhin (2023). In more abstract trade models with exogenous deficits following Dekle, Eaton, and Kortum (2007), the literature has found that the choice of the numeraire is surprisingly consequential for the optimal tariff, as we study below.

[11] This section is related to the contemporaneous analysis of Aguiar, Amador, and Fitzgerald (2025) that also allows countries to produce both goods, trade flows to reverse in response to tariffs, and the two regions to coordinate their trade policies to close the imbalances.

thus, an equilibrium with balanced trade.[12] At the same time, there are many values of the terms of trade that are consistent with $NX = 0$, namely $\mathcal{S} = C_H^*/C_F$ with the endogenous export and import quantities C_H^* and C_F.

Violation of Lerner symmetry, discussed in Proposition 2, implies that two tariffs jointly can generate arbitrary asset valuation effects for any target value of the terms of trade. In other words, this allows the policymaker to engineer an international transfer of wealth large enough to eliminate the net foreign asset position. Furthermore, the policymaker can implement any of the continuum of allocations consistent with $C_H^* = g_0(C_F) \equiv g(C_F; 0, 0)$ from Lemma 1, which ensures $NX = 0$ (see the proof in Appendix A2).[13] This includes the trade autarky equilibrium, $C_F = C_H^* = 0$, as a special case when home imposes prohibitive tariffs, as well as the balanced-trade equilibrium with the most favorable terms of trade characterized in Section 4.

3.1 Import tariff

Motivated by the fact that import tariffs are a much more common instrument in practice with export taxes outlawed in some countries, including the U.S., we next ask whether a balanced-trade equilibrium can be achieved when τ^I is the only available tool, restricting $\tau^E = 1$. We start with the empirically-relevant case when home — the U.S. — runs a long-run trade deficit.

Proposition 4 *Suppose home runs a trade deficit under free trade, $NX_0 = -NFA_0 < 0$. Then there exists a unique balanced-trade equilibrium that can be implemented with an import tariff, $\tau^I > 1$, which results in a real exchange rate appreciation $\Delta\mathcal{Q}/\mathcal{Q} = NX_0/(P_{H0}B_0) < 0$.*

We provide a formal proof in Appendix A2, yet this result can be best understood using the Edgeworth box shown in Figure 1. As usual, the coordinates of each point show the allocation of the two goods between home and foreign, and the blue solid line is the contract curve showing all Pareto efficient allocations satisfying $u_F/u_H = u_F^*/u_H^*$. The red solid line shows the trade possibilities frontier $C_H^* = g_0(C_F)$ under financial autarky, $B_0 = B_0^* = 0$. Point E denotes the countries' endowments and point P corresponds to their portfolios represented as claims on foreign goods. When $B_0 = B_0^* = 0$, these two points coincide ($P = E$) with the corresponding free-trade equilibrium in point A given by the intersection of the contract and the balanced-trade curves.

[12] Proposition 3 implies that trade balance can only be achieved if $B_0/B_0^* > 0$. If the economy is a net borrower or saver in both home and foreign assets (currencies) — that is, B_0 and B_0^* have the same sign — no movements in relative prices of the two assets can bring the country's net asset position to zero.

[13] Recall that the import tariff and the export tax have the same effects on the country's terms of trade, but the opposite effects on its real exchange rate. As a result, the two tariffs are sufficient to separately target the two international prices in (9). In particular, the overall trade distortion $\tau = \tau^I\tau^E$ can be kept at any fixed level, while the tariffs are chosen to target implement the target value of the real exchange rate.

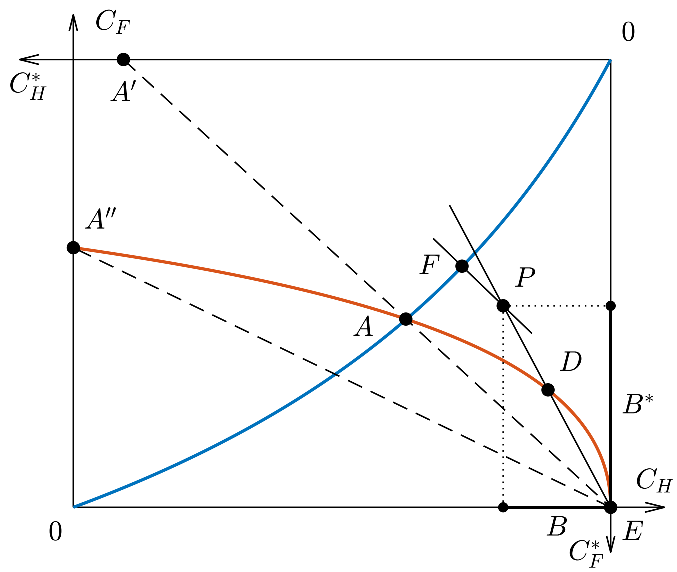

Figure 1: Closing imbalances with an import tariff

Note: The figure plots the Edgeworth box for the home economy and the rest of the world with $E = (Y, 0)$ denoting the home endowment point and $P = (Y - B_0, B_0^*)$ corresponding to the endowment combined with the external asset position. The blue curve is the free-trade contract curve and the red curve is the balanced trade frontier $C_H^* = g_0(C_F) \equiv g(C_F; 0, 0)$. A is the free-trade equilibrium when $B_0 = B_0^* = 0$ and F is the free-trade equilibrium for the asset position P. D is the unique balanced-trade equilibrium attainable with an import tariff starting from P.

The line segment EA' corresponds to the country's budget constraint which results in balanced trade under free trade (with equilibrium in point A). The slope of this line (against the vertical axis) is equal to the terms of trade $\mathcal{S}_0 = P_{F0}^*/P_{H0}^*$ under free trade, and hence also to the corresponding real exchange rate $\mathcal{Q}_0 = P_{F0}^*/P_{H0}$. Any portfolio P that lies on EA' results in the same balanced-trade equilibrium under free trade in point A. Furthermore, any portfolio P that lies above the EA' line results in a free-trade equilibrium on a contract curve above point A, like point F, and therefore implies that home runs a trade deficit, $NX_0 < 0$, in the absence of tariffs (and vice versa). We restrict our attention to such portfolios P.

Suppose that the policymaker aims to close imbalances in both goods and asset markets. Graphically, the new equilibrium D can be found as the intersection of the balanced trade red curve and the ray EP that shows the set of points that satisfy $C_H^*/C_F = B_0/B_0^*$. According to the characterization in Lemma 1, the red line $C_H^* = g_0(C_F)$ is monotonic, convex, and under CES preferences satisfies the Inada condition at point E, which guarantees that a zero-imbalance equilibrium always exists and is unique. The move from a free-trade to a balanced-trade equilibrium is achieved by imposing a large enough import tariff that shifts the contract curve to go through point D. Because the new equilibrium lies below point A on the balanced-trade curve, the tariff required to close the imbalance is unambiguously positive, i.e., $\tau^I > 1$.

The real exchange rate plays a central role in the transmission of trade policy and has several interesting features. In particular, the imbalances can only be closed if the real exchange rate appreciates. Indeed, a positive initial net foreign asset position implies that $\mathcal{Q}_0 =$

$P^*_{F0}/P_{H0} > B_0/B^*_0$ under free trade, and thus $\mathcal{Q} = P^*_F/P_H$ needs to go down (corresponding to a real appreciation) to close the financial imbalance. Graphically, the free-trade real exchange rate, $\mathcal{Q}_0 = \mathcal{S}_0$, is given by the slope of PF (against the vertical axis), which is always higher than the slope of ED — equal to the real exchange rate after the tariff is imposed.

This observation goes against the conventional expenditure-switching logic that a depreciation of the exchange rate is required to improve trade balance. This partial-equilibrium logic ignores the country budget constraint and the fact that it is not possible to increase net exports in a long-run equilibrium without deteriorating the net foreign asset position. The latter requires an exchange rate appreciation which increases the value of home's external liabilities relative to its external assets.[14]

Furthermore, as established in Propositions 3 and 4, the required exchange rate appreciation is fully independent of trade shares, global input-output linkages, or trade elasticities — including whether the Marshall-Lerner condition holds — and is entirely determined by the international asset position of the country. In contrast, computing the level of the import tariff that results in such an appreciation generally requires a fully calibrated structural model. Nonetheless, there exists a simple first-order approximation that maps tariffs into the resulting real exchange rate that provides an accurate global approximation in the context of our model (see derivation in Appendix A2, along with a parallel result for the export tax):

Lemma 2 *A first-order approximation of the response of the real exchange to an import tariff is:*

$$\Delta \log \mathcal{Q} = -\frac{(1-\gamma)\theta}{(1-\gamma)\theta + (1-\gamma^*)\eta - (1-\gamma-\gamma^*)\frac{EX-\mathcal{B}}{EX}} \cdot \Delta\tau^I, \tag{14}$$

where $EX \equiv P^*_H C^*_H$ is the value of exports and $\mathcal{B} \equiv P^*_H B_0$ is the value of external liabilities.

In partial equilibrium, the import tariff only reduces country's imports, while the exchange rate appreciation simultaneously increases imports and decreases exports, with the effects proportional to the respective trade elasticities. Provided sufficient symmetry on the import and export sides, equilibrium requires that

$$\Delta \log \mathcal{Q} \approx -\frac{1}{2} \cdot \Delta\tau^I,$$

that is, the exchange rate appreciates by approximately half a percentage point for each percentage point increase in the tariff. This is true under financial autarky to keep trade balanced irrespective of the value of the tariff, but it is also approximately true with cross-border asset holdings which somewhat mute the response of the exchange rate to the tariff, according

[14]The empirical evidence in Gourinchas and Rey (2007b) shows that the depreciation of the dollar indeed generates positive valuation effects for the U.S. economy, contributing to the sustainability of an equilibrium trade deficit (see also Lane and Shambaugh 2010, Forbes, Hjortsoe, and Nenova 2017).

to (14). We provide a quantification of the tariff and the associated exchange rate appreciation needed to close the U.S. trade deficit in Section 3.3.

3.2 Export subsidy

The limitations of the partial-equilibrium intuition are even more evident when considering the alternative implementations of closing the imbalances with an export tax. Proposition 4 shows that an import tariff is necessary to eliminate a long-run trade deficit. A standard Lerner-symmetry logic would suggest that the same outcome could be achieved with an export tax. However, Lerner symmetry does not apply when $B_0 > 0$. In fact, imposing an export tax $\tau^E > 1$ depreciates the real exchange and improves the home's net foreign asset position amplifying the original imbalances, that is, $NX < NX_0 < 0$ instead of $NX = 0$. It follows that an export subsidy $\tau^E < 1$, not a tax, is required to close the trade deficit.

Proposition 5 *Suppose home runs a trade deficit under free trade, $NX_0 = -NFA_0 < 0$. Then in addition to an import tariff $\tau^I > 1$, there exists a unique export subsidy $\tau^E < 1$ that also closes a trade deficit, resulting in an equilibrium with $NX = 0$, but under different terms of trade and with different quantities of exports and imports.*

This turns Lerner symmetry on its head, making an import tariff similar to an export subsidy in terms of their implications for trade deficits. This is the case because the two tariffs have the opposite implications for the real exchange rate, which is a sufficient statistic for the imbalances, in line with Proposition 3. At the same time, the two tariffs move the terms of trade in the same direction. In contrast to the import tariff, a trade subsidy deteriorates the terms of trade, which is still relevant for the allocation, and the two trade policies result in very different balanced-trade equilibria.

Figure 2 illustrates this by plotting allocations implementable with an import tariff $C_H^* = g^I(C_F)$ and allocations implementable with an export tax $C_H^* = g^E(C_F)$ for a given portfolio position P.[15] Under financial autarky, $B_0 = B_0^* = 0$, the two sets coincide with the balanced-trade curve, $g^I = g^E = g_0$. Furthermore, as follows from Lemma 2, $B_0 = 0$ is sufficient for Lerner symmetry to hold, and the two instruments are equivalent when home assets are not traded internationally. That is, when $B_0 = 0$, we have $g^I = g^E = g$ for any value of B_0^*, with this curve shifting upwards relative to g_0 for $B_0^* > 0$, and vice versa.

More generally, import and export tariffs result in very different allocations and the implied equilibria coincide only under free trade, $\tau^I = \tau^E = 1$, in point F. While the two tariffs result in the same shift of the contract curve (11), export tax drives a wedge between the real exchange rate and the terms of trade (recall (10)), generating additional valuation effects in

[15] Note that $C_H^* = g^I(C_F)$ and $C_H^* = g^E(C_F)$ are both defined implicitly by (12) with $\tau^E = 1$ in the former case and by substituting $\tau^E = \frac{u_H/u_F}{u_F^*/u_H^*}$ in the latter case.

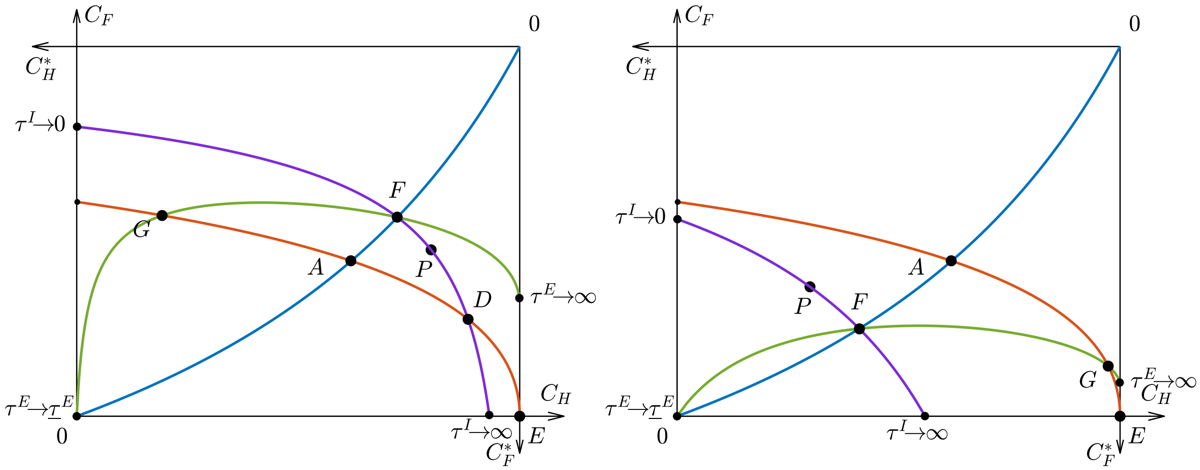

Figure 2: Effects of import and export tariffs under imbalances

Note: The blue curve is the free-trade contract curve and the red curve is the balanced trade frontier $C_H^* = g_0(C_F)$ for $B_0 = B_0^* = 0$. Point P in the left and right panels features portfolio (B_0, B_0^*) such that $NFA \gtrless 0$, respectively. Purple (green) curve corresponds to g^I (g^E), i.e., the set of implementable allocations for a given portfolio (B_0, B_0^*) using an import tariff (an export tax).

the budget constraint (9).[16] As can be seen from the intersections of function g with g^I and g and g^E, respectively, the balanced-trade equilibrium implemented with an import tariff $\tau^I > 1$ features less trade — both exports C_H^* and imports C_F — than the balanced-trade equilibrium implemented with an export subsidy $\tau^E < 1$ (*cf.* points G and D, which are on opposite sides of point A along the g_0 curve in the left panel of Figure 2). Intuitively, while both instruments ensure balanced trade, the import tariff curbs the gross volume of trade, while the export subsidy increases it.

3.3 Quantifying the balanced-trade tariffs

With our theoretical results in hand, we can now quantify import and export tariffs that can close a long-run trade deficit for a country that is calibrated to mimic the salient features of the U.S. economy. Propositions 1 and 3 emphasize the sufficient-statistic properties of the real exchange rate in quantifying the tariff-induced valuation effects which, in turn, must account for the entire equilibrium trade rebalancing effect. Lemma 2 provides the approximate size of the exchange rate adjustment given the size of the tariff; we use the non-linear calibrated model to quantify this effect and evaluate the quality of the approximation in (14).

Calibration We use the standard values of the long-run trade elasticity, $\theta = \eta = 4$. Interpreting data as an outcome of a free-trade equilibrium, we calibrate γ, γ^* using the U.S. share

[16] In particular, this explains why point $P = (Y - B_0, B_0^*)$ with consumption of each good equal to corresponding yields from endowment and asset positions can be implemented as an equilibrium only with an import tariff, but not with an export tax (which results in $\mathcal{Q} \neq \mathcal{S}$ in (9)).

in the world output of 20% and its exports and imports of 12% and 14% of GDP with an implied 2% long-run trade deficit. In other words we assume the average value of the U.S. trade deficit as a share of GDP over the last 50 years to reflect the long run.[17]

The calibrated value of the long-run trade deficit pins down the value of the net foreign asset position, according to the country budget constraint (7), where all asset values are converted into annual flow terms in shares of GDP. Additionally, we calibrate the external liabilities of the U.S. in terms of home assets, $P_{H0}B_0$, to be approximately 175% of the U.S. GDP, or 7% in flow value terms using the annual interest rate of 4% (close to the mean real returns on U.S. equity and debt combined). The size of the U.S. external assets are harder to measure because of hidden wealth and profits (see Zucman 2013, Coppola, Maggiori, Neiman, and Schreger 2021, Guvenen, Mataloni Jr, Rassier, and Ruhl 2022). However, given the value of net foreign assets implied by the long-run trade deficit of 2% of GDP, we obtain that $P^*_{F0}B^*_0$ must equal 9% of the U.S. GDP in flow terms. As the recorded U.S. external assets are smaller than external liabilities — corresponding to a negative net foreign asset position — this approach to calibration implies that $P^*_{F0}B^*_0$ features "dark matter", i.e., an unobserved external wealth of the U.S. (Hausmann and Sturzenegger 2007). Section 5 relaxes this assumption and allows part of the trade deficit to be financed by the exorbitant privilege.

For comparison, we also consider predictions of a standard trade model without cross-border positions, $B_0 = B^*_0 = 0$, and a model with gross asset positions but no imbalances such that $P^*_{F0}B^*_0 = P_{H0}B_0 > 0$. In both of these counterfactual cases, imports and exports are calibrated to equal 14% of GDP. Appendix A3 provides details on how the model is solved. Given that the model abstracts from several important features of international trade (e.g., input-output linkages), the numbers should be taken as illustrative with the focus on the relative magnitudes of the effects. Table 1 summarizes all quantitative results across various specifications.

Import tariff We first quantify the import tariff τ^I that can close the long-run 2% trade deficit of the U.S. under free trade. The first row of Table 1 shows the results under our baseline calibration. Closing the U.S. trade deficit requires a high import tariff of 63.8%, which appreciates the real exchange rate by 22.2% and decreases household welfare by 1.6% equal to the permanent decline in real consumption. Perhaps surprisingly, the associated welfare losses in the rest of the world are two orders of magnitude smaller (0.04%), as we explain in the following section where we consider the welfare-maximizing tariff. Finally, such import tariff generates a substantial fiscal revenue equal to 2.9% of GDP.

[17] An alternative assumption is that the past 50 years correspond to a slow transition which eventually leads to a long-run equilibrium with a trade surplus. We evaluate such an alternative scenario in the following subsection.

Table 1: Tariffs, allocation and welfare

	τ	τ^*	C	C^*	$\mathcal{Q}$	$\mathcal{S}$	T	NX
Baseline calibration								
Closing imbalance τ^I	63.81	0.00	−1.60	−0.04	−22.22	−22.22	2.86	0.00
Closing imbalance τ^E	−41.71	0.00	−12.90	1.87	−22.22	33.44	−15.31	0.00
Optimal τ^I	8.76	0.00	0.10	−0.05	−4.37	−4.37	1.02	−1.61
Optimal τ^E	67.47	0.00	1.80	−0.93	19.45	−28.67	2.13	−3.75
Trade war τ^I	6.75	6.78	−0.08	−0.02	−0.22	−0.22	0.76	−1.98
Fiscal tariff τ^I	64.93	0.00	−1.64	−0.03	−22.47	−22.47	2.86	0.02
Financial autarky								
Optimal τ	35.31	0.00	0.95	−0.46	−15.14	−15.14	2.60	0.00
Trade war τ	34.90	40.48	−1.61	−0.29	3.55	3.55	1.48	0.00
Fiscal tariff τ	81.62	0.00	0.41	−0.74	−27.98	−27.98	3.14	0.00
No imbalances								
Improving balance τ^I	122.53	0.00	−3.49	−0.05	−28.57	−28.57	2.15	2.00
Exorbitant privilege								
Closing imbalance τ^I	14.36	0.00	−1.58	0.36	−5.11	−5.11	1.41	0.00
Optimal τ^I	−31.19	0.00	2.53	−1.25	17.96	17.96	−10.47	−11.19
Optimal τ^E	−1.82	0.00	0.02	−0.01	−0.92	0.92	−0.23	−2.12

Note: The table shows import tariffs τ^I and export taxes τ^E (in percent), in home (first column) and foreign (second column, with $*$), and their implications for welfare C and C^* (percentage change in permanent real consumption), the real exchange rate $\mathcal{Q}$ and the terms of trade $\mathcal{S}$ (percent change), fiscal revenues T (percent of GDP), and the trade balance NX (percent of GDP). "Baseline calibration" features trade deficit $NX_0 < 0$ equal to 2% of GDP with $B_0, B_0^* > 0$ as described in the text; "Financial autarky" corresponds to $B_0 = B_0^* = 0$; "No imbalances" assumes $NX_0 = 0$; "Exorbitant privilege" features $\Omega > 0$ defined in Section 5.

Export tax The second row of Table 1 shows that an export tax that closes the U.S. trade deficit is, in fact, an export subsidy of 41.7%, in line with Proposition 5. This export subsidy generates the same real appreciation of 22.2% as the 63.8% import tariff, and thus similarly eliminates the net foreign asset position of the U.S. that supported the long-run trade deficit, in line with Proposition 3. The welfare cost of such an export subsidy is an order of magnitude larger relative to that of the import tariff, with the government fiscal deficit of 15.3% of GDP. The reason is that achieving such a sizeable real appreciation requires subsidizing large quantities of trade (exports and indirectly imports) resulting in trade shares in GDP much in excess of their optimal levels.

3.4 Trade surplus

Our baseline analysis focuses on the case of a long-run trade deficit under free trade, defined as $NX_0 = -NFA_0 < 0$ in the context of the one-period (long-run) model and generalized to the infinite-period dynamic model in Section 5. In the empirical context of the U.S., it is notoriously difficult to distinguish the case of a permanent trade deficit from the case of a persistent yet temporary trade deficit which is eventually followed by a long-run trade balance or trade surplus. Therefore, we now consider two alternative scenarios.

Closing a temporary trade deficit First, consider the case where starting with $NFA_0 \le 0$, the short-run trade deficits must be succeeded by a long-run trade surplus. In the context of our long-run (one-period) model, we address this question by evaluating the ability of a policymaker to use tariffs to further improve an already non-negative long-run trade balance, $NX_0 = -NFA_0 \ge 0$. In the context of a dynamic model, this corresponds to an upward shift in the entire path of all trade balances (see Section 5).

In this case, as with $NX_0 < 0$, both the import tariff and the export subsidy still lead to an appreciation of the home currency generating a negative valuation effect, reducing NFA and increasing NX, i.e., "improving" the long-run trade balance. However, this does not mean that any temporary deficit can be necessarily closed with a permanent tariff as there is an upper bound on the international transfer that can be induced by either tariff. This point can be clearly seen in the right panel of Figure 2. When a prohibitive import tariff is imposed, $\tau^I \to \infty$, the resulting equilibrium features zero imports $C_F = 0$, but positive exports $C_H^* > 0$. Both the terms of trade and the real exchange rate are finite and the long-run trade balance attains the maximum value implementable with τ^I.

In contrast, there exists a finite export subsidy, $0 < \underline{\tau}^E < 1$, that results in complete home immiserization with the exchange rate appreciation sufficient to transfer all resources to the rest of the world. In this case, export and import quantities converge to $C_H^* = Y^*$ and $C_F = 0$, respectively. Even in this case though the value of $NX > 0$ is finite. The following lemma provides a formal characterization of all the limiting cases, illustrated in Figure 2, with the proof contained in Appendix A2:

Lemma 3 (a) *Export and import quantities, C_H^* and C_F, are decreasing in the import tariff τ^I and converge to $C_H^* \in (0, B_0)$, $C_F = 0$ such that $NX > 0$ when $\tau^I \to \infty$, and to $C_H^* = Y$, $C_F \in (B_0^*, Y^*)$ such that $NX < 0$ when $\tau^I \to 0$.* (b) *In contrast, if $B_0 > 0$, export and import quantities converge to $C_H^* = 0$, $C_F > 0$ such that $NX < 0$ when the export tax $\tau^E \to \infty$, and to $C_H^* = Y$, $C_F = 0$ such that $NX > 0$ when $\tau^E \to \underline{\tau}^E$, where $\underline{\tau}^E \in (0, 1)$ is the export subsidy that results in complete home immiserization due to a negative valuation effect.*

Finally, we use our calibration introduced in the previous subsection to provide a quantitative illustration for the case when the policymaker aims to close a *temporary* trade deficit of 2% of GDP, misinterpreting it for a permanent one, while in fact $NX_0 = -NFA_0 = 0$. Specifically, we ask what import tariff can shift the trade balance up by 2% of GDP starting from $NX_0 = 0$. The "No Imbalances" panel of Table 1 displays the results: the required import tariff is large, equal to 123%. This is twice as high as the 64% import tariff that was need to close the permanent 2% trade deficit in the baseline case with $NFA_0 > 0$. The associated welfare costs are more than doubled, equal to a 3.5% loss in the permanent real consumption.

Closing a long-run trade surplus Finally, we consider the remaining case in which a policymaker aims to close a long-run trade surplus starting with $NX_0 = -NFA_0 > 0$. Whether this is possible depends on how indebted the country is and which tariff is used. For any initial portfolio below EA' and above EA'' in Figure 1, an import subsidy or an export tax can both be used to depreciate the real exchange rate sufficiently to generate a large enough positive valuation effect to close the imbalances, resulting in $NX = -NFA = 0$. However, if the initial asset position is below EA'', i.e., the country's external liabilities significantly exceed its assets ($P_{H0}B_0 \gg P^*_{F0}B^*_0$), an import subsidy cannot close the imbalance, and all attainable equilibria feature $NX > 0$, as depicted in the right panel of Figure 2. Indeed, even as $\tau^I \to 0$, the exchange rate depreciation is not sufficiently large, and even in this limit $NFA < 0$ and $NX > 0$.[18] In contrast, a sufficiently large export tax can always attain the necessary depreciation rate to eliminate the imbalances (point G in the right panel of Figure 2).

Proposition 6 *A country running a trade surplus, $NX_0 > 0$ under initial $NFA_0 < 0$, can always achieve a balanced trade using an export tariff, $\tau^E > 1$. Only if the initial imbalances are not too large, there also exists an import subsidy, $\tau^I < 1$, that can close the imbalances.*

3.5 Production and manufacturing jobs

Another often discussed objective of trade policy is to bring home manufacturing jobs that have been outsourced to countries with lower labor costs. The argument is based on political reasons, security concerns, or growth externalities.[19] In Appendix A4, we extend our analysis to a production economy with a tradable and non-tradable sector and labor mobility across sectors, assuming that manufacturing jobs are concentrated in the tradable sector.

We show that under financial autarky and balanced trade, an import tariff unambiguously reduces tradable-sector jobs. Indeed, this is a tax on the tradable sector, reducing both the quantities of imports and exports, even if the terms of this exchange are improved. This unambiguously shifts labor away from the tradable (manufacturing) sector and towards the non-

[18]Formally, Figure 1 provides the partition of the space of initial portfolios $(-B_0, B^*_0)$ for which trade balance is implementable with an import tariff, or subsidy, or not at all. Line EA' is the locus of initial portfolios that result in $NX_0 = NFA_0 = 0$ under free trade (point A on this line). Any portfolio above this line features $NFA_0 > 0$ and $NX_0 < 0$, and $NFA = NX = 0$ is implementable with an import tariff, $\tau^I > 1$, resulting in points below A on the trade balance curve. All portfolios that lie below EA' feature $NFA_0 < 0$ and $NX_0 > 0$; if, in addition, the portfolio is above EA'', then $NFA = NX = 0$ is implementable with an import subsidy, $\tau^I < 1$, resulting in points above A on the trade balance curve. Otherwise, for portfolios below EA'', there exists no import subsidy (or tariff) that can implement $NFA = NX = 0$ (line EA'' itself corresponds to the locus of portfolios that require infinite subsidy, $\tau^I = 0$, to achieve $NFA = NX = 0$ in point A'').

[19]See Krugman (1987), Venables (1987), Rodrik (1998), Ossa (2011), Itskhoki and Moll (2019) and Benigno, Fornaro, and Wolf (2025).

tradable (service) sector: the jobs created in import substitution are more than offset by the export-oriented jobs lost. Therefore, if the goal of trade policy is to increase broad tradable-sector employment, a trade subsidy — not a trade tax — should be used, even when jobs are lost to an external trade shock (e.g., the "China shock" as in Autor, Dorn, and Hanson 2013).[20]

Perhaps a more intriguing question is whether tradable jobs can increase when an import tariff is used to reduce a long-run deficit, as discussed above. Under these circumstances, imports are reduced by more than exports, at least in value terms, hence the tradable production may increase on net. Proposition A2 in the appendix shows that, indeed, such situation is possible, but it requires both extreme values of parameters (close to a Leontief utility function between tradables and non-tradables) and an extreme external liability position $B_0 \gg 0$, in excess of the country's exports in flow-value terms. Neither of these is realistic in the data. In our calibration described above, the large external liabilities of the U.S. are still quite a bit smaller than exports (7% versus 12% of GDP in flow-value terms). Therefore, manufacturing jobs are unlikely to increase with a broad tariff even when it reduces the trade deficit.

4 Optimal Tariff

Under balanced trade, it is generically optimal for countries to impose a (positive) tariff to improve their terms of trade. This section explores whether the size of the optimal tariff increases under a permanent trade deficit. We show that it is the gross asset positions — rather than the net deficits — that affect the size of the optimal tariff via the associated valuation effects, with the empirically-observed cross-border asset positions lowering the optimal import tariffs.

4.1 Import tariff

We first solve for the optimal import tariff $\tau^I = \tau$ in the absence of an export tax, $\tau^E = 1$. In this case, from (10) we have $\mathcal{Q} = \mathcal{S}$, and the implicit definition of the trade possibilities frontier $C_H^* = g(C_F)$ from Lemma 1 simplifies to:

$$u_H^* \cdot (C_H^* - B_0) = u_F^* \cdot (C_F - B_0^*). \tag{15}$$

The problem of a local policymaker is to choose a point on this implementability constraint that maximizes home welfare given by $u(C_H, C_F)$. We can write this problem compactly as:

$$\max_{C_F} \; u\big(Y - g(C_F), C_F\big).$$

[20]The underlying assumptions are homogenous labor and decreasing returns to employment in production. Heterogeneity of labor inputs or tradable sectors may lead to employment increasing in some segments of the tradable sector (for a quantitative analysis, see Rodriguez-Clare, Ulate, and Vasquez 2025).

The first-order optimality condition for this problem is $u_H \cdot g' = u_F$, or equivalently:

$$\frac{u_F}{u_H} = \frac{g'(C_F) \cdot C_F}{g(C_F)} \cdot \frac{C_H^*}{C_F}.$$

From the household optimality condition (5), the ratio on the left-hand side is equal to the home relative price P_F/P_H. Dividing both sides by P_F^*/P_H^* yields the value of the equilibrium tariff distortion τ on the left-hand side, according to (4), and therefore the optimal tariff can be expressed as:

$$\tau = \varepsilon \cdot \frac{EX}{IM}, \qquad \text{where} \qquad \varepsilon \equiv \frac{d \log C_H^*}{d \log C_F} = \frac{g'(C_F) \cdot C_F}{g(C_F)}, \tag{16}$$

where $EX = P_H^* C_H^*$ and $IM = P_F^* C_F$ are the values of home's exports and imports expressed in international prices.

Condition (16) is the generalization of the celebrated Johnson (1950) formula to the case of imbalanced trade. In particular, ε is the elasticity of the feasibility constraint faced by the planner which equals the percentage increase in export quantity C_H^* required to obtain an extra one percent of import quantity C_F. This elasticity reflects both foreign import demand (preferences), foreign export supply (endowment and, more generally, production), as well as the international asset positions. Even if the mapping from the model's parameters to the optimal tariff may be complex, the elasticity of the $g(\cdot)$ function together with a measure of trade imbalance EX/IM form a sufficient statistic, and no other information about home or foreign economies is required for the optimal tariff. Under balanced trade, $EX = IM$, the optimal tariff is simply the elasticity of the trade possibilities frontier, $\tau = \varepsilon$.

The immediate corollary of equation (16) is that tariffs are suboptimal when home is a price taker in international goods markets, i.e., takes its terms of trade $\mathcal{S} = P_F^*/P_H^*$ as exogenous. In this case, (15) can be rewritten as $C_H^* - B_0 = \mathcal{S} \cdot (C_F - B_0^*)$, such that the associated trade possibilities frontier has a constant slope equal to $\mathcal{S}$, and hence an elasticity given by $\varepsilon = \mathcal{S} C_F / C_H^* = IM/EX$. It follows that the optimal tariff distortion is nil, $\tau = 1$, and free trade maximizes welfare, irrespectively of the asset position and the trade deficit.

Without market power, the country cannot manipulate its terms of trade and extract surplus from foreign consumers and producers. Less obviously, neither can the country manipulate its exchange rate to generate cross-border valuation effects. Indeed, in the absence of export taxes, the real exchange rate coincides with the terms of trade and is also exogenous from the perspective of the economy. With both goods and asset prices determined internationally, the country cannot benefit from a tariff, and the optimal trade policy is free trade.

More generally, it may appear from (16) that the optimal tariff is decreasing in the trade deficit, IM/EX. However, the value of the elasticity ε is endogenous to the trade balance. Solving for ε using (15) under CES demand (2) provides a simple characterization of the optimal tariff in terms of measurable sufficient statistics in the presence of imbalances:

Proposition 7 *The welfare-maximizing import tariff is given by:*

$$\tau^W = 1 + \frac{1}{\eta\left(1 + \frac{\mathcal{B}}{EX - \mathcal{B}}\right) - 1} \cdot \frac{1}{\Lambda^*}, \tag{17}$$

where $EX \equiv P_H^* C_H^*$ *and* $\mathcal{B} \equiv P_H^* B_0$ *are the values of home exports and external liabilities in a tariff equilibrium, and* $\Lambda^* \equiv \frac{P_F^* C_F^*}{P_H^* C_H^* + P_F^* C_F^*}$ *is the foreign local expenditure share.*

To unpack the implications of Proposition 7 for the relationship between trade deficits and the optimal tariff, we consider a number of special cases. First, consider the textbook case when there are no cross-border financial positions, $B_0 = B_0^* = 0$, and trade is balanced. Formula (17) then simplifies to:[21]

$$\tau^W = \varepsilon = 1 + \frac{1}{\eta - 1}\frac{1}{\Lambda^*} > 1, \tag{18}$$

as $\eta \in (1, \infty)$. Therefore, it is generically optimal to impose a tariff to exploit the terms-of-trade externality and shift welfare from the rest of the world by distorting trade.

The size of the optimal tariff depends on two statistics — the foreign elasticity of substitution η and local expenditure share Λ^* — determining the market power of the home economy. Even when home is small and $\Lambda^* = 1$, imperfect substitutability of the home goods results in an optimal tariff, $\eta/(\eta - 1)$, which decreases in η. As home goods account for a larger market share abroad, $\Lambda^* < 1$, the optimal tariff increases, resembling the oligopoly markup in Atkeson and Burstein (2008). Note that the policymaker would still impose a tariff even if firms set prices with markups, as the policy aims to distort relative prices between the markets.

The left panel of Figure 3 provides a graphical illustration of the result. The policymaker can choose any allocation on the red trade possibilities frontier $C_H^* = g_0(C_F)$ and aims to shift the indifference curve up. At the optimal point, the two curves are tangent, with the tariff distorts the contract curve shifting it to the dashed blue line (see Aguiar, Itskhoki, and Mukhin 2025). The optimal tariff is always positive as long as $g_0(\cdot)$ has curvature, i.e., whenever the terms of trade are not fixed.

Next, suppose that only home assets B_0 are traded internationally allowing for external imbalances, while $B_0^* = 0$. The home budget constraint then simplifies to $EX = IM + \mathcal{B}$, and the optimal tariff is given by $\tau = 1 + \frac{1}{\eta(1+NX/IM)-1}\frac{1}{\Lambda^*}$. In this case, the optimal tariff is increasing in the (aggregate) trade deficit.[22] This result relies on the assumption that trade

[21]This result is derived in Gros (1987) in the context of the Krugman model; Alvarez and Lucas (2007), Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung, and Larch (2013) derive related results in the Eaton-Kortum and Melitz models; Caliendo and Parro (2022) provide a survey of these and other related results in the literature, while Humphrey (1995) reviews the early contributions based on the geometric approach. Note that Λ^* corresponds to the own share in the gains-from-trade formula of Arkolakis, Costinot, and Rodríguez-Clare (2012).

[22]When $\Lambda^* \approx 1$, the first-order approximation around $EX = IM$ yields $\tau \approx \frac{\eta}{\eta-1} + \frac{\eta}{\eta-1}\frac{IM-EX}{IM}$, an expression that resembles the formula $\Delta\tau = \max\left\{10\%, \frac{IM-EX}{IM}\right\}$ used to rationalize the "Liberation Day" tariffs. Of course, our formula is derived for a uniform tariff on all trade partners, not for bilateral trade tariffs.

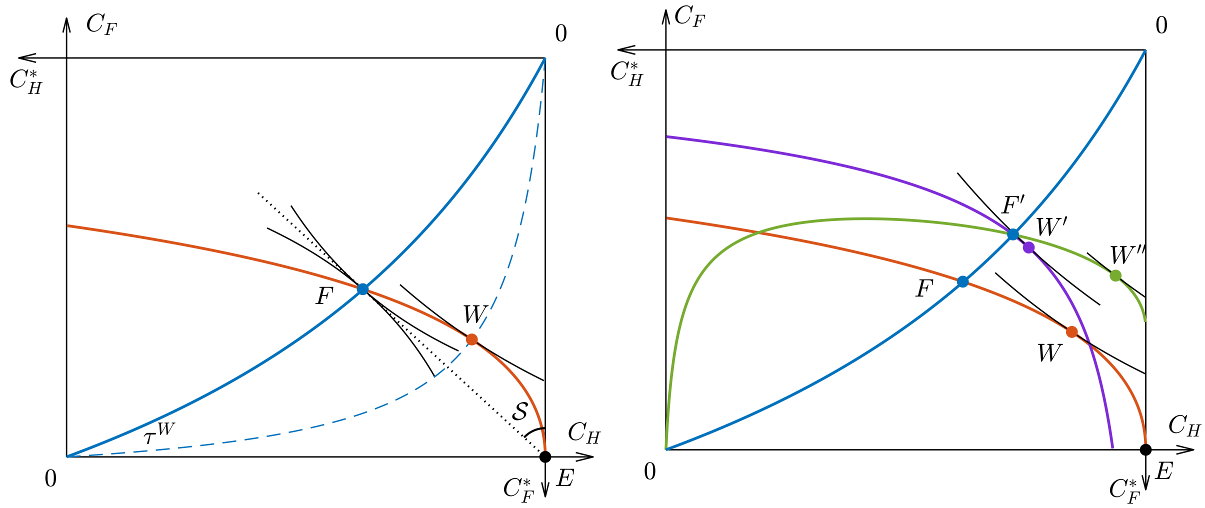

Figure 3: Laissez faire allocation and the optimal tariff

Note: The blue curve is the contract curve, the red curve is the balanced trade frontier $C_H^* = g_0(C_F)$; the purple (green) curve corresponds to g^I (g^E), i.e., the set of implementable allocations using an import (export) tariff for a given asset position (B_0, B_0^*). The dotted black line is the budget constraint without tariffs and the black solid lines are the indifference curves. Points F, F' denote the free-trade equilibrium with and without imbalances. The allocation under the optimal unilateral policy is denoted with W for balanced trade, and with W' and W'' for an import tariff and an export tax, respectively, under financial imbalances.

deficits emerge from an international asset position in terms of home assets, or in our notation $B_0 < 0$ such that $NX = \mathcal{B} \equiv P_H^* B_0 < 0$. This counterfactually implies that an exchange rate appreciation generates a positive valuation effect for the home economy. Thus, the reason the optimal tariff is increasing in the trade deficit in this case has little to do with trade flows or terms of trade, and is due to the valuation effect associated with the gross asset position.

Finally, away from the knife-edge case of $B_0^* = 0$, larger imbalances are neither necessary nor sufficient to rationalize higher tariffs relative to the balanced-trade case. In contrast, it is possible to have imbalances that do not affect the optimal tariff. In the limiting case when U.S. assets are traded only locally, $B_0 = 0$, and all international positions are in terms of foreign assets, the expression for the optimal tariff becomes $\tau = 1 + \frac{1}{\eta-1}\frac{1}{\Lambda^*}$, just as in the baseline model with balanced trade. Thus, conditional on Λ^*, the optimal tariff is independent of whether the country runs a trade deficit or a trade surplus. On the other hand, it is possible to have no imbalances and yet, the optimal tariff that depends on the international portfolio. This happens when countries have significant gross positions $B_0, B_0^* \gg 0$, yet the equilibrium net foreign assets are zero, $NFA = P_F^* B_0^* - P_H^* B_0 = 0$.

Corollary 1 *Trade imbalance is neither necessary nor sufficient to affect the optimal tariff. In the empirically-relevant case with $B_0 \gg 0$, the optimal import tariff is lower than under financial autarky, $B_0 = B_0^* = 0$, with $NX = 0$.*

To understand the general implications of imbalances for the optimal tariff (17), notice from the budget constraint (9) that there is a trade-off between the terms-of-trade effect and the valuation effect. Ideally, the policymaker would like to *improve* the terms of trade (reduce $\mathcal{S}$) and *depreciate* the real exchange rate (reduce $\mathcal{Q}$). However, this cannot be simultaneously achieved with an import tariff under which $\mathcal{S} = \mathcal{Q}$ by (10). It follows that, as long as $B_0 > 0$, the favorable terms-of-trade effect in the goods market is partially offset by an unfavorable valuation effect on external liabilities, and the optimal tariff is lower than under financial autarky. More generally, formula (17) implies that conditional on trade shares, the optimal tariff is decreasing in home's external liabilities, reaches zero when $\mathcal{B} = EX$ (or, equivalently, $B_0 = C_H^*$), and even turns into an import subsidy when external liabilities exceed exports.[23]

Empirically, both U.S. equities and dollar-denominated debt held by the foreigners are important components of the large external liabilities $B_0 \gg 0$ (see Gourinchas and Rey 2014). Thus, despite the trade deficit, the optimal tariff is lower relative to the balanced-trade case with no gross foreign assets and liabilities.[24] Quantitatively, Table 1 shows that under our baseline calibration, the optimal import tariff is equal 8.8%, which is three times lower than the optimal tariff of 35.3% in a model with no cross-border positions. The implied welfare gains equal 0.1% of aggregate consumption and are an order of magnitude smaller than under financial autarky. The negative valuation effects due to the associated 4% appreciation of the exchange rate largely offset the gains from the proportionate improvement in the goods-market terms of trade.For larger tariffs, valuation losses can dominate welfare gains from the improved terms of trade. Under the optimal tariff, trade balance improves relative to the initial deficit of 2% of GDP, but remains negative at 1.6% of GDP.

We close this section with a brief discussion of a strategic motive in foreign asset accumulation. As the optimal tariff decreases with external liabilities B_0, at least for a given value of exports EX and foreign share Λ^*, foreign countries have incentives to accumulate home assets as a hedge against the trade war. First, the appreciation of the home currency in response to an import tariff compensates foreign countries with a valuation effect in proportion to their holdings of home's assets. Second, larger home's external liabilities reduce both its optimal tariff in case of a trade war, as well as incentives to begin a trade war altogether. While only large trade partners internalize the latter strategic motive, all countries and even individual foreign investors have incentives to buy home's assets as a hedge against trade war (*cf.* Dooley, Folkerts-Landau, and Garber 2004).

Corollary 2 *The U.S. trade partners have incentives to accumulate U.S. assets B_0 as a hedge against an import tariff and trade war.*

[23] The optimal tariff formula (17) emphasizes the home's external liabilities $\mathcal{B} = P_H^* B_0$ as a sufficient statistic from the perspective of the asset market. Of course, both B_0 and B_0^* shape equilibrium outcomes, including exports EX and foreign share Λ^* that also enter the optimal tariff formula. Nevertheless, the valuation effect from the tariff is proportional to B_0, in line with Proposition 1, explaining the asymmetry in the formula.

[24] This conclusion contrasts with the results of Pujolas and Rossbach (2024) who focus on the special case of $B_0^* = 0$ and assume that $B_0 < 0$ to reproduce the U.S. trade deficit. See also Ossa (2016) and Ignatenko, Lashkaripour, Macedoni, and Simonovska (2025).

4.2 Export tax

An export tax is an instrument rarely used in practice. Nevertheless, it is instructive to solve for the optimal export tax in the presence of imbalances and contrast it with the equivalence of the two tax instruments under Lerner symmetry. We consider first the case when the policymaker can use both trade taxes.

Proposition 8 *When $B_0 > 0$, the optimal policy combines unbounded export tax $\tau^E \to \infty$ and import subsidy $\tau^I \to 0$ to both depreciate away the country's external liabilities with $\mathcal{Q} \to \infty$ and keep the terms of trade $\mathcal{S}$ at the optimal level with a combined trade wedge $\tau = \tau^I \tau^E = 1 + \frac{1}{\eta-1}\frac{1}{\Lambda^*}$.*

A country with $B_0 \neq 0$ and access to both trade taxes can combine them to engineer a pure transfer from the rest of the world. In particular, when $B_0 > 0$, home can depreciate away its local-currency debt B_0 by imposing an export tariff $\tau^E > 1$ and an import subsidy $\tau^I < 1$, without constraining the overall trade wedge $\tau \equiv \tau^E \tau^I$ or the resulting terms of trade $\mathcal{S}$. In fact, it is optimal to take this to the limit, fully eliminating external liabilities while keeping the overall trade distortion at the optimal level given by formula (17) with $B_0 = 0$.[25]

If the policymaker cannot use the import tariff, $\tau^I = 1$, it is no longer possible to simultaneously generate an unbounded valuation effect and achieve the optimal terms of trade with a single export tax. Nonetheless, and in contrast with the case of the import tariff, the export tax moves both the terms of trade and the real exchange rate in the desirable directions — improving the former and depreciating the latter — relaxing the home's budget constraint (9). This can be clearly seen once the optimal tax is expressed in terms of sufficient statistics:

Proposition 9 *The welfare-maximizing export tariff is given by:*

$$\tau^E = 1 + \frac{1}{\eta - 1}\frac{1}{\Lambda^*} + \frac{1}{\theta}\frac{\eta}{\eta - 1}\frac{\mathcal{B}}{IM}\frac{1}{\Lambda}, \tag{19}$$

where $IM \equiv P_F^* C_F$, $\mathcal{B} \equiv P_H^* B_0$, and $\Lambda \equiv \frac{P_H C_H}{P_H C_H + P_F C_F}$ is the home's local expenditure share.

When $B_0 = 0$, Lerner symmetry holds, and the formula coincides with the optimal import tariff in (17). With $B_0 > 0$, a positive valuation effect from an export tariff increases the size of the optimal export tax beyond the optimal terms-of-trade manipulation level given by (18). Furthermore, as long as $B_0 > 0$, the optimal export tax is unambiguously higher than the optimal import tariff given by (17). The right panel of Figure 3 provides a graphical illustration showing the optimal allocation for each trade tax as a tangency point between the indifference curves and the corresponding implementability constraints g^E and g^I. The difference between

[25] Conversely, a country with a positive asset position in its own currency, $B_0 < 0$, can appreciate its value with a combination of an export subsidy $\tau^E < 1$ and an import tariff $\tau^I > 1$. As $\tau^E \to 0$ and $\tau^I \to \infty$, a country generates an infinite transfer from the rest of the world and immiserates it. Finally, when $B_0 = 0$, Lerner symmetry applies and one tariff instrument is redundant as no international wealth transfer can be engineered.

the two is particularly stark in the limiting case of a price-taking economy $\eta \to \infty$. In this case, an import tariff cannot affect either $\mathcal{S}$ or $\mathcal{Q} = \mathcal{S}$, and the optimal import tariff is zero. In contrast, taking $\mathcal{S}$ as given, the export tax can still depreciate the real exchange rate, $\mathcal{Q} = \tau^E \mathcal{S}$, and generate a positive transfer from the rest of the world.[26]

Given empirical magnitudes of the U.S. external liabilities, the difference between the optimal export tax and import tariff is quantitatively large. Table 1 shows that the optimal export tax is 67.5% in our baseline calibration, almost twice as high as under financial autarky and 8 times higher than the optimal import tariff. The policy improves the country's terms of trade by 29% and depreciates its liabilities by 19%. The associated welfare gains are equal to 1.8% of real consumption and greatly exceed the gains from the import tariff. At the same time, the positive valuation effect results in a deterioration of the trade balance from 2% to 3.8% of GDP.

4.3 Retaliation

Our baseline model abstracts from foreign trade policy. This section shows how our results extend to a setting where both countries impose tariffs. As a starting point, notice that while foreign welfare is monotonically decreasing in home tariffs under financial autarky, $B_0 = B_0^* = 0$, it may in fact increase when $B_0, B_0^* > 0$. The reason is that the appreciation of the real exchange rate generates a positive transfer to the foreign economy that can more than offset the negative terms-of-trade effect. Foreigners then have little incentives to retaliate. Although this cannot happen under the welfare-maximizing tariff, it may happen when the country aims to use a tariff to close a large trade deficit.

Consider next the case when home and foreign choose *import* tariffs τ and τ^*, abstracting from export taxes for simplicity. Equation (4) for prices faced by households is then replaced by:

$$P_F = \tau P_F^* \qquad \text{and} \qquad P_H^* = \tau^* P_H, \tag{4'}$$

while the market clearing (3), household demand (5)-(6) and budget constraint (7) remain unchanged. It follows that the relative prices in the two countries are distorted by both home and foreign tariffs, and condition (11) is replaced by:

$$\frac{u_F}{u_H} = \tau\tau^* \cdot \frac{u_F^*}{u_H^*}.$$

We consider a Nash equilibrium with each country setting trade policy taking foreign trade policy as given. As before, equations (6) and (7) result in the implementability constraint for the home planner, $C_H^* = g(C_F; \tau^*)$, which now depends on the foreign trade policy:

$$u_H^*(C_H^*, Y^* - C_F)(C_H^* - B_0) = \tau^* u_F^*(C_H^*, Y^* - C_F)(C_F - B_0^*).$$

[26] Taking international prices as given, an export tax shifts revenues away from domestic producers to the government, which results in a negative international valuation effect for foreign owners of domestic assets.

Given this generalized trade possibilities frontier $g(\cdot)$, the planner's problem remains the same as before, and the expression for the optimal tariff (17) is unchanged in terms of sufficient statistics. Symmetrically, we can define the foreign trade possibilities frontier $C_F = g^*(C_H^*, \tau)$ and arrive at the same conclusion for the foreign optimal tariff (see Appendix Figure A1).

Proposition 10 *The Nash equilibrium tariffs have the same structure as the unilateral tariffs:*

$$\tau = 1 + \frac{1}{\eta\left(1 + \frac{\mathcal{B}}{EX - \mathcal{B}}\right) - 1} \cdot \frac{1}{\Lambda^*} \qquad \textit{and} \qquad \tau^* = 1 + \frac{1}{\theta\left(1 + \frac{\mathcal{B}^*}{IM - \mathcal{B}^*}\right) - 1} \cdot \frac{1}{\Lambda}, \tag{20}$$

where $\mathcal{B}^* \equiv P_F^* B_0^*$, $\Lambda \equiv \frac{P_H C_H}{P_F^* C_F + P_H C_H}$ *and* $\Lambda^* \equiv \frac{P_F^* C_F^*}{P_H C_H^* + P_F^* C_F^*}$ *are the local expenditure shares.*

Even though the structure of the trade-war tariffs in (20) is the same as for the unilateral tariffs in (17), the values of these tariffs are different. A positive foreign tariff implies that both economies become more closed to trade: C_H^* and C_F decline. This increases both $\mathcal{B}/EX = B_0/C_H^*$ and Λ^*, and thus lowers the optimal home tariff in (20). With a symmetric argument for foreign, the best response functions in the space of tariffs are negatively slopped, hence tariff policies are strategic substitutes (*cf.* Bagwell and Staiger 1999). Quantitiatevely, Table 1 shows that the optimal U.S. tariff goes down from 8.8% to 6.8% when the foreign economy retaliates. While a U.S. unilateral tariff raises the U.S. welfare by 0.1% of aggregate consumption, the U.S. welfare *declines* by 0.1% in a trade war. These effects are muted by gross asset positions. Under financial autarky, the trade-war tariffs are 35% for the U.S. and 40% for the rest of the world, with the U.S. welfare declining by 1.6% relative to a gain of 1% under a unilateral tariff.

Proposition 10 further emphasizes the point that it is *gross* asset positions (which may be symmetric) rather than net positions and trade deficits (which are necessarily asymmetric) that matter for the optimal tariff. In particular, in a world where countries hold large cross-border asset positions against each other, the optimal import tariffs are low for everyone, and the probability of a trade war declines. Thus, the increase of gross foreign asset holding around the world starting in 1980s may have contributed to the persistence of the low-tariff equilibrium and prevented the emergence of earlier trade wars (e.g., in response to large unemployment after the 2008 Global Financial Crisis).

Corollary 3 *Large cross-border asset positions lower the optimal tariff for every country irrespective of the net asset position and trade deficit, supporting a low-tariff Nash equilibrium.*

Net positions, however, do matter for chances of "winning" a trade war. From (20), the country's optimal tariff is higher $\tau > \tau^*$ and its exposure to a trade war is lower when the foreign economy is more open $\Lambda^* < \Lambda$, has lower elasticity of substitution $\eta < \theta$, and/or larger external liabilities $B^* > B$. In the extreme case, when countries are sufficiently asymmetric, it is even possible for home to improve its welfare in the Nash equilibrium relative to the free trade (Johnson 1953).

In practice, the first source of asymmetry does not favor the U.S. — even though it is larger than most of its trade partners, it is small and open relative to the rest of the world as a whole. Table 1 shows that the optimal tariff is lower and the welfare costs are higher for the U.S. than for the RoW in a model without cross-border positions, $B_0 = B_0^* = 0$. However, this differential effect is mostly offset by the large cross border asset positions, making the trade-war tariffs and the associated welfare losses much more similar for the two regions in our baseline calibration. Notice also that both the real exchange rate and the trade balance remain largely unchanged in a trade war.

4.4 Fiscal revenues

Our baseline analysis assumes that the policymaker maximizes welfare of a representative household. An alternative objective of trade policy may be fiscal revenues, as was often emphasized in the U.S. policy debate.[27] This section considers this alternative objective and solves for the revenue-maximizing import tariff, i.e., the peak of the tariff Laffer curve. We focus on fiscal revenues in terms of domestic purchasing power (i.e., in units of the home good, or home currency) and assume that all government debt is in terms of home currency as well.[28]

Proposition 11 *The revenue-maximizing import tariff is given by*

$$\tau^R = \frac{\theta}{\theta - 1 - \tau^W \frac{1-\Lambda}{\Lambda}} \cdot \left[\tau^W + (\tau^W - 1) \frac{\mathcal{B}^*}{IM - \mathcal{B}^*} \right], \tag{21}$$

where $\tau^W = 1 + \frac{1}{\eta\left(1+\frac{\mathcal{B}}{EX-\mathcal{B}}\right)-1} \cdot \frac{1}{\Lambda^*}$ *as in* (17), $\mathcal{B}^* = P_F^* B_0^*$, $IM = P_F^* C_F$, *and* $\Lambda \equiv \frac{P_H C_H}{P_F^* C_F + P_H C_H}$ *is the home's local expenditure share. Assuming* $B_0, B_0^* \geq 0$ *and* $\mathcal{B}^* < IM$, *we have* $\tau^R > \tau^W \geq 1$. *When* $\eta \to \infty$, $\tau^W = 1$ *and* $\tau^R = \frac{\theta}{\theta - 1/\Lambda} > 1$.

The government that aims to collect maximum fiscal revenues puts zero weight on the distortionary effects of tariffs. The trade-off and the bounded solution arise from a standard Laffer-curve logic: a higher tariff decreases gross trade flows and, therefore, diminishes the tax base. A higher home elasticity — determined by θ and the local expenditure share Λ — means that households switch more easily to domestic goods when import prices go up due to tariffs, which lowers the revenue-maximizing tariff τ^R.

[27] Although the standard Diamond and Mirrlees (1971) argument implies that it is suboptimal to tax differentially home and foreign producers to collect fiscal revenues, it does not necessarily apply when consumption tax is not available and the government uses other distortionary taxes. See Alessandria, Ding, Khan, and Mix (2025) and Kocherlakota (2025) for the contemporaneous work that revisits the fiscal role of tariffs in the world with distortionary taxation, and Chari, Nicolini, and Teles (2023) for the optimal cooperative taxation.

[28] We use the equilibrium conditions and the trade possibilities frontier to write the fiscal revenues objective as $(P_F - P_F^*)C_F/P_H = \frac{u_F}{u_H} C_F - \frac{g(C_F)-B_0}{C_F - B_0^*} C_F$, assuming the policy tool is the import tariff. In the absence of international asset positions, Lerner symmetry applies to government revenues, and the maximum levels of tariffs and revenues are the same for τ^I and τ^E (see Itskhoki and Mukhin 2026).

Examining the formulas (21) and (17), we can verify that $\tau^R > \tau^W \geq 1$ for given values of trade and expenditure shares and empirically-relevant external asset positions. Intuitively, $\tau^R > \tau^W$ because the planner aims to extract revenues from both the home and foreign households with τ^R, while τ^W is used to only extract rents from the foreigners. In the limit with $\eta \to \infty$, it is impossible to manipulate the term of trade and the welfare-maximizing trade policy is free trade, $\tau^W = 1$. In contrast, the revenue-maximizing policy is $\tau^R = \frac{\theta}{\theta - 1/\Lambda} > 1$, with the government extracting surplus from the domestic households.

Lastly, we quantify the effects in Table 1. The revenue-maximizing tariff is an order of magnitude higher than the welfare-maximizing tariff, 64.9% against 8.8%. As mentioned above, this is partly because the planner does not care about distortionary effects of tariffs on home households in the former case. But even more importantly, a revenue-maximizing policymaker ignores the negative international valuation effects. Indeed, the difference between the two tariffs, τ^R and τ^W, is substantially smaller in the calibration without global asset positions, 81.6% versus 35.3%. This also explains a much more detrimental effect of τ^R on welfare in the baseline model: the maximum fiscal revenues that can be raised with import tariffs are equal 2.9% of GDP and come at a cost of 1.6% fall in household welfare.

5 Dynamic Model

This section generalizes our analysis of the long-run trade deficit and the optimal tariff to a fully dynamic environment. First, this allows us to distinguish between temporary and permanent trade deficits, show how they depend on trade policies, and prove the robustness of the baseline results in a dynamic setting. Second, we introduce "exorbitant privilege" explicitly, distinguishing it from accumulated asset positions. In particular, we show that exorbitant privilege is both endogenous to trade policy and affects the optimal tariff. Details of the derivations and proofs can be found in Appendix A2.

5.1 Equilibrium environment

The aim of this section is to offer the most direct dynamic extension of the baseline setup. Portfolio choice with incomplete international markets is generally intractable. We adopt a number of assumptions that deliver tractability and allow us to generalize the analysis to a full-fledged dynamic environment. Furthermore, our dynamic model with tariffs and portfolio choice establishes a natural benchmark that can be used for future generalizations and extensions.

First, we assume the same elasticities across countries, $\eta = \theta$, and equalize inter- and intratemporal elasticities of substitutions to make household preferences *separable* across periods and goods. Specifically, the expected utility is $U \equiv (1-\beta)\mathbb{E}\sum_{t=0}^{\infty}\beta^t u_t$ at home and similarly U^* in foreign, with period flow utilities u_t and u_t^* defined as in (2). Therefore, θ is also the elasticity of intertemporal substitution and, hence, $1/\theta$ is the relative risk aversion.[29]

Second, endowment shocks $\{Y_t, Y_t^*\}$ are the only source of uncertainty and equities issued on these endowment processes (also known as Lucas trees) are the only internationally traded assets.[30] We denote positions (shares) of home and foreign investors in the home asset with $1 - b_t$ and b_t, respectively. Similarly, they hold shares b_t^* and $1 - b_t^*$ in the foreign asset. Consequently, the dividends on these positions are $P_{Ht}Y_t(1 - b_t) + P_{Ft}^*Y_t^*b_t^*$ for the home portfolio and $P_{Ht}Y_tb_t + P_{Ft}^*Y_t^*(1 - b_t^*)$ for the foreign portfolio, where as before P_{Ht} and P_{Ft}^* are the local (producer) prices.

Third, we introduce an exogenous convenience yield on the home asset, $\chi \in (\beta, 1]$, as a source of exorbitant privilege. Formally, we write the flow budget constraint for home households as:

$$P_{Ht}C_{Ht} + P_{Ft}C_{Ft} + \chi\mathcal{V}_t(1 - b_{t+1}) + \mathcal{V}_t^*b_{t+1}^* = (\mathcal{V}_t + P_{Ht}Y_t)(1 - b_t) + (\mathcal{V}_t^* + P_{Ft}^*Y_t^*)b_t^* + T_t,$$

where $\mathcal{V}_t$ and $\mathcal{V}_t^*$ are the ex-dividend values of the home and foreign assets and

$$T_t = (\tau^I - 1)P_{Ft}^*C_{Ft} + (\tau^E - 1)P_{Ht}C_{Ht}^* - (1 - \chi)\mathcal{V}_t(1 - b_{t+1})$$

is the lump-sum transfer of tariff revenues and convenience fees supporting $\chi < 1$.[31]

The foreign budget constraint is similar and also features convenience yield χ on the home asset. The foreign government collects no tariffs, and the transfer from households to the government within the foreign region reflects convenience fees only, $T_t^* = -(1 - \chi)\mathcal{V}_tb_{t+1}$. The common (and constant over time) convenience yield on home assets perceived by both home and foreign agents is essential for the tractable solution of the model, which can be easily extended to also feature a convenience yield (or discount) on foreign assets χ^*, as well as foreign trade policy. We focus on the time-invariant tariffs τ^I and τ^E imposed by the home economy, and characterize comparative statics across equilibria with different levels of tariffs and convenience yields.

Dynamic equilibrium Many equilibrium conditions from the static economy still apply in the dynamic environment. In particular, the market clearing condition (3), the law of one price (4), and relative good demand (static household optimality) at home (5) and abroad (6)

[29]This is a weaker assumption than the Cole and Obstfeld (1991)'s logarithmic preferences, with $\theta = 1$, in which case the portfolio choice is indeterminate. We still assume $\theta > 1$ for the optimal tariff analysis.

[30]The same solution emerges in a non-stochastic environment, where Y_t and Y_t^* evolve deterministically over time, with real (or nominal) bonds traded instead of the equities. With stochastic endowments, trade in equities allows to fully share risk from endowment shocks, but does not provide first-best insurance against tariff (or convenience-yield) shocks (*cf.* Caliendo, Kortum, and Parro 2025).

[31]Convenience yield χ is similar to a cash-in-advance liquidity services provided by home assets. The liquidity fees are paid locally without altering the overall country budget constraint. Qualitatively this modeling is equivalent to the home asset delivering direct utility benefits (as money-in-the-utility). See, e.g., Jiang, Krishnamurthy, Lustig, and Sun (2024) for a reduced-form modeling approach and Bianchi, Bigio, and Engel (2021) for a more spelled-out model of convenience yields in an international context.

still apply and hold state-by-state and period-by-period (with time index $t \geq 0$).

Substituting in the transfer into the household budget constraint and simplifying using the market clearing conditions, we obtain the country's flow budget constraint describing the evolution of its external asset position:

$$\mathcal{V}_t^* \Delta b_{t+1}^* - \mathcal{V}_t \Delta b_{t+1} = (P_{Ht}^* C_{Ht}^* - P_{Ft}^* C_{Ft}) + (P_{Ft}^* Y_t^* b_t^* - P_{Ht} Y_t b_t). \tag{22}$$

The first term on the right-hand side is net exports NX_t and the second term is net factor payment (dividends) from abroad. Together they form the current account equal to the financial account on the left-hand side. When international positions are unchanged, $\Delta b_{t+1} = \Delta b_{t+1}^* = 0$, equation (22) corresponds directly to the static budget constraint (7), using notation $B_t \equiv Y_t b_t$ and $B_t^* \equiv Y_t^* b_t^*$ for international dividend claims in real terms.

The only new type of equilibrium conditions is the optimal portfolio choice Euler equations for each representative household and each asset. We use auxiliary notation $V_t \equiv \mathcal{V}_t / P_{Ht}$ and $V_t^* \equiv \mathcal{V}_t^* / P_{Ft}^*$ for the real ex-dividend values of assets. Using this notation, we can write the home and foreign Euler equations for the home asset and for the foreign asset as respectively:

$$\beta \mathbb{E}_t \left[\frac{u_{Ht+1}}{u_{Ht}} \frac{V_{t+1} + Y_{t+1}}{V_t} \right] = \beta \mathbb{E}_t \left[\frac{u_{Ht+1}^*}{u_{Ht}^*} \frac{V_{t+1} + Y_{t+1}}{V_t} \right] = \chi, \tag{23}$$

$$\beta \mathbb{E}_t \left[\frac{u_{Ft+1}}{u_{Ft}} \frac{V_{t+1}^* + Y_{t+1}^*}{V_t^*} \right] = \beta \mathbb{E}_t \left[\frac{u_{Ft+1}^*}{u_{Ft}^*} \frac{V_{t+1}^* + Y_{t+1}^*}{V_t^*} \right] = 1. \tag{24}$$

These conditions determine the optimal portfolio choice, b_{t+1} and b_{t+1}^* for $t \geq 0$, for both households, as well as asset market clearing, which is implicitly embedded in our notation. The initial asset positions are b_0 and b_0^*. Asset market clearing determines the equilibrium values of the two assets, V_t and V_t^* for $t \geq 0$.

Lemma 4 *The equilibrium consumption allocation is time-invariant fractions of endowments:*

$$C_{Ht} = (1-a) Y_t, \qquad C_{Ht}^* = a Y_t, \qquad C_{Ft} = a^* Y_t^*, \qquad C_{Ft}^* = (1-a^*) Y_t^*, \tag{25}$$

and the equilibrium portfolios are constant shares of home and foreign equities: $b_{t+1} = \tau^E a$ *and* $b_{t+1}^* = a^*$ *for all* $t \geq 0$. *The constants* $a, a^* \in [0,1]$ *solve a system of two equations which ensure the household optimality of the consumption allocation:*

$$\frac{a^*}{1-a^*} = (\tau^I \tau^E)^{-\theta} \frac{\gamma^*}{1-\gamma^*} \frac{\gamma}{1-\gamma} \frac{1-a}{a}, \tag{26}$$

and the intertemporal budget constraint of the countries:

$$a^* - b_0^* = \left(\frac{\gamma^*}{1-\gamma^*} \frac{1-a^*}{a} \right)^{\frac{1}{\theta}} \frac{\mathbb{E} \sum_{t=0}^{\infty} (\beta/\chi)^t Y_t^{\frac{\theta-1}{\theta}}}{\mathbb{E} \sum_{t=0}^{\infty} \beta^t Y_t^{*\frac{\theta-1}{\theta}}} \left(a - \frac{b_0}{\tau^E} \right). \tag{27}$$

Given separable preferences, stochastic endowment Y_t and Y_t^* of home and foreign goods is optimally allocated to consumption in constant proportions. The two "frictions" — namely, the time-invariant tariffs and convenience yield — distort the equilibrium consumption shares, but do not change the fact that they are constant over time and states. The consumption allocation in (25) satisfies both feasibility (3) and private consumption optimality (5) and (6), given the distorted contract curve (26) which applies period-by-period and state-by-state in terms of consumption shares.

Lastly, portfolio shares $b_t = \tau^E a$ and $b_t^* = a^*$ for $t \geq 1$ with a and a^* that satisfy (27) given initial portfolios b_0 and b_0^* ensure simultaneously the optimal savings and portfolio choice decisions of the households (23)–(24), as well as the country budget constraints (22) for all $t \geq 0$. The equilibrium is supported with the real equity prices $V_t/Y_t = \mathbb{E}_t \sum_{j=1}^{\infty} (\beta/\chi)^j \left(Y_{t+j}^*/Y_t^*\right)^{\frac{\theta-1}{\theta}}$ and $V_t^*/Y_t^* = \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \left(Y_{t+j}^*/Y_t^*\right)^{\frac{\theta-1}{\theta}}$.[32] Note that, given no foreign tariffs, the home's consumption share in foreign output, a^*, coincides with its portfolio share in foreign equity, b^*; in contrast, the foreign share in home equity, b needs to be higher than its share in consumption of the home good, a, to cover the export tax.

To summarize, the equilibrium allocation is fully summarized by a pair of constants, a and a^*, which satisfy two equations — the distorted contract curve (26) and the intertemporal budget constraint (27) — the direct counterparts of equations (11) and (12) in the static model. As a result, we can replace the Edgeworth box in the space of goods with an Edgeworth box in the space of portfolio shares (see Figure 4 below). As before, tariffs shift the contract curve (and, when $b_0 \neq 0$, also shift the budget constraint via the valuation effect). Additionally, greater convenience yields on the U.S. assets (lower χ) shifts out the budget constraint (27) and allows the U.S. to sustain a higher consumption of both goods (greater $1-a$ and a^*).

5.2 Mapping from dynamic to long-run model

The equilibrium characterization in Lemma 4 allows us to aggregate the dynamic model into a single-period long-run model of Section 2 using the following definition:

Definition 1 *The permanent component of quantities $X \in \{Y, C, C_H, C_F\}$, prices P_J for $J \in \{H, F\}$, and trade values $M \in \{EX, IM\}$ are given, respectively, by:*

$$X \equiv \left((1-\beta)\mathbb{E}\sum_{t=0}^{\infty}\beta^t X_t^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}, \quad P_J \equiv \left((1-\beta)\mathbb{E}\sum_{t=0}^{\infty}\beta^t P_{Jt}^{1-\theta}\right)^{\frac{1}{1-\theta}}, \quad M \equiv (1-\beta)\mathbb{E}\sum_{t=0}^{\infty}\beta^t M_t,$$

and similarly for foreign. Standard relationships $IM = P_F^* C_F$ and $EX = P_H^* C_H^*$ hold for

[32] Under the conjectured consumption allocation, the home and foreign stochastic discount factors (SDFs) satisfy $u_{Ht+1}/u_{Ht} = u_{Ht+1}^*/u_{Ht}^* = (Y_t/Y_{t+1})^{1/\theta}$ and $u_{Ft+1}/u_{Ft} = u_{Ft+1}^*/u_{Ft}^* = (Y_t^*/Y_{t+1}^*)^{1/\theta}$, and thus the endowment risk is perfectly shared. Given these SDFs and the equity prices $\{V_t, V_t^*\}$, all four Euler equations are satisfied for all $t \geq 0$. Furthermore, constant portfolio shares ensure that flow budget constraints hold for all $t \geq 1$, and (27) corresponds to the budget constraint at $t = 0$ given the initial portfolios.

permanent components, and the long-run trade deficit is $NX = EX - IM$. The permanent component of the real exchange rate and terms of trade are $\mathcal{Q} \equiv P_F^*/P_H$ and $\mathcal{S} \equiv P_F^*/P_H^*$.

This aggregation mimics the standard cross-sectional aggregation under CES preferences and applies for all objects in our analysis — quantities, prices, and values of exports and imports. For terms of trade and the real exchange rate, we use the corresponding long-run price averages and then take the ratios.[33] Note that whenever there is no time subscript, the variable is the long-run aggregate.

The permanent components defined above map directly into the static equilibrium conditions of the baseline model. In particular, it follows immediately that consumption aggregates form the long-run welfare function as in (2), and the resource constraints (3) hold for quantity aggregates. Given time-invariant tariffs, the law of one price deviations in (4) also apply for price aggregates, which in particular implies $\mathcal{Q} = \tau^E \mathcal{S}$ as in (10). Static consumption optimality (5) and (6) holds for permanent components of consumption and prices, in light of Lemma 4. The only static equilibrium condition that is altered by convenience yield is the country budget constraint (7).

Proposition 12 *The permanent components of allocations and prices from Definition 1 satisfy the static equilibrium system (2)–(6) and the country budget constraint (7) generalized as:*

$$\big(C_H^*/\mathcal{S} - C_F\big) + \big(B_0^* - B_0/\mathcal{Q}\big) + \Omega \cdot \big(C_H^*/\mathcal{S} - B_0/\mathcal{Q}\big) = 0, \tag{28}$$

where $B_0 = b_0 Y$ and $B_0^* = b_0^* Y^*$ given the initial portfolio shares b_0 and b_0^*, and

$$\Omega \equiv \frac{\mathbb{E}\sum_{t=0}^{\infty}(\beta/\chi)^t Y_t^{\frac{\theta-1}{\theta}}}{\mathbb{E}\sum_{t=0}^{\infty}\beta^t Y_t^{\frac{\theta-1}{\theta}}} - 1 \geq 0 \tag{29}$$

is a cumulative measure of exorbitant privilege stemming from convenience yield $\chi \leq 1$.

In the absence of convenience yields, $\chi = 1$, we have $\Omega = 0$, and the dynamic model is fully nested within our single-period long-run framework of Section 2. In particular, the intertemporal budget constraint (28) corresponds exactly to the long-run budget constraint (7), and the result of Proposition 1 holds. Therefore, our characterization in the static framework applies for the long-run outcomes in the dynamic model, including for the optimal tariff, given that the models also agree on welfare.

Corollary 4 *When $\chi = 1$ and $\Omega = 0$, the welfare, permanent allocations, the optimal long-run tariffs, and tariffs closing the long-run imbalance are the same in the dynamic model as in the static model. Lerner symmetry holds if and only if $B_0 = 0$ irrespective of the value of χ and Ω.*

[33] To aggregate nominal prices over time, we adopt as a numeraire the nominal aggregate demand in the home economy, $P_{Ht} \equiv C_{Ht}^{-1/\theta}$. This is similar to a constant-money-supply rule and it implies a constant marginal value of the dollar in each period and ensures that nominal variables, just like real variables, are discounted at rate β.

When $\chi < 1$ and $\Omega > 0$, the budget constraint (28) features a new "exorbitant privilege" term corresponding to the last term in (1). Indeed, convenience yield $\chi < 1$ translates into a higher price V_t and lower future returns on domestic assets for a given stochastic dividend stream $\{Y_t\}$, as summarized by Ω. The international wealth transfer associated with convenience yield equals $\Omega\big(C^*_H/\mathcal{S} - B_0/\mathcal{Q}\big)$, and it depends on the value of exports relative to the value of the foreigner's initial position in terms of home assets. A positive gap in these values implies that the foreigners need to increase their exposure to home assets, which are expensive in light of their convenience yield. Even though Ω is exogenous to tariffs, the presence of the exorbitant-privilege term has important implications for the optimal tariff, as we show below. Despite this fact, Lemma 2 holds whether or not $\Omega = 0$, and Lerner symmetry applies whenever $B_0 = 0$, as the long-run equilibrium system depends only on the terms of trade $\mathcal{S}$ and not on the real exchange rate $\mathcal{Q}$ in this case.

Trade deficit dynamics The static equilibrium system in Proposition 12 characterizes comparative statics for the long-run allocations, prices and trade values in response to a permanent change in tariffs. We can now characterize comparative dynamics in response to a permanent change in tariffs, with a particular focus on trade deficit.

Proposition 13 *Consider an unexpected permanent change in (τ^I, τ^E). Denote with $\hat{X} = \frac{\Delta X}{X}$ the proportional change in the long-run component of variable X, corresponding to an allocation, a price, or a trade value. The change in the dynamic path of X_t is given by $\hat{X}_t = \hat{X}$. The change in the path of trade deficit is given by:*

$$\Delta NX_t = \Delta EX_t - \Delta IM_t = EX_t \cdot \widehat{EX} - IM_t \cdot \widehat{IM}. \tag{30}$$

Tariffs that reduce the permanent gross trade flows EX and IM also compress the transitory trade imbalances $NX_t - NX$ around the permanent trade balance $NX = (1-\beta)\mathbb{E}\sum_{t=0}^{\infty}\beta^t NX_t$.

The dynamic path of the economy is shaped by output shocks Y_t and Y^*_t. A permanent change in tariffs alters the entire equilibrium path by shifting allocations and prices proportionally with their permanent components which are determined from the long-run equilibrium system in Proposition 12. In this sense, the long-run components are sufficient summary statistics for the entire dynamic impact of permanent tariff shocks.

We next zoom in on the dynamics of trade deficits. The long-run trade balance NX is the weighted average of individual trade deficits $NX_t = EX_t - IM_t$. A permanent change in tariffs shifts the path of exports and imports proportionately with their permanent components, EX and IM. The long-run trade deficit adjusts accordingly, $\Delta NX = \Delta EX - \Delta IM$, consistent with our long-run analysis in Section 3. However, the dynamics of trade deficits period-by-period can not be recovered from ΔNX alone, as it depends on the dynamics of exports and imports separately, given by (30). On average, trade deficit NX_t shift by ΔNX, but there is an additional force to compress transitory trade imbalances, $NX_t - NX$, towards zero as tariffs reduce gross trade flows EX and IM.[34]

5.3 Retrenchment and exorbitant privilege

In the absence of exorbitant privilege, $\Omega = 0$, the previous section shows that the results from our long-run analysis in Sections 3 and 4 fully extend to the dynamic model. We now focus on the case with $\chi < 1$ and $\Omega > 0$ to explore the differential implications of a dynamic model with exorbitant privilege for the imbalances, the optimal tariff, and the exchange rate.

The exorbitant privilege manifests itself as the last term in the country budget constraint (28). By Lemma 4, this term equals to $\Omega(b - b_0)Y/\mathcal{Q}$, and it is positive when $\Omega > 0$ and $b_0 < b$, where b is the equilibrium share of home assets in the foreign portfolio.[35] Therefore, convenience yield on the home asset, $\chi < 1$, results in the exorbitant privilege if and only if the foreigners want to increase their equilibrium holdings of home assets b relative to their initial endowment b_0. In turn, tariffs generally result in greater home bias and reduce the external portfolio share b, a *retrenchment* effect. This decreases the international wealth transfer associated with a given convenience yield $\chi < 1$, and hence endogenously eliminates a part of the exorbitant privilege. This insight is at the core of the results that we describe next.

Imbalances First, we consider the case with zero initial external positions, $b_0 = b_0^* = 0$. In this case, the second (valuation) term in the country budget constraint (28) is absent, and we can rewrite (28) simply as:

$$IM = (1 + \Omega)EX,$$

where $IM/EX = \mathcal{S}C_F/C_H^*$. Therefore, $\Omega > 0$ results in a trade deficit, $NX = -\Omega \cdot EX < 0$, even in the absence of initial net foreign assets. Furthermore, even though Ω defined in (29) is entirely exogenous to tariffs, a tariff compresses both trade values towards zero, and in the limit eliminates exorbitant privilege altogether as country portfolios become fully home biased.

Second, consider the general case with $b_0, b_0^* > 0$. In this case, an import tariff has a twofold effect on the trade deficit: it reduces it both via the negative valuation effect associated with the exchange rate appreciation, as characterized in Section 3, and via the loss of exorbitant privilege, as just discussed. Therefore, it generally requires a smaller import tariff and a smaller associated real appreciation to rebalance trade, relative to the case without privilege when $\Omega = 0$.[36]

[34] From (30), it follows that $\Delta(NX_t - NX) = (EX_t - EX)\widehat{EX} - (IM_t - IM)\widehat{IM}$. For example, consider the case with $B_0 = B_0^* = \Omega = 0$, such that $NX = 0$ both before and after the tariff and $\widehat{EX} = \widehat{IM}$ is the proportional reduction in gross trade flows from the tariff. Then, $\Delta NX_t = NX_t \cdot \widehat{EX}$, and $\widehat{EX} < 0$ compresses NX_t towards zero without affecting $NX = 0$. This mechanism is studied by Obstfeld and Rogoff (2001), Reyes-Heroles (2016) and Costinot and Werning (2025), and we abstract from it in our long-run analysis.

[35] In the derivation, we use the facts that $b = \tau^E a$, $C_H^* = aY$, $B_0 = b_0 Y$, and $\mathcal{Q} = \tau^E \mathcal{S}$.

[36] Recall that $\Omega > 0$ results in exorbitant privilege if and only if $b_0 < b$, which is also a sufficient condition for an import tariff to reduce exorbitant privilege. The same condition implies that the required real appreciation

We next quantify these effects using the same calibration as described in Section 3.3, except that $B_0^* = 0.05$ now matches U.S. foreign assets and $\Omega = 0.8$ is inferred as a residual from the intertemporal budget constraint (28).[37] The bottom panel of Table 1 shows the results: a smaller import tariff of 14.4% is sufficient to close the imbalances, against 63.8% in the baseline case without exorbitant privilege. Consistent with our analytical results, this rebalancing is achieved with a smaller real exchange rate movement of 5% against 22% in the baseline. However, the resulting welfare loss is similar in the two cases, equal to 1.6% of real consumption.

Optimal tariff We now characterize the optimal tariff in the dynamic model. This amounts to maximizing home welfare U, given by (2) in terms of permanent components, subject to constraints (3)–(6) and (28). Given the initial portfolio positions, the government chooses the tariff, while the households respond with rebalancing their portfolios with the tariff in place.

Proposition 14 *The optimal permanent import tariff is decreasing in gross external liabilities $\mathcal{B} \equiv P_H Y b_0$ and in exorbitant privilege Ω:*

$$\tau^I = 1 + \frac{1}{\theta\big(1 + \frac{\mathcal{B}}{EX - \mathcal{B}}\big) - 1}\frac{1}{\Lambda^*} - \frac{\theta}{\theta - 1 + \frac{\mathcal{B}}{EX}}\frac{\Omega}{1+\Omega}. \tag{31}$$

The optimal permanent export tariff is increasing in $\mathcal{B}$ and decreasing in Ω:

$$\tau^E = 1 + \frac{1}{\theta - 1}\frac{1}{\Lambda^*} + \frac{1}{\theta - 1}\frac{1}{\Lambda}\frac{\mathcal{B}}{IM} - \frac{\theta}{\theta - 1}\frac{\Omega}{1+\Omega}. \tag{32}$$

As before (in Propositions 7 and 9), the optimal tariff is driven by the the terms-of-trade manipulation motive, which is offset (amplified) by the valuation effect for the import (export) tariff. The valuation effect operates via the real exchange rate in proportion with the initial external liabilities Yb_0. In addition, there is now an extra force associated with exorbitant privilege, $\Omega > 0$. Tariffs lead to a retrenchment of cross-border asset positions, eliminating part of the international wealth transfer associated with convenience yield on the home assets. Consequently, the optimal tariffs are decreasing in Ω.[38]

Table 1 shows that the implications of exorbitant privilege for the optimal trade policy are quantitatively as important as those of the valuation effects. In particular, the portfolio retrenchment effect decreases both the import and the export tariffs, turning them into subsidies of 31.2% and 1.8%, respectively. As a result, the exchange rate depreciates by 18% in the former

to rebalance trade is smaller under $\Omega > 0$ than under $\Omega = 0$. From (28), the real exchange rate that rebalances trade, $C_F = C_H^*/\mathcal{S}$, is given by $\mathcal{Q} = (1+\Omega)B_0/[B_0^* + \Omega C_F]$; this expressions reduces to (13) when $\Omega = 0$.

[37] Assuming a constant growth rate of output $g = \Delta Y_{t+1}/Y_t$, from (29) we calculate $\Omega \approx \frac{1-\chi}{\chi/\beta - 1 - (1-1/\theta)g\chi}$. It follows that for $\theta = 4$, $\beta = 0.96$ and $g = 0.025$, the value $\Omega = 0.8$ corresponds to a 1% convenience yield on U.S. assets $(1 - \chi = 0.01)$, a number close to empirical estimates (Gourinchas and Rey 2014).

[38] This channel can be seen most clearly for the import tariff in the special case when $\mathcal{B} = EX$, or equivalently $b_0 = b = a$. In this case, all gains from the improved terms of trade are fully offset by the valuation losses. As a result, the optimal import tariff is given by $\tau^I = 1/(1+\Omega)$, and hence it is an import subsidy when $\Omega > 0$. While the last term in (28) is $\Omega(b - b_0)Y/\mathcal{Q} = 0$, its derivative with respect to τ^I is negative.

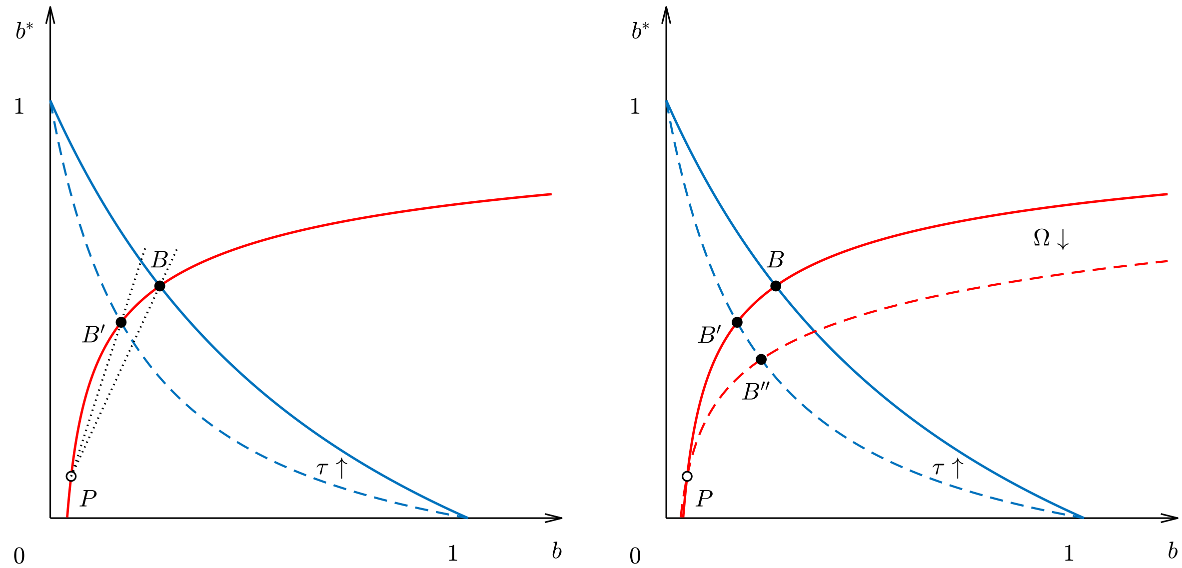

Figure 4: Equilibrium portfolios with import tariff and convenience yield

Note: The axes correspond to the foreign share in the home asset b and the home share in the foreign assets b^* (note that $b = a$ and $b^* = a^*$ according to Lemma 4 when $\tau^E = 1$). The blue curve is the contract curve (26): solid for $\tau^I = 1$ and dashed for $\tau^I > 1$. The red curve is the home budget constraint (27): solid for the baseline $\chi < 1$ and $\Omega > 0$; dashed for a larger $\chi \leq 1$ and smaller $\Omega \geq 0$. Point P is the initial portfolio (b_0, b_0^*), which is always budget feasible; points B, B', B'' are the equilibrium portfolios corresponding to a given (τ^I, χ). The slopes of intervals PB, PB' and PB'' reflect the equilibrium terms of trade and the real exchange, $\mathcal{Q} = \mathcal{S}$ given $\tau^E = 1$, with steeper intervals corresponding to more appreciated real exchange rate (see Lemma 4 and footnote 39).

case and appreciates by 1% in the latter case, and the associated valuation effects flip the sign relative to our benchmark. The import subsidy results in a sizable welfare gain, supported by substantially increased government spending, and resulting in a widening trade deficit.

Real exchange rate Lastly, we describe the implications of tariff and convenience yield shocks for the real exchange rate. We assume, for concreteness, that $b_0 < b$, such that convenience yield on home assets results in exorbitant privilege to the home economy, and we rely on Lemma 4 to characterize comparative statics, as illustrated in Figure 4.[39] An import tariff shifts the contract curve (26) such that $a/(1-a^*)$ decreases, improving the home's terms of trade and appreciating the real exchange rate, irrespective of the value of χ. Therefore, even in the presence of exorbitant privilege, import tariffs lead to a real appreciation, despite the endogenous loss of the privilege associated with portfolio retrenchment effect discussed above. In contrast, an increase in χ and a loss of exorbitant privilege Ω shifts the budget constraint (27) such that $a/(1-a^*)$ increases and the real exchange rate depreciates. Therefore, a tariff announcement that leads to a loss of the convenience yield χ can result in a real depreciation of the exchange rate.

[39]We use the facts that $\mathcal{Q} = \tau^E \mathcal{S}$ and $\mathcal{S}^\theta = \frac{(1-\gamma^*)aY}{\gamma^*(1-a^*)Y^*}$; this follows from the equilibrium consumption allocation and conditions (10) and (8) which hold for the respective permanent components in the dynamic model. Lemma 4 then provides the equilibrium system to perform the comparative statics for (a, a^*).

6 Conclusion

This paper studies the two-way interaction between global imbalances and trade policy. Cross-border asset positions lead to exchange-rate-driven valuation effect and international wealth transfers triggered by tariffs. With the current size of cross-border asset holdings, this force becomes first order in shaping welfare consequences of tariffs and trade wars. Large external liabilities in terms of domestic assets reduce the welfare benefit of an import tariff, with a negative valuation effect from real appreciation largely offsetting the improvement in the terms of trade. For the same reason, external asset holdings provide a hedge against the trade war. Importantly, it is gross asset positions rather than net positions and trade deficits that shape properties of the optimal tariff. The optimal tariff is, further, reduced in the presence of convenience yield on domestic assets, as tariff-induced retrenchment of international portfolio positions endogenously eliminates part of the associated "exorbitant privilege".

We show that the same asset-valuation mechanism is the force behind the ability of tariffs to alter long-run trade deficits. Eliminating a long-run trade deficit requires a negative valuation effect on the country's external asset position which results in a wealth transfer abroad. In contrast to conventional intuition, such rebalancing requires an appreciation of the exchange rate, with its magnitude pinned down solely by the external balance sheet of the country, and irrespective of trade shares and elasticities in the import and export markets. In equilibrium, trade balance adjusts endogenously to the change in the external wealth of the country. A reduction in the long-run trade deficit can be generally achieved with either an import tariff or an export subsidy, as both are associated with an equilibrium exchange rate appreciation.

Existing empirical evidence confirms that import tariffs appreciate the exchange rate in the vast majority of cases (Ostry, Lloyd, and Corsetti 2025). Nonetheless, the dollar depreciated sharply in response to the "Liberation Day" tariff announcement, and it remained persistently weaker against most other currencies even as other asset markets recovered from the initial shock. This suggests that additional forces were at play. Through the lens of our model, this episode is consistent with a decline in convenience yields on dollar assets and the associated U.S. "exorbitant privilege" (see also Jiang, Krishnamurthy, Lustig, Richmond, and Xu 2025). Whether this depreciation was a direct consequence of higher tariffs or instead reflected other simultaneous shocks, such as shifts in expectations about other government policies, remains an open question. Irrespective of the answer to this question, financial valuation effects are an essential channel of trade policy transmission which cannot be ignored at the current levels of financial globalization.

A Appendix

A1 Appendix Figures

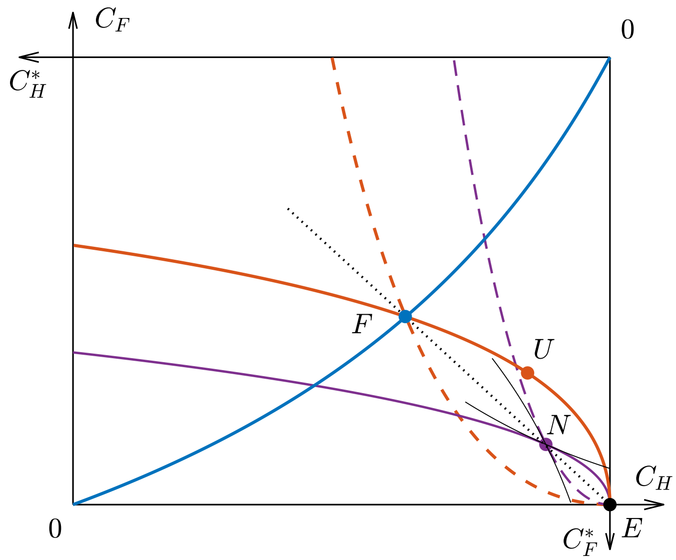

Figure A1: Free trade allocation and the tariff war Nash equilibrium

Note: see notes to Figure 3. The dashed red curve is the trade possibilities frontier from the perspective of the foreign country when home tariff $\tau = 1$, i.e., $C_F = g^*(C_H^*; \tau = 1)$, while the solid red curve is still $C_H^* = g(C_F; \tau^* = 1)$. The intersection of these two red curves is the free trade equilibrium F. The corresponding purple curves plot $C_F = g^*(C_H^*; \tau = \varepsilon)$ and $C_H^* = g(C_F; \tau^* = \varepsilon^*)$, and their intersections corresponds to the trade war Nash equilibrium N, as discussed in the text. Point U corresponds to the unilateral home tariff.

A2 Proofs and Derivations

Derivations for Section 1 Consider the home economy in a fully dynamic environment with a general portfolio choice. In any period t, home starts with vector of asset holdings B^j_{t-1} for $j \in J_{t-1}$ inherited from period $t-1$, where $B^j_{t-1} < 0$ means a liability. At t, asset j pays dividends D^j_t and its post-dividend price is denoted Q^j_t. Therefore, $R^j_t \equiv (Q^j_t + D^j_t)/Q^j_{t-1}$ is the realized one-period return on asset $j \in J_{t-1}$ at t. We denote with $\bar{R}_t$ the risk-free interest rate between periods t and $t+1$ (and known at t, hence the subscript). All asset returns, payouts and prices are quoted in the home currency (dollar) for concreteness, but the underlying assets can pay fixed income in foreign currency as well. The new asset positions at time t are denoted with B^j_t for $j \in J_t$, and $\mathcal{B}_t \equiv \sum_{j\in J_t} Q^j_t B^j_t$ denotes the new net foreign asset position that is carried from period t into period $t+1$. We also write the pay-out on the entire net foreign asset position as $\mathcal{R}_t\mathcal{B}_{t-1} \equiv \sum_{j\in J_{t-1}}(Q^j_t + D^j_t)B^j_{t-1}$, where $\mathcal{R}_t$ can be thought of as the realized return on the entire NFA position.[40]

In general, the return $\mathcal{R}_t$ is different from the risk-free rate $\bar{R}_{t-1}$, both ex ante in expectation and ex post realization-by-realization. However, if there exists no arbitrage that can be constructed using available returns $\{R^j_{t+1}\}_{j\in J_t}$, then the following result links $\mathcal{R}_{t+1}$ and $\bar{R}_t$:[41]

Lemma A1 *If there is no arbitrage for asset returns in J_t, then the pay-out on the international asset portfolio $\mathcal{R}_{t+1}$ satisfies:*

$$\mathbb{E}_t\{\Theta_{t+1}(\mathcal{R}_{t+1} - \bar{R}_t)\} = 0, \tag{A1}$$

where Θ_{t+1} is any stochastic discount factor that prices all assets $j \in J_t$.

This lemma shows that — in expectation and appropriately adjusted for risk — the return on the NFA position $\mathcal{R}_{t+1}$ is still linked to the risk-free rate $\bar{R}_t$, unless portfolio returns offer arbitrage opportunity. Of course, the latter possibility must not be excluded for a country like the United States which arguably enjoys an "exorbitant privilege" on its international portfolio.

With the general notation introduced above, the balance of payments requires that:

$$\mathcal{B}_t - \mathcal{R}_t\mathcal{B}_{t-1} = NX_t, \tag{A2}$$

where NX_t is the dollar value of the country's trade balance, that is, the value of home exports minus the value of home imports in period t net of domestic tariffs (i.e., calculated using the rest-of-the-world trade prices). Aggregating this flow country budget constraint and discounting using the risk-free interest rate, we obtain the intertemporal budget constraint that

[40] Note that $\mathcal{R}_t$ is well-defined from the value of $\mathcal{R}_t\mathcal{B}_{t-1}$ whenever $\mathcal{B}_{t-1} \neq 0$, and otherwise we work directly with $\mathcal{R}_t\mathcal{B}_{t-1}$ which is always well-defined and is generally non-zero even when $\mathcal{B}_{t-1} = 0$.

[41] No arbitrage means that there exists no real vector $\{\alpha_{jt}\}_{j\in J_t}$ such that $\mathrm{var}_t(\sum_j \alpha_{jt}R^j_{t+1}) = 0$ and $\mathbb{E}_t \sum_j \alpha_{jt}R^j_{t+1} > \bar{R}_t$. Then there exists a stochastic discount factor Θ_{t+1} that prices all assets $j \in J_t$ such that $\mathbb{E}_t\{\Theta_{t+1}R^j_{t+1}\} = 1$. In particular, the risk-free rate satisfies $\bar{R}_t = 1/\mathbb{E}_t\Theta_{t+1}$, where this is either a result if the risk-free rate is available at t (i.e., part of set J_t) or otherwise the definition of the shadow risk-free rate. See, for example, Campbell (2017).

must hold along any future path of histories:

$$\mathcal{R}_t\mathcal{B}_{t-1}+\sum_{s=t}^{\infty}q_{t,s}NX_s+\sum_{s=t}^{\infty}q_{t,s+1}\left(\mathcal{R}_{s+1}-\bar{R}_s\right)\mathcal{B}_s-\lim_{s\to\infty}q_{t,s}\mathcal{B}_s=0, \qquad \text{(A3)}$$

where $q_{t,t}\equiv 1$ and $q_{t,s}\equiv\left(\prod_{\ell=t}^{s-1}\bar{R}_\ell\right)^{-1}$ for $s\geq t+1$ is the risk-free discount factor based on linked one-period risk-free rates $\bar{R}_t$. If the risk-free rate is constant over time, $\bar{R}_s=1/\beta$ for all s, then $q_{t,s}=\beta^{s-t}$ is geometric discounting. The last term $\lim_{s\to\infty}q_{t,s}\mathcal{B}_s=0$ by the no-bubble condition on the international asset position (non-explosive NFA).[42]

We can rewrite the intertemporal budget constraint (A3) resulting in:

Proposition A1 *The long-run trade deficit (more generally, imbalance) is fully determined by the financial position of a country:*

$$\underbrace{-\sum_{s=t}^{\infty}q_{t,s}NX_s}_{\text{long-run trade deficit}}=\underbrace{\bar{R}_{t-1}\mathcal{B}_{t-1}}_{\substack{\text{exogenous}\\\text{initial NFA}}}+\underbrace{(\mathcal{R}_t-\bar{R}_{t-1})\mathcal{B}_{t-1}}_{\substack{\text{on-impact}\\\text{valuation effect}}}+\underbrace{\sum_{s=t+1}^{\infty}q_{t,s}\left(\mathcal{R}_s-\bar{R}_{s-1}\right)\mathcal{B}_{s-1}}_{\text{future realized excess returns}}, \qquad \text{(A4)}$$

that is, it does not change in response to a policy unless the policy results in a valuation effect on the existing net foreign asset position or a change in the future excess returns on the international portfolio of the country.

Equation (1) is the special case of (A4) when the risk-free rate is constant (or using the unconditional average risk-free rate in the discounting) and focusing on the initial period $t=0$. A corollary of equation (A4) is that the long-run trade deficit can change only as a result of either the valuation effect on impact $\mathcal{R}_t$ on the pre-determined initial NFA position $\mathcal{B}_{t-1}$, or as a result of changes in future excess returns reflected in the terms $\left(\mathcal{R}_s-\bar{R}_{s-1}\right)\mathcal{B}_{s-1}$ for $s>t$.

Equation (A4) further simplifies when all assets j are priced by the same stochastic discount factor, as in Lemma A1, in which case the last term in (A4) must be zero in expectation using this discount factor as there can be no expected risk-adjusted excess returns. As a result, the only change in the expected long-run trade deficit can come as a result of a surprise valuation effect on impact of the policy announcement at t.

Corollary A1 *If there is no arbitrage and all assets $j\in J_s$ can be priced by Θ_{s+1} for $s\geq t$, then:*

$$\underbrace{-\sum_{s=t}^{\infty}\mathbb{E}_t\{\Theta_{t,s}NX_s\}}_{\text{expected long-run trade deficit}}=\bar{R}_{t-1}\mathcal{B}_{t-1}+(\mathcal{R}_t-\bar{R}_{t-1})\mathcal{B}_{t-1}, \qquad \text{(A5)}$$

where $\Theta_{t,t}\equiv 1$ and $\Theta_{t,s}\equiv\prod_{\ell=t}^{s-1}\Theta_{\ell+1}$ for all $s\geq t+1$, hence $\mathbb{E}_t\Theta_{t,s}=q_{t,s}$. Therefore, the change in the expected long-run trade deficit at t equals the on-impact valuation effect $\mathcal{B}_{t-1}\mathrm{d}\mathcal{R}_t$.

Proposition A1 and Corollary A1 establish our claim that the long-run trade deficit is pinned down by the financial position of the country, not by relative prices in the goods market. Equations (A4) and (A5) describe the set of possible outcomes: the financial position of the

[42] Some models may violate the no-bubble condition and then these deviations should be included in the financial position as part of the excess returns. However, most models with "$r<g$" satisfy this condition with the third term in (A3) absorbing convenience yields and excess returns (see Reis 2021).

country can change as a result of the valuation effect on the initial NFA position or the change in the future path of excess returns on international assets and liabilities. In models where there are no arbitrage opportunities, this latter channel is closed, and the expected long-run trade deficit only responds to on-impact valuation effects. If such valuation effects are absent, $\mathcal{B}_{t-1}\mathrm{d}\mathcal{R}_t = 0$, the long-run trade deficit is exogenous to the policy altogether. This, of course, does not mean that trade policy is inconsequential: holding trade (im)balance constant, a tariff affects the terms of trade as well as the resulting values of imports and exports.

Proof of Proposition 1 To prove the result, it is convenient to introduce the nominal exchange rate $\mathcal{E}$ and prices of goods in buyers' currency p_H, p_F and p_H^*, p_F^*. We interpret both foreign stocks and foreign direct investment as shares in a Lucas tree abroad and, for brevity, combine them into "equity". Let B^e, B^n, B^r denote initial net positions of the U.S. against the rest of the world in home equity, home currency nominal bonds and loans, and home real bonds and loans, respectively. The net positions of home in foreign equity, foreign-currency nominal debt and foreign real debt are denoted respectively with B^{e*}, B^{n*}, B^{r*}. The dollar value of nominal bonds is given by B^n and $\mathcal{E}B^{n*}$. Assuming that real debt is indexed by the local PPI inflation, its dollar value is equal $p_H B^r$ and $\mathcal{E}p_F^* B^{r*}$. Finally, assume that the law of one price holds, so that Lucas trees earn respectively p_H and p_F in local currency on each unit of good sold in any market independently from tariffs. Normalizing total supply of equity in each country to one, the dollar dividends on shares in Lucas trees are then given by $p_H Y B^e$ and $\mathcal{E}p_F^* Y^* B^{e*}$.[43] Combine these positions together to get home's NFA:

$$NFA = \mathcal{E}p_F^*\big(B^{n*}/p_F^* + B^{r*} + Y^*B^{e*}\big) + p_H\big(B^n/p_H + B^r + YB^e\big) = P_F^* B_0^* - P_H B_0,$$

where we used $p_H \equiv P_H$ and $\mathcal{E}p_F^* \equiv P_F^*$ and denoted gross external assets and liabilities respectively with $B_0^* \equiv B^{n*}/p_F^* + B^{r*} + Y^*B^{e*}$ and $B_0 \equiv -\big(B^n/p_H + B^r + YB^e\big)$. Given exogenous Y, Y^* and predetermined asset positions, it follows that B_0, B_0^* do not depend on tariffs when comprised of real bonds, stocks and FDI. To extend the result to nominal bonds, specify monetary policy to stabilize producer prices in local currency, i.e., p_H, p_F^* are exogenous to trade policies. It follows that nominal bonds can also be absorbed into tariff-invariant positions B_0 and B_0^*. Therefore,

$$\frac{NFA}{P_F^*} = B_0^* - \frac{B_0}{\mathcal{Q}}, \qquad \text{where} \qquad \mathcal{Q} \equiv \frac{P_F^*}{P_H}.$$

Relative to the free-trade equilibrium with $\mathcal{Q}_0 \equiv P_{F0}^*/P_{H0}$, we get

$$\Delta(NFA/P_F^*) = \left(B_0^* - \frac{B_0}{\mathcal{Q}}\right) - \left(B_0^* - \frac{B_0}{\mathcal{Q}_0}\right) = B_0\left(\frac{1}{\mathcal{Q}_0} - \frac{1}{\mathcal{Q}}\right) = \frac{\mathcal{Q}-\mathcal{Q}_0}{\mathcal{Q}}\frac{B_0}{\mathcal{Q}_0} = \frac{\Delta\mathcal{Q}}{\mathcal{Q}}\frac{B_0}{\mathcal{Q}_0},$$

consistent with the equation from the proposition. ■

[43] Section 5 extends the argument to a multi-period setting with convenience yields (see equation (A6)).

Proof of Lemma 1 Substitute out P_H from the budget constraint (7) using the pricing condition (4), P_F^*/P_H^* using foreign demand (6) and C_F^* using the market clearing (3) to obtain the implementability condition (12). The remaining conditions (4), (5) are side equations that pin down τ^I and P_F/P_H. Thus, the planner can choose any allocation that satisfies (12) and (3).

Under CES preferences, (12) can be written as

$$\underbrace{C_H^{*-\frac{1}{\eta}}\left(C_H^* - \frac{B_0}{\tau^E}\right)}_{\equiv f(C_H^*)} = \underbrace{\left(\frac{1-\gamma^*}{\gamma^*}\right)^{\frac{1}{\eta}} \frac{C_F - B_0^*}{(Y^* - C_F)^{\frac{1}{\eta}}}}_{\equiv h(C_F)}.$$

Function f is increasing and concave:

$$f'(x) = \frac{\eta-1}{\eta}x^{-\frac{1}{\eta}} + \frac{1}{\eta}\frac{B_0}{\tau^E}x^{-\frac{1+\eta}{\eta}} > 0, \qquad f''(x) = -\frac{\eta-1}{\eta^2}x^{-\frac{1+\eta}{\eta}} - \frac{1+\eta}{\eta^2}\frac{B_0}{\tau^E}x^{-\frac{1+2\eta}{\eta}} < 0,$$

while h is increasing and convex:

$$h'(x) = \left(\frac{1-\gamma^*}{\gamma^*}\right)^{\frac{1}{\eta}} \frac{(Y^* - B_0^*) + (\eta-1)(Y^* - x)}{\eta(Y^* - x)^{\frac{1+\eta}{\eta}}} > 0,$$

$$h''(x) = \left(\frac{1-\gamma^*}{\gamma^*}\right)^{\frac{1}{\eta}} \frac{(\eta+1)(Y^* - B_0^*) + (\eta-1)(Y^* - x)}{\eta^2(Y^* - x)^{\frac{1+2\eta}{\eta}}} > 0,$$

where we use $\eta > 1$, $x < Y^*$, and $0 \le B_0^* \le Y^*$. Finally, since f^{-1} is increasing and convex and h is increasing and convex, $g = f^{-1}(h)$ is strictly increasing and convex. ■

Proof of Proposition 2 Suppose $B_0 = 0$. Then from equilibrium system (3), (11), (12), a sufficient statistic for the equilibrium allocation (C_H, C_F, C_H^*, C_F^*) is the product of two tariffs $\tau^I\tau^E$. Thus, the effects of import and export taxes are the same and Lerner symmetry holds. In contrast, when $B_0 \neq 0$ the effects of the two instruments are no longer symmetric. In particular, adjusting τ^I and τ^E so as to keep $\tau^I\tau^E$ unchanged does affect the equilibrium allocation through the last term in the implementability condition (12). ■

Proof of Proposition 3 From the country budget constraint (7), $NX = 0$ if and only if $NFA = 0$. By Proposition 1, $NFA = P_F^* B_0^* - P_H B_0$ and therefore

$$NX = 0 \iff NFA = 0 \iff \mathcal{Q} \equiv \frac{P_F^*}{P_H} = \frac{B_0}{B_0^*},$$

which proves (13). Given the foreign household demand (6), the balanced-trade condition $NX = 0$ is isomorphic to the balanced-trade implementability constraint (12) with $B_0 = B_0^* = 0$ and describes the set of possibilities $C_H^* = g(C_F; 0, 0)$. For any interior allocation on this set, one can compute a corresponding export tariff

$$\tau^E \equiv \frac{P_H^*}{P_H} = \frac{P_F^*}{P_H} \cdot \frac{P_H^*}{P_F^*} = \frac{B_0}{B_0^*} \cdot \frac{C_F}{C_H^*},$$

which ensures (13). The overall trade wedge $\tau = \tau^I \tau^E$ is then pinned down by (11), so τ^I adjusts residually to implement the target allocation. Finally, trade autarky, $C_F = C_H^* = 0$, is obtained as a limiting case with prohibitive tariffs. ■

Proof of Proposition 4 Under $\tau^E = 1$, balanced trade is characterized by the frontier $C_H^* = g(C_F; 0, 0)$ from Proposition 3. By part (a) of Lemma 3, the equilibrium allocation implemented by an import tariff traces a monotone locus: at free trade $\tau^I = 1$ we have $NX_0 < 0$, while for as $\tau^I \to \infty$, we have $NX > 0$. Therefore, this locus crosses the balanced-trade frontier exactly once, which implies a unique balanced-trade equilibrium. Since the economy starts from a trade deficit under free trade, the balancing tariff must satisfy $\tau^I > 1$.

To calculate the size of the exchange rate appreciation associated rebalancing trade, note that pre-tariff and post-tariff trade deficits are given, respectively, by:

$$NX_0/P_{F0}^* = -NFA_0/P_{F0}^* = -\big(B_0^* - B_0/\mathcal{Q}_0\big) < 0,$$
$$NX/P_F^* = -NFA/P_F^* = -\big(B_0^* - B_0/\mathcal{Q}\big) = 0.$$

Subtracting the second line from the first line, we have:

$$NX_0/P_{F0}^* = B_0/\mathcal{Q}_0 - B_0/\mathcal{Q} = B_0/\mathcal{Q}_0 \cdot \frac{\mathcal{Q} - \mathcal{Q}_0}{\mathcal{Q}} \qquad \Rightarrow \qquad \frac{\Delta\mathcal{Q}}{\mathcal{Q}} = \frac{NX_0/P_{F0}^*}{B_0/\mathcal{Q}_0} = \frac{NX_0}{P_{H0}B_0} < 0,$$

where $\Delta\mathcal{Q} \equiv \mathcal{Q} - \mathcal{Q}_0$ and we used $\mathcal{Q}_0 \equiv P_{F0}^*/P_{H0}$. ■

Proof of Lemma 2 Under CES preferences (2), household optimality (5) and (6) results in standard constant elasticity demand:

$$C_H = (1-\gamma)\left(\frac{P_H}{P}\right)^{-\theta} C, \qquad C_F = \gamma\left(\frac{P_F}{P}\right)^{-\theta} C$$

and

$$C_H^* = \gamma^*\left(\frac{P_H^*}{P^*}\right)^{-\eta} C^*, \qquad C_F^* = (1-\gamma^*)\left(\frac{P_F^*}{P^*}\right)^{-\eta} C^*,$$

where price indexes are given by:

$$P = \left[(1-\gamma)P_H^{1-\theta} + \gamma P_F^{1-\theta}\right]^{\frac{1}{1-\theta}} \quad \text{and} \quad P^* = \left[\gamma^* P_H^{*1-\eta} + (1-\gamma^*)P_F^{*1-\eta}\right]^{\frac{1}{1-\eta}},$$

with the relationship between prices across markets given by (4).

Substituting the demand schedules above into the market clearing conditions (3) and the country budget constraint (9), we have a system of three equations, which can be written as:

$$
\begin{aligned}
Y &= (1-\gamma)\left[(1-\gamma)+\gamma(\tau\mathcal{S})^{1-\theta}\right]^{\frac{\theta}{1-\theta}} C + \gamma^*\left[(1-\gamma^*)\mathcal{S}^{1-\eta}+\gamma^*\right]^{\frac{\eta}{1-\eta}} C^*,\\
Y^* &= \gamma\left[(1-\gamma)(\tau\mathcal{S})^{\theta-1}+\gamma\right]^{\frac{\theta}{1-\theta}} C + (1-\gamma^*)\left[(1-\gamma^*)+\gamma^*\mathcal{S}^{\eta-1}\right]^{\frac{\eta}{1-\eta}} C^*,\\
0 &= \gamma^*\left[(1-\gamma^*)\mathcal{S}^{1-\eta}+\gamma^*\right]^{\frac{\eta}{1-\eta}} C^*/\mathcal{S} - \gamma\left[(1-\gamma)(\tau\mathcal{S})^{\theta-1}+\gamma\right]^{\frac{\theta}{1-\theta}} C + B_0^* - B_0/\mathcal{Q},
\end{aligned}
$$

where $\mathcal{Q} = \tau^E\mathcal{S}$ by (10), and we used the definitions of the price indexes above and the terms of trade $\mathcal{S} = P_F^*/P_H^*$. This system can be solved for $(C, C^*, \mathcal{S})$ given (Y, Y^*), (τ^I, τ^E) and (B_0, B_0^*), and $\mathcal{Q}$ is then obtained from equilibrium $\mathcal{S}$.

As a point of approximation, we consider a symmetric equilibrium such that $\gamma Y = \gamma^* Y^*$ and $B_0 = B_0^* \geq 0$ with $\tau^I = \tau^E = 1$, i.e., no tariffs. In this symmetric equilibrium, all relative prices equal 1, in particular $\mathcal{Q} = \mathcal{S} = 1$; $C = Y$, $C^* = Y^*$; $NX = 0$, home export quantity equals $C_H^* = \gamma Y = \gamma^* C^*$, and home import quantity equals $C_F = \gamma C = \gamma^* Y^*$. Define $b_0 \equiv B_0/(\gamma Y) = \mathcal{B}/EX$, where the second equality uses the notation introduced in the lemma.

Log-linearizing the equilibrium system around this equilibrium point, we obtain:

$$
\begin{aligned}
y &= (1-\gamma)[c+\gamma\theta(s+\hat{\tau})] + \gamma[c^* + (1-\gamma^*)\eta s],\\
y^* &= \gamma^*[c-(1-\gamma)\theta(s+\hat{\tau})] + (1-\gamma^*)[c^* - \gamma^*\eta s],\\
0 &= c^* - c - s + (1-\gamma^*)\eta s + (1-\gamma)\theta(s+\hat{\tau}) + b_0 q,
\end{aligned}
$$

where $q = s + \hat{\tau}^E$, $\hat{\tau} \equiv \hat{\tau}^E + \hat{\tau}^I$, and $\hat{\tau}^m \equiv \log\tau^m$ for $m \in \{I, E\}$, and for any variable X we have $x \equiv \log(X/\bar{X})$.

We solve for s and $c - c^*$:

$$
s = \frac{(c-c^*) - (1-\gamma)\theta\hat{\tau} - b_0\hat{\tau}^E}{(1-\gamma)\theta + (1-\gamma^*)\eta - 1 + b_0},
$$

$$
c - c^* = \frac{(y-y^*) - (\gamma+\gamma^*)(1-\gamma)\theta\hat{\tau} - (\gamma+\gamma^*)[(1-\gamma)\theta + (1-\gamma^*)\eta]s}{1-\gamma-\gamma^*},
$$

and therefore

$$
s = \frac{(y-y^*) - (1-\gamma)\theta\hat{\tau} - (1-\gamma-\gamma^*)b_0\hat{\tau}^E}{(1-\gamma)\theta + (1-\gamma^*)\eta + (1-\gamma-\gamma^*)(b_0-1)}.
$$

Setting $y = y^* = 0$, we solve for RER's response to tariffs:

$$
q = s + \tau^E = \frac{[\gamma + (1-\gamma^*)(\eta-1)]\hat{\tau}^E - (1-\gamma)\theta\hat{\tau}^I}{(1-\gamma)\theta + (1-\gamma^*)\eta + (1-\gamma-\gamma^*)(b_0-1)},
$$

which corresponds to the result in the lemma using the definition of b_0 above. Note that $\theta, \eta \geq 1$ and $b_0 \geq 0$ is sufficient for $\partial \log \mathcal{Q}/\partial\tau^I < 0$ and $\partial \log \mathcal{Q}/\partial\tau^E > 0$. ■

Proof of Proposition 5 At $\tau^E = 1$, we have $NX = NX_0 < 0$. By part (b) of Lemma 3, as τ^E falls towards $\underline{\tau}^E \in (0,1)$, the equilibrium converges to $C_H^* = Y$ and $C_F = 0$, in which case $NX > 0$. Therefore, by continuity, there exists $\tau^E \in (\underline{\tau}^E, 1)$ such that $NX = 0$. Uniqueness follows from the same monotonicity in Lemma 3(b). Hence, the balanced-trade equilibrium is implemented with an export subsidy, $\tau^E < 1$, rather than with an export tax. Finally, by Proposition 2, the export subsidy is not equivalent to the import tariff when $B_0 > 0$, so the resulting balanced-trade equilibrium generally features different terms of trade and different trade quantities. ■

Proof of Lemma 3 **Part a**: For import tariffs, notice that the set of implementable allocations is determined by two conditions: the household demand

$$\frac{u_F}{u_H} = \tau^I \frac{u_F^*}{u_H^*}$$

and the country's budget constraint

$$u_F^*(C_F - B^*) = u_H^*(C_H^* - B).$$

When $\tau^I \to \infty$, there are two possibilities. Suppose first that u_F/u_H stays finite. Then the former condition requires $u_F^*/u_H^* \to 0$ and hence, $u_H^* \to \infty$ and $C_H^* \to 0$. This is inconsistent with the budget constraint. The only other possibility is that $u_F/u_H \to \infty$, which implies $u_F \to \infty$ and $C_F \to 0$. Substituting into the budget constraint, we get

$$\Phi(C_H^*) \equiv \left(\frac{1-\gamma^*}{\gamma^*}\frac{C_H^*}{Y^*}\right)^{\frac{1}{\eta}} - \frac{B - C_H^*}{B^*} = 0.$$

As long as $B > 0$, function $\Phi(\cdot)$ is continuous, strictly increasing with $\Phi(0) < 0$ and $\Phi(B) > 0$, which guarantees a unique solution $0 < C_H^* < B$. Since $C_F = 0$, we get $NX > 0$. And as $B \to 0$, the limiting point converges to autarky, $C_H^* = C_F = 0$.

Suppose $\tau^I \to 0$. If $u_F/u_H \not\to 0$, then $u_F^* \to \infty$ and $C_F^* \to 0$, $C_F \to Y^*$, which is inconsistent with the budget constraint. It follows that $u_F/u_H \to 0$ and hence, $C_H \to 0$, $C_H^* \to Y$. The equilibrium value of C_F is then uniquely pinned down by the budget constraint

$$\left(\frac{1-\gamma^*}{\gamma^*}\frac{Y}{Y^* - C_F}\right)^{\frac{1}{\eta}} = \frac{Y - B}{C_F - B^*}$$

and satisfies $B^* < C_F < Y^*$. Both imports and exports are positive in this limit with $NX < 0$. Indeed, monotonicty proven below implies that C_H^* rises and C_F falls relative to a free-trade equilibrium, which depreciates the real exchange rate, i.e., reduces $P_F^*/P_H^* = u_F^*/u_H^*$. Thus, as long as $NFA > 0$ under free trade, the trade balance remains negative in the limit of $\tau^I \to 0$.

To prove monotonicity, substitute marginal utilities into the budget constraint, take logs, differentiate and simplify:

$$\frac{Y^* - B^* + (\eta - 1)(Y^* - C_F)}{(Y^* - C_F)(C_F - B^*)} dC_F = \frac{B + (\eta - 1)C_H^*}{(C_H^* - B)C_H^*} dC_H^*.$$

Notice that $C_F - B^*$ and $C_H^* - B$ always have the same sign. Given that $\eta > 1$ and $C_F, B^* \leq Y^*$, all other terms are positive and we get $dC_H^*/dC_F > 0$. As τ^I changes from zero to infinity, the economy moves along the budget constraint and, given previous limiting cases, both C_F and C_H^* decrease.

Part b: For export tariffs, the equilibrium conditions are given by household demand

$$\frac{u_H^*}{u_F^*} = \tau^E \frac{u_H}{u_F}$$

and the budget constraint

$$u_F(C_F - B^*) = u_H(\tau^E C_H^* - B).$$

Suppose $\tau^E \to 0$. The budget constraint implies $\frac{u_H}{u_F} = \frac{B^* - C_F}{B} < \infty$ and hence, from the former condition $u_H^*/u_F^* \to 0$ and $C_F^* \to 0$. But then $C_F \to Y^*$ and $B^* - C_F < 0$, which is inconsistent with $u_H/u_F > 0$. Thus, there is no equilibrium for $\tau^E \to 0$ and there is a maximum feasible subsidy $\underline{\tau}^E$.

Consider next $\tau^E \to \underline{\tau}^E$. The allocation must then converge to the boundaries of the Edgeworth box with either $C_F = Y^*$ or $C_H = 0$. If $C_F \to Y^*$, then $C_F^* \to 0$ and $u_F^* \to \infty$. Given that $\tau^E u_H/u_F \not\to 0$, the optimal demand requires $u_H^* \to \infty$ and $C_H^* \to 0$. This is inconsistent with the budget constraint $\frac{u_F}{u_H} = -\frac{B}{Y^* - B^*} < 0$. Suppose then $C_H \to 0$ and $C_H^* \to Y$. It follows from household demand $u_F/u_F^* \to \infty$ and hence, $C_F \to 0$ and $C_F^* \to Y^*$. Substitute into the budget constraint

$$\frac{u_F^*}{u_H^*} = \left(\frac{1 - \gamma^*}{\gamma^*} \frac{Y}{Y^*} \right)^{\frac{1}{\eta}} = \frac{B/\tau^E - Y}{B^*},$$

which determines $\underline{\tau}^E$. As long as $B < Y$, we get $0 < \underline{\tau}^E < 1$. The fact that $C_H^* \to Y$ and $C_F \to 0$ implies that $NX > 0$.

Finally, suppose $\tau^E \to \infty$. If $u_H^*/u_F^* \not\to \infty$, then $u_F \to \infty$ and $C_F \to 0$, $C_F^* \to Y^*$, which is inconsistent with the budget constraint. It follows that $u_H^*/u_F^* \to \infty$ and $C_H^* \to 0$, $C_H \to Y$. The budget constraint implies

$$\left(\frac{\gamma}{1 - \gamma} \frac{Y}{C_F} \right)^{\frac{1}{\theta}} = \frac{B}{B^* - C_F}$$

and thus, there is a unique solution $0 < C_F < B^*$. There is zero exports, positive imports and $NX < 0$ in this limit. ■

Proof of Proposition 6 At $\tau^E = 1$, home starts with $NX_0 > 0$. By Proposition 3, balanced trade is characterized by $NX = 0 \iff NFA = 0$. By part (b) of Lemma 3, along the export-tariff equilibrium locus we have $NX < 0$ in the limit as $\tau^E \to \infty$. Therefore, by continuity, there exists an export tariff $\tau^E > 1$ such that $NX = 0$.

Consider next an import subsidy with $\tau^E = 1$. Again, $NX_0 > 0$ at $\tau^I = 1$. By part (a) of Lemma 3, $\tau^I \to 0$ delivers the maximal real depreciation attainable with the import instrument. Define

$$\bar{\mathcal{Q}} \equiv \lim_{\tau^I \to 0} \mathcal{Q}(\tau^I) = \lim_{\tau^I \to 0} \mathcal{S}(\tau^I) < \infty,$$

where finiteness follows from Lemma 3(a), which implies $C_H^* \to Y$ and $C_F \to \bar{C}_F \in (B_0^*, Y^*)$. By Proposition 3, balanced trade requires $\mathcal{Q} = \frac{B_0}{B_0^*}$. Therefore, an import subsidy $\tau^I < 1$ can close the imbalance if and only if $\frac{B_0}{B_0^*} < \bar{\mathcal{Q}}$. Thus, an import subsidy achieves balanced trade if and only if the initial surplus is not too large, in the sense that the required depreciation $\frac{B_0}{B_0^*}$ is below the maximal feasible depreciation $\bar{\mathcal{Q}}$. ■

Proof of Proposition 7 From Lemma 1, condition (15) defines a mapping $C_H^* = g(C_F)$ which is strictly increasing and convex when $B_0, B_0^* > 0$. The planners problem of maximizing $u(C_H, C_F)$ subject to the implementability and resource constraints reduces then to $\max_{C_F} u\big(Y - g(C_F), C_F\big)$, which has the necessary and sufficient optimality condition $u_H \cdot g' = u_F$. Recall that the import tariff (when $\tau^E = 1$) can be expressed as $\tau = \tau^I = (P_F/P_H)/(P_F^*/P_H^*)$ from (4) and (5). Therefore, the optimal tariff equals

$$\tau = \frac{u_F}{u_H} \cdot \frac{P_H^*}{P_F^*} = \underbrace{\frac{g'(C_F)C_F}{g(C_F)}}_{\equiv \varepsilon} \cdot \underbrace{\frac{P_H^* C_H^*}{P_F^* C_F}}_{EX/IM},$$

where the second equality substitutes in the tariff optimality condition and multiplies and divides the expression by $C_F/g(C_F) = C_F/C_H^*$. In the special case of balanced trade $P_H^* C_H^* = P_F^* C_F$, the optimal tariff satisfies $\tau = \varepsilon$ as in Johnson (1950).

Under CES preferences, the elasticity ε of the trade possibilities frontier function $g(\cdot)$ derives from the full differential of (12) and is given by:

$$\varepsilon = \frac{d \log C_H^*}{d \log C_F} = \frac{\eta \frac{C_F}{C_F - B_0^*} + \frac{C_F}{Y^* - C_F}}{\eta \frac{C_H^*}{C_H^* - B_0} - 1}.$$

Substituting this expression into the optimal tariff, we have:

$$\tau = \frac{\eta \frac{C_F}{C_F - B_0^*} + \frac{C_F}{Y^* - C_F}}{\eta \frac{C_H^*}{C_H^* - B_0} - 1} \frac{P_H^* C_H^*}{P_F^* C_F} = \frac{\eta \frac{P_H^* C_H^*}{P_F^* C_F - P_F^* B_0^*} + \frac{P_H^* C_H^*}{P_F^*} \frac{1}{Y^* - C_F}}{\eta \frac{P_H^* C_H^*}{P_H^* C_H^* - P_H^* B_0} - 1} = 1 + \frac{1 + \frac{P_H^* C_H^*}{P_F^* C_F^*}}{\eta \frac{P_H^* C_H^*}{P_H^* C_H^* - P_H B_0} - 1},$$

where the last equality uses the fact that $P_F^* C_F - P_F^* B_0^* = P_H^* C_H^* - P_H B_0$ by the budget constraint (7) and that $P_H = P_H^*$ when $\tau^E = 1$. Finally, we note that

$$1 + \frac{P_H^* C_H^*}{P_F^* C_F^*} = \frac{P_H^* C_H^* + P_F^* C_F^*}{P_F^* C_F^*} = \frac{P^* C^*}{P_F^* C_F^*} = \frac{1}{\Lambda^*},$$

where Λ^* is the foreign local expenditure share. With this, write the optimal import tariff as:

$$\tau = 1 + \frac{1}{\eta}\left(1 + \frac{P_H B_0}{P_H^* C_H^* - P_H B_0}\right)^{-1} \cdot \frac{1}{\Lambda^*}.$$

This corresponds to (17), where $\bar{B} \equiv P_H B_0$. ■

Proof of Proposition 8 By Lemma 1, for a given combined trade wedge $\tau \equiv \tau^I \tau^E$, the contract curve (11) depends only on τ, while the implementability condition (12) depends on τ^E only through B_0/τ^E. Therefore, holding τ fixed and increasing τ^E relaxes the implementability constraint by lowering the effective external liabilities term B_0/τ^E. Hence, welfare is maximized by taking τ^E as large as possible. In the limit $\tau^E \to \infty$, $B_0/\tau^E \to 0$, and therefore the planner's problem converges to the problem in Proposition 7 with $B_0 = 0$. The optimal import tariff τ^I is then chosen following (18) to implement $\tau = \tau^I \tau^E = 1 + \frac{1}{\eta-1}\frac{1}{\Lambda^*}$. ■

Proof of Proposition 9 Substitute τ^E from the contract curve (11) into the budget constraint (12) to get the following implementability condition $C_H^* = g(C_F)$:

$$C_F - B_0^* = \frac{u_H^*}{u_F^*} C_H^* - \frac{u_H}{u_F} B_0.$$

Use CES preferences and the market clearing condition:

$$C_F - B_0^* = \left(\frac{\gamma^*}{1-\gamma^*}\right)^{\frac{1}{\eta}} (Y^* - C_F)^{\frac{1}{\eta}} C_H^{*\frac{\eta-1}{\eta}} - \left(\frac{1-\gamma}{\gamma}\frac{C_F}{Y - C_H^*}\right)^{\frac{1}{\theta}} B_0.$$

Take a full differential in logs:

$$C_F \mathrm{d}\log C_F = \left(\frac{\gamma^*}{1-\gamma^*}\right)^{\frac{1}{\eta}} (Y^* - C_F)^{\frac{1}{\eta}} C_H^{*\frac{\eta-1}{\eta}} \left[-\frac{1}{\eta}\frac{C_F}{Y^* - C_F}\mathrm{d}\log C_F + \frac{\eta-1}{\eta}\mathrm{d}\log C_H^*\right]$$
$$- \left(\frac{1-\gamma}{\gamma}\frac{C_F}{Y - C_H^*}\right)^{\frac{1}{\theta}} B_0 \frac{1}{\theta}\left[\mathrm{d}\log C_F + \frac{C_H^*}{Y - C_H^*}\mathrm{d}\log C_H^*\right].$$

Combine terms and solve for the elasticity:

$$\varepsilon = \frac{\mathrm{d}\log C_H^*}{\mathrm{d}\log C_F} = \frac{C_F + \frac{1}{\eta}\frac{P_H^* C_H^*}{P_F^*}\frac{C_F}{Y^* - C_F} + \frac{P_H}{P_F}\frac{1}{\theta}B_0}{\frac{\eta-1}{\eta}\frac{P_H^* C_H^*}{P_F^*} - \frac{P_H}{P_F}\frac{C_H^*}{Y - C_H^*}\frac{1}{\theta}B_0}.$$

Because the planner's problem written in terms of $g(\cdot)$ remains unchanged, the optimality condition is the same as in the case of the import tariff. Use the definition of export tariff to show $\tau^E = \varepsilon \frac{EX}{IM}$. Substitute in the elasticity and use the fact that $\frac{P_H}{P_F} = \frac{1}{\tau^E}\frac{P^*_H}{P^*_F}$:

$$\tau^E = \frac{EX}{IM}\frac{C_F + \frac{1}{\eta}\frac{P^*_H C^*_H}{P^*_F}\frac{C_F}{Y^*-C_F} + \frac{P_H}{P_F}\frac{1}{\theta}B_0}{\frac{\eta-1}{\eta}\frac{P^*_H C^*_H}{P^*_F} - \frac{1}{\tau^E}\frac{P^*_H}{P^*_F}\frac{C^*_H}{Y-C^*_H}\frac{1}{\theta}B_0}.$$

Multiply by the denominator and simplify using sufficient statistics:

$$\frac{\eta-1}{\eta}\tau^E = 1 + \frac{1}{\eta}\frac{P^*_H C^*_H}{IM}\frac{C_F}{C^*_F} + \frac{1}{\theta}\frac{P_H B_0}{P_H C_H} + \frac{1}{\theta}\frac{P_H B_0}{P_F C_F}.$$

Finally, use definitions of spending shares and value of liabilities to get formula (19). ■

Proof of Proposition 10 Given $C^*_H = g(C_F, \tau^*)$ defined in the main text, the planner's problem boils down to

$$\max u(Y - g(C_F,\tau^*), C_F) \qquad \Rightarrow \qquad \frac{u_F}{u_H} = g_{C_F}(C_F, \tau^*)$$

and from the pricing block, we get

$$\tau = \frac{u_F/u_H}{\tau^* u^*_F/u^*_H} = \frac{g_{C_F} C_F/g}{P^*_F C_F/P_H C^*_H} = \varepsilon\,\frac{P_H C^*_H}{P^*_F C_F} = \varepsilon \frac{EX}{IM}.$$

Use CES preferences and the market clearing conditions:

$$\tau^* \left(\frac{1-\gamma^*}{\gamma^*}\frac{C^*_H}{Y^* - C_F}\right)^{\frac{1}{\eta}} (C_F - B^*_0) = (C^*_H - B_0).$$

It follows that the elasticity ε does not directly depend on τ^* and can be expressed in the same way as under a unilateral tariff. Substitute back into the optimal tariff and simplify using the budget constraint:

$$\tau = \frac{\frac{P_H C^*_H}{P^*_F C^*_F} + \eta\frac{P_H C^*_H}{P^*_F C_F - P^*_F B^*_0}}{\eta\left(1 + \frac{B_0}{C^*_H - B_0}\right) - 1} = 1 + \frac{1}{\eta\left(1+\frac{\mathcal{B}}{P_H C^*_H - \mathcal{B}}\right) - 1}\frac{1}{\Lambda^*},$$

where $\Lambda^* \equiv \frac{P^*_F C^*_F}{P_H C^*_H + P^*_F C^*_F}$. This expression is isomorphic to the formula for τ in (20). The derivations for the optimal foreign tariff are symmetric.

To prove that optimal tariffs are lower than in unilateral case, suppose this is not the case. It is starightforward to show that $\tau > \tau^W$ and $\tau^* > 1$ imply that trade flows C^*_H, C_F are smaller in Nash equilibrium than under a unilateral tariff. It follows that both $\mathcal{B}/EX = B_0/C^*_H$ and Λ^* are higher in Nash equilibrium:

$$\Lambda^{*-1} = 1 + \frac{P_H C^*_H}{P^*_F(Y^* - C_F)} = 1 + \frac{1}{\tau^*}\frac{P^*_H C^*_H}{P^*_F(Y^*-C_F)} = 1 + \frac{1}{\tau^*}\left(\frac{\gamma^*}{1-\gamma^*}\right)^{\frac{1}{\eta}}\left(\frac{C^*_H}{Y^*-C_F}\right)^{\frac{\eta-1}{\eta}}.$$

But then formula (20) implies that the optimal tariff is lower $\tau < \tau^W$, which contradicts the initial conjecture. A symmetric argument applies to foreign tariff, so that $\tau^* < \tau^{W*}$. ■

Proof of Proposition 11 Consider a planner maximizing fiscal revenues in units of home currency $\max \quad \frac{P_F - P_F^*}{P_H} C_F$ using the import tariff as the only instrument. Substitute out prices using conditions (4)-(6) and the implementability condition (15):

$$\max \quad \frac{u_F(Y - g(C_F), C_F)}{u_H(Y - g(C_F), C_F)} C_F - \frac{g(C_F) - B_0}{C_F - B_0^*} C_F.$$

The first-order condition for separable preferences is given by

$$\frac{u_F}{u_H} + \frac{u_{FF} C_F}{u_H} + \frac{u_{HH} u_F g' C_F}{u_H^2} + \frac{(g - B_0) C_F}{(C_F - B_0^*)^2} - \frac{g - B_0}{C_F - B_0^*} - \frac{g' C_F}{C_F - B_0^*} = 0.$$

Use home iso-elastic preferences and rearrange terms:

$$\frac{u_F}{u_H}\left(\theta - 1 - \varepsilon \frac{C_H^*}{C_H}\right) = \theta \left[\frac{C_H^* - B_0}{C_F - B_0^*} - \frac{(C_H^* - B_0) C_F}{(C_F - B_0^*)^2} + \frac{\varepsilon C_H^*}{C_F - B_0^*}\right].$$

Substitute in $u_F/u_H = \tau P_F^*/P_H$ and the budget constraint:

$$\tau^R = \frac{\theta}{\theta - 1 - \varepsilon \frac{C_H^*}{C_H}} \left[\frac{\varepsilon C_H^*}{C_H^* - B_0} - \frac{B_0^*}{C_F - B_0^*}\right],$$

where elasticity ε is the same as before. Rewrite in terms of values and use $\tau^W = \varepsilon \frac{EX}{IM}$ to substitute out ε:

$$\tau^R = \frac{\theta}{\theta - 1 - \tau^W \frac{IM}{P_H C_H}} \cdot \frac{\tau^W IM - \mathcal{B}^*}{IM - \mathcal{B}^*},$$

which is equivalent to (21), as $\frac{\tau^W IM - \mathcal{B}^*}{IM - \mathcal{B}^*} = \tau^W + (\tau^W - 1)\frac{\mathcal{B}^*}{IM - \mathcal{B}^*}$ and $\frac{IM}{P_H C_H} = \frac{1-\Lambda}{\Lambda}$.

Finally, by (17), we have $\tau^W \geq 1$ as $B_0 \geq 0$. In addition, since $B_0^* \geq 0$ and assuming that $\mathcal{B}^* < IM$, it follows that $\tau^W + (\tau^W - 1)\frac{\mathcal{B}^*}{IM - \mathcal{B}^*} \geq \tau^W$. At the same time, $\Lambda \in (0,1)$ implies $\frac{\theta}{\theta - 1 - \tau^W \frac{IM}{P_H C_H}} > 1$. Hence, $\tau^R > \tau^W \geq 1$. For any values of $B_0, B_0^* \geq 0$, $\tau^W \to 1$ as $\eta \to \infty$, and therefore $\tau^R = \frac{\theta}{\theta - 1 - \frac{IM}{P_H C_H}} = \frac{\theta}{\theta - 1/\Lambda}$. ■

Proof of Lemma 4 Conjecture that endowments are split in constant proportion between two countries according to (25) in all periods. The market clearing conditions are automatically satisfied. Substitute out prices from the static demand conditions

$$\frac{\gamma}{1-\gamma} \frac{C_{Ht}}{C_{Ft}} = (\tau^I \tau^E)^\theta \frac{1-\gamma^*}{\gamma^*} \frac{C_{Ht}^*}{C_{Ft}^*}$$

and substitute in conjectured consumption profiles to get the contract curve in terms consumption shares (26). Conjecture that portfolio shares are constant. For all future periods

$t > 0$, the flow budget constraint (22) simplifies to

$$P_{Ht}Y_t(\tau^E a - b) - P^*_{Ft}Y^*_t(a^* - b^*) = 0.$$

Then $b = \tau^E a$, $b^* = a^*$ is required for it to be satisfied for any realizations of endowment shocks Y_t, Y^*_t. The same budget consraint in the initial period $t = 0$ simplifies

$$b^* - b^*_0 = \frac{P_{H0}}{P^*_{F0}}\left(\frac{Y_0}{Y^*_0}\right)^{\frac{1}{\theta}} \frac{\mathbb{E}\sum_{t=0}^{\infty}(\beta/\chi)^t Y_t^{\frac{\theta-1}{\theta}}}{\mathbb{E}\sum_{t=0}^{\infty}\beta^t Y_t^{*\frac{\theta-1}{\theta}}}(b - b_0),$$

where we used the fact that both home and foreign Euler equations (23)-(24) imply the following stock prices:

$$V_t = Y_t^{\frac{1}{\theta}}\mathbb{E}_t\sum_{j=1}^{\infty}(\beta/\chi)^j Y_{t+j}^{\frac{\theta-1}{\theta}}, \qquad V^*_t = Y_t^{*\frac{1}{\theta}}\mathbb{E}_t\sum_{j=1}^{\infty}\beta^j Y_{t+j}^{*\frac{\theta-1}{\theta}}. \tag{A6}$$

Substituing in static demand and switching to consumption shares, the budget constraint reduces to (27). ■

Proof of Proposition 12 By Lemma 4, the dynamic equilibrium allocation is given by constant shares of endowments, so Definition 1 immediately implies:

$$C_H = (1-a)Y, \qquad C^*_H = aY, \qquad C_F = a^*Y^*, \qquad C^*_F = (1-a^*)Y^*.$$

Therefore, the resource constraints (3) hold for permanent components. The lemma and definition also imply that the permanent consumption aggregates preserve the CES utility form in (2). And since the law of one price (4) is satisfied for any history in the dynamic model, it also holds for the corresponding permanent components and therefore, $\mathcal{Q} = \tau^E\mathcal{S}$.

Consider next the static optimality conditions for households:

$$u_{Ht} = \lambda_t P_{Ht}, \qquad u_{Ft} = \lambda_t P_{Ft}, \qquad u^*_{Ht} = \lambda^*_t P^*_{Ht}, \qquad u^*_{Ft} = \lambda^*_t P^*_{Ft},$$

where $\beta^t\lambda_t$ and $\beta^t\lambda^*_t$ are Lagrange multipliers on flow budget constraints of home and foreign households. From Lemma 4 and the law of one price, it follows that

$$\frac{\lambda^*_t}{\lambda_t} = \frac{u^*_{Ht}}{u_{Ht}}\frac{P_{Ht}}{P^*_{Ht}} = \left(\frac{\gamma^*}{1-\gamma}\frac{1-a}{a}\right)^{\frac{1}{\theta}}\frac{1}{\tau^E}.$$

We use λ_t as a numeraire in each period making it constant in time, which implies that λ^*_t is time invariant as well. Integrate home household optimality conditions with respect to C_{Ht}

across periods to get

$$(1-\gamma)^{\frac{1-\theta}{\theta}} C_{Ht}^{\frac{\theta-1}{\theta}} = (\lambda P_{Ht})^{1-\theta} \quad \Rightarrow \quad (1-\gamma)^{\frac{1-\theta}{\theta}} (1-\beta) \mathbb{E} \sum_{t=0}^{\infty} \beta^t C_{Ht}^{\frac{\theta-1}{\theta}} = (1-\beta) \mathbb{E} \sum_{t=0}^{\infty} \beta^t (\lambda P_{Ht})^{1-\theta},$$

which can be rewritten in terms of permanent consumption and prices as

$$(1-\gamma)^{\frac{1-\theta}{\theta}} C_H^{\frac{\theta-1}{\theta}} = \lambda^{1-\theta} P_H^{1-\theta} \quad \Rightarrow \quad u_H = \lambda P_H.$$

The derivations for C_{Ft} are symmetric and result in $u_F = \lambda P_F$. Combining the two conditions together, we arrive at static optimality condition (5). The derivations for C^*_{Ht} and C^*_{Ft} are analoguos and use the fact that $\lambda^*_t = \lambda^*$ resulting in static optimality condition (6).

It remains to derive the country budget constraint. Rewrite (27) using $B_0 = b_0 Y$, $B^*_0 = b^*_0 Y^*$, the definition of Ω in (29), and the foreign demand condition (6) for permanent components. Substituting $C^*_H = aY$, $C_F = a^* Y^*$, and $C^*_F = (1-a^*)Y^*$, we obtain:

$$C_F - B^*_0 = (1+\Omega) \left(\frac{C^*_H}{\mathcal{S}} - \frac{B_0}{\mathcal{Q}} \right),$$

which is equivalent to (28). ■

Proof of Corollary 4 When $\chi = 1$, we have $\Omega = 0$ by (29). Then Proposition 12 implies that the permanent components in the dynamic model satisfy exactly the same static equilibrium system as in Section 2.1, including the same budget constraint (7). Therefore, the welfare, permanent allocations, optimal long-run tariffs, and tariffs closing the long-run imbalance are the same in the dynamic and static models.

To characterize Lerner symmetry, note that Proposition 12 implies $\mathcal{Q} = \tau^E \mathcal{S}$ and the generalized budget constraint (28). If $B_0 = 0$, this budget constraint no longer depends on $\mathcal{Q}$ and hence does not depend separately on τ^E. The long-run equilibrium system then depends on tariffs only through the overall wedge $\tau = \tau^I \tau^E$, so the argument of Proposition 2 applies and Lerner symmetry holds. Conversely, when $B_0 \neq 0$, both (28) and $\mathcal{Q} = \tau^E \mathcal{S}$ imply that τ^E affects the long-run equilibrium separately from τ^I, even for a given $\tau = \tau^I \tau^E$. Therefore, Lerner symmetry fails. ■

Proof of Proposition 13 By Lemma 4, the dynamic equilibrium allocation is given by constant shares of contemporaneous endowments in every period. Therefore, an unexpected permanent change in tariffs changes only the constant shares a, a^* and the corresponding price wedges, while the stochastic paths of Y_t, Y^*_t remain unchanged. It follows that every allocation, price, and trade value X_t shifts proportionally in all periods by the same factor as its permanent component from Definition 1, so $\hat{X}_t = \hat{X}$.

For trade values, we then have $\Delta EX_t = EX_t \cdot \hat{EX}$ and $\Delta IM_t = IM_t \cdot \hat{IM}$. Hence,

$$\Delta NX_t = \Delta EX_t - \Delta IM_t = EX_t \cdot \hat{EX} - IM_t \cdot \hat{IM},$$

which proves (30). Averaging over time and states gives $\Delta NX = \Delta EX - \Delta IM$. Finally,

$$\Delta(NX_t - NX) = (EX_t - EX) \cdot \hat{EX} - (IM_t - IM) \cdot \hat{IM},$$

so when tariffs reduce the permanent gross trade flows, $\hat{EX} < 0$ and $\hat{IM} < 0$, they also compress transitory trade imbalances around the permanent trade balance NX. ■

Proof of Proposition 14 Substitute out prices from the budget constraint (28) using the optimal static demand

$$\left(\frac{1-\gamma^*}{\gamma^*}\frac{C_H^*}{C_F^*}\right)^{\frac{1}{\theta}} = \mathcal{S}$$

and obtain an implicit function $C_H^* = g(C_F)$:

$$C_F - B_0^* = \left(\frac{\gamma^*}{1-\gamma^*}\frac{Y^* - C_F}{C_H^*}\right)^{\frac{1}{\theta}}(1+\Omega)(C_H^* - B_0).$$

Log-differentiating and rearranging yields

$$g' = \frac{\mathrm{d}C_H^*}{\mathrm{d}C_F} = \frac{\theta C_F^* + C_F - B_0^*}{C_F^*(C_F - B_0^*)}\frac{C_H^*(C_H^* - B_0)}{(\theta - 1)C_H^* + B_0}.$$

The planner solves

$$\max\ u(Y - g(C_F), C_F) \qquad \Rightarrow \qquad \frac{u_F}{u_H} = g' \qquad \Rightarrow \qquad \tau^I = \frac{u_F/u_F^*}{u_H/u_H^*} = g'\frac{u_H^*}{u_F^*},$$

so substituting g' and using $g(\cdot)$ to eliminate $(C_F - B_0^*)/(C_H^* - B_0)$ gives

$$\tau^I = \frac{C_H^*(\theta C_F^* + C_F - B_0^*)}{C_F^*((\theta-1)C_H^* + B_0)}\frac{1}{1+\Omega}.$$

Using again the budget constraint in the form $P_F^*(C_F - B_0^*) = (1+\Omega)P_H^*(C_H^* - B_0)$, we get

$$\frac{C_F - B_0^*}{C_F^*} = (1+\Omega)\frac{P_H^*(C_H^* - B_0)}{P_F^*C_F^*}, \qquad \frac{B_0}{C_H^*} = \frac{P_H^*B_0}{P_H^*C_H^*} = \frac{\mathcal{B}}{EX}, \qquad \frac{P_H^*C_H^*}{P_F^*C_F^*} = \frac{1-\Lambda^*}{\Lambda^*}.$$

Substituting these identities into the previous expression and simplifying yields (31).

The static contract curve implies that the export tariff can be expressed as

$$\tau^E = \left(\frac{\gamma^*}{1-\gamma^*}\frac{\gamma}{1-\gamma}\frac{C_H}{C_H^*}\frac{C_F^*}{C_F}\right)^{\frac{1}{\theta}}.$$

Substitute into the budget constraint (28) to get $C_H^* = g(C_F)$:

$$(1+\Omega)^{-1}(C_F - B_0^*) = \left(\frac{\gamma^*}{1-\gamma^*}(Y^* - C_F)\right)^{\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} - \left(\frac{1-\gamma}{\gamma}\frac{C_F}{Y - C_H^*}\right)^{\frac{1}{\theta}} B_0.$$

Take a full differential:

$$(1+\Omega)^{-1}\mathrm{d}C_F = \left(\frac{\gamma^*}{1-\gamma^*}\right)^{\frac{1}{\theta}} (Y^* - C_F)^{\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} \left[-\frac{1}{\theta}\frac{\mathrm{d}C_F}{Y^* - C_F} + \frac{\theta-1}{\theta}\frac{\mathrm{d}C_H^*}{C_H^*}\right]$$

$$-\frac{1}{\theta}\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\theta}} \left(\frac{C_F}{Y - C_H^*}\right)^{\frac{1}{\theta}} B_0 \left[\frac{\mathrm{d}C_F}{C_F} + \frac{dC_H^*}{Y - C_H^*}\right].$$

Rearrange terms, use prices and simplify:

$$g' = \frac{\mathrm{d}C_H^*}{\mathrm{d}C_F} = \frac{\theta(1+\Omega)^{-1} + \frac{P_H^* C_H^*}{P_F^* C_F^*} + \frac{P_H B_0}{P_F C_F}}{(\theta-1)\frac{P_H^*}{P_F^*} - \frac{P_H B_0}{P_F C_H}}.$$

The planner's problem is solved by

$$\max\ u(Y - g(C_F), C_F) \qquad \Rightarrow \qquad \frac{u_F}{u_H} = g' \qquad \Rightarrow \qquad \tau^E = \frac{u_H^*/u_H}{u_F^*/u_F} = g'\frac{u_H^*}{u_F^*}.$$

It follows that

$$\tau^E = \frac{\theta(1+\Omega)^{-1} + \frac{P_H^* C_H^*}{P_F^* C_F^*} + \frac{P_H B_0}{P_F C_F}}{(\theta-1) - \frac{B_0}{\tau^E C_H}}.$$

Multiply both sides by the denominator:

$$(\theta-1)(\tau^E - 1) = -\theta\left(1 - (1+\Omega)^{-1}\right) + \frac{P_F^* C_F^* + P_H^* C_H^*}{P_F^* C_F^*} + \frac{P_H B_0}{P_F C_F}\frac{P_F C_F + P_H C_H}{P_H C_H}.$$

Rewriting in terms of sufficient statistics, we get formula (32). ■

A3 Numerical Solution

This section provides further details on solving numerically for optimal tariffs and the resulting allocation. The challenge is twofold. First, the propositions above characterize tariffs in terms of sufficient statistics, i.e. trade shares, but the latter are themselves endogenous to trade policy, and one needs to solve for a fixed point. Second, it is not possible to calibrate γ and γ^* separately from real quantities and prices in the initial equilibrium. Following the exact hat algebra approach from Dekle, Eaton, and Kortum (2007), we address both issues rewriting the system of equilibrium conditions in terms of (i) gross changes of variables $\hat{X} \equiv X'/X$ between the initial free-trade equilibrium X and the counterfactual equilibrium with tariffs X' and (ii) initial shares under free trade directly measurable in the data. Because tariffs are absent in the initial equilibrium, their new values coincide with gross changes. For simplicity, we assume $\theta = \eta$, a relevant case under our calibration.

Equilibrium system We first generalize the model to allow for both convenience yields and import and export tariffs in both countries to nest all results from above as special cases. For brevity, we skip the derivations for the full dynamic model, which rely on constant consumption and portfolio shares and follow the same steps as in Section 5, and show only the resulting static conditions for permanent prices and quantities. The pricing block is given by

$$P_F = \tau^I \tau^{E*} P_F^*, \qquad P_H^* = \tau^E \tau^{I*} P_H$$

and the market clearing is

$$C_H + C_H^* = Y, \qquad C_F + C_F^* = Y^*.$$

The terms of trade are given by $\mathcal{S} = \frac{\tau^{E*} P_F^*}{\tau^E P_H}$ and the real exchange rate by $\mathcal{Q} = \frac{P_F^*}{P_H}$ and therefore, $\mathcal{Q} = \frac{\tau^E}{\tau^{E*}} \mathcal{S}$. The optimal static demand for local and imported goods can then be expressed as

$$\frac{u_F}{u_H} = \left(\frac{\gamma}{1-\gamma} \frac{C_H}{C_F} \right)^{\frac{1}{\theta}} = \tau^I \tau^E \mathcal{S}, \qquad \frac{u_F^*}{u_H^*} = \left(\frac{1-\gamma^*}{\gamma^*} \frac{C_H^*}{C_F^*} \right)^{\frac{1}{\theta}} = \frac{\mathcal{S}}{\tau^{I*} \tau^{E*}}.$$

Finally, the country's budget constraint is given by

$$C_F - \frac{B_0^*}{\tau^{E*}} = \mathcal{S}^{-1} (1+\Omega) \left(C_H^* - \frac{B_0}{\tau^E} \right),$$

where $B_0^* \equiv b_0^* Y^*$ and $B_0 \equiv b_0 Y$.

To write the system in changes, we first introduce the following notation for the pre-tariff variables. The export shares $\bar{\alpha} \equiv \frac{C_H^*}{Y} = \frac{P_H C_H^*}{GDP}$ and $\bar{\alpha}^* \equiv \frac{C_F}{Y^*} = \frac{P_F^* C_F}{GDP^*}$ can be measured in quantities or values given that $GDP = P_H Y$ and $GDP^* = P_F^* Y^*$. The spending shares are measured using border prices for imported goods $\Lambda^* \equiv \frac{P_F^* C_F^*}{\tau^E P_H C_H^* + P_F^* C_F^*}$, $\Lambda \equiv \frac{P_H C_H}{\tau^{E*} P_F^* C_F + P_H C_H}$ and coincide with consumption shares in the pre-tariff equilibrum $\bar{\Lambda}^* = 1 - \frac{EX}{P^* C^*}$, $\bar{\Lambda} = 1 - \frac{IM}{PC}$. In addition, it is convenient to define the ratio of imports to GDP, $\bar{\gamma} \equiv \frac{IM}{GDP} = \frac{P_F C_F}{P_H Y}$ and $\bar{\gamma}^* \equiv \frac{EX}{GDP^*} = \frac{P_H^* C_H^*}{P_F^* Y^*}$. The gross asset positions $\bar{b} \equiv \frac{P_H B_0}{GDP}$ and $\bar{b}^* \equiv \frac{P_F^* B_0^*}{GDP}$ are measured as a share of home GDP. It follows that the initial budget constraint $IM - \mathcal{B}^* = (1+\Omega)(EX - \mathcal{B})$ can be rewritten in terms of units of home's GDP, $\bar{\gamma} = (1+\Omega)(\bar{\alpha} - \bar{b}) + \bar{b}^*$.

Algorithm For given tariffs, the equilibrium can be found following those steps:

1. Conjecture $\hat{\mathcal{S}}$.

2. Rewrite market clearing conditions using hat algebra:

$$(1-\bar{\alpha})\hat{C}_H + \bar{\alpha}\hat{C}_H^* = \hat{Y}, \qquad \bar{\alpha}^*\hat{C}_F + (1-\bar{\alpha}^*)\hat{C}_F^* = \hat{Y}^*.$$

   Use the fact that $\hat{Y} = \hat{Y}^* = 1$ and the optimal demand conditions

$$\hat{C}_H = (\hat{\tau}^I \hat{\tau}^E \hat{\mathcal{S}})^\theta \hat{C}_F, \qquad \hat{C}_F^* = \left(\frac{\hat{\mathcal{S}}}{\hat{\tau}^{I*}\hat{\tau}^{E*}}\right)^{-\theta} \hat{C}_H^*.$$

   Substitute into the market clearing conditions and solve the linear system:

$$\hat{C}_F = \frac{(1-\bar{\alpha}^*)\left(\hat{\tau}^{I*}\hat{\tau}^{E*}\hat{\mathcal{S}}^{-1}\right)^\theta - \bar{\alpha}}{(1-\bar{\alpha})(1-\bar{\alpha}^*)\left(\hat{\tau}^I\hat{\tau}^E\hat{\tau}^{I*}\hat{\tau}^{E*}\right)^\theta - \bar{\alpha}\,\bar{\alpha}^*}, \quad \hat{C}_H^* = \frac{(1-\bar{\alpha})\,(\hat{\tau}^I\hat{\tau}^E\hat{\mathcal{S}})^\theta - \bar{\alpha}^*}{(1-\bar{\alpha})(1-\bar{\alpha}^*)\left(\hat{\tau}^I\hat{\tau}^E\hat{\tau}^{I*}\hat{\tau}^{E*}\right)^\theta - \bar{\alpha}\,\bar{\alpha}^*}.$$

3. Rewrite the budget constraint using hat algebra and use it to update the terms of trade

$$\bar{\gamma}\hat{C}_F - \frac{\bar{b}^*}{\hat{\tau}^{E*}} = \hat{\mathcal{S}}^{-1}(1+\Omega)\left(\bar{\alpha}\hat{C}_H^* - \frac{\bar{b}}{\hat{\tau}^E}\right) \qquad \Rightarrow \qquad \hat{\mathcal{S}} = (1+\Omega)\,\frac{\bar{\alpha}\hat{C}_H^* - \frac{\bar{b}}{\hat{\tau}^E}}{\bar{\gamma}\hat{C}_F - \frac{\bar{b}^*}{\hat{\tau}^{E*}}}$$

   iterating untill convergence.

4. Once the fixed point is estimated, one can compute changes in welfare

$$\hat{C} = \left[\bar{\Lambda}\hat{C}_H^{\frac{\theta-1}{\theta}} + (1-\bar{\Lambda})\hat{C}_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \qquad \hat{C}^* = \left[\bar{\Lambda}^*\hat{C}_F^{*\frac{\theta-1}{\theta}} + (1-\bar{\Lambda}^*)\hat{C}_H^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

   and consumption shares evaluated at border prices

$$\hat{\Lambda} = \frac{\bar{\Lambda}\hat{C}_H}{\bar{\Lambda}\hat{C}_H + (1-\bar{\Lambda})\hat{\tau}^E\hat{\mathcal{S}}\hat{C}_F}, \qquad \hat{\Lambda}^* = \frac{\bar{\Lambda}^*\hat{C}_F^*}{\bar{\Lambda}^*\hat{C}_F^* + (1-\bar{\Lambda}^*)\hat{\tau}^{E*}\hat{\mathcal{S}}^{-1}\hat{C}_H^*}.$$

   The new trade balance can be found as

$$\frac{NX'}{P_H'Y'} = \frac{EX\cdot\hat{\tau}^E\hat{P}_H\hat{C}_H^* - IM\cdot\hat{\tau}^{E*}\hat{P}_F^*\hat{C}_F}{GDP\cdot\hat{P}_H} = \bar{\alpha}\hat{\tau}^E\hat{C}_H^* - \bar{\gamma}\hat{\tau}^E\hat{\mathcal{S}}\hat{C}_F,$$

where one can think of the denominator $P'_H Y'$ as initial GDP using home goods prices as a numeraire. Finally, tax revenues measured in the same units are equal

$$\frac{T'}{P'_H Y'} = \frac{IM \cdot (\hat{\tau}^I - 1)\hat{\tau}^{E*}\hat{P}^*_F \hat{C}_F + EX \cdot (\hat{\tau}^E - 1)\hat{P}_H \hat{C}^*_H}{GDP \cdot \hat{P}_H} = \bar{\gamma}(\hat{\tau}^I - 1)\hat{\tau}^E \hat{\mathcal{S}}\hat{C}_F + \bar{\alpha}(\hat{\tau}^E - 1)\hat{C}^*_H.$$

Tariffs Depending on the application, the expressions for tariffs are different:

- Optimal import tariff: assuming no export tariffs, the expression for the optimal tariff can be rewritten in terms of gross changes as

$$\hat{\tau}^I = 1 + \frac{1 - \frac{\Omega}{1+\Omega}\theta\bar{\Lambda}^*\hat{\Lambda}^* \frac{\bar{\alpha}\hat{C}^*_H}{\bar{\alpha}\hat{C}^*_H - \bar{b}}}{\theta\frac{\bar{\alpha}\hat{C}^*_H}{\bar{\alpha}\hat{C}^*_H - \bar{b}} - 1} \frac{1}{\bar{\Lambda}^*\hat{\Lambda}^*}.$$

- Optimal export tariff: similarly, the unilateral export tax can be expressed as

$$\hat{\tau}^E = 1 + \frac{1}{\theta - 1}\frac{1}{\bar{\Lambda}^*}\frac{1}{\hat{\Lambda}^*} + \frac{1}{\theta - 1}\frac{\bar{b}}{\bar{\gamma}\bar{\Lambda}}\frac{1}{\hat{\Lambda}\hat{\mathcal{Q}}\hat{C}_F} - \frac{\theta}{\theta - 1}\frac{\Omega}{1 + \Omega}.$$

- Closing imbalances: the import and export tariffs imposed by the home economy to close the trade deficit can be estimated imposing

$$\frac{NX'}{P'_H Y'} = \bar{\alpha}\hat{\tau}^E \hat{C}^*_H - \bar{\gamma}\hat{\tau}^E \hat{\mathcal{S}}\hat{C}_F = 0.$$

- Fiscal tariff: assuming $\Omega = 1$, the import tariff maximizing fiscal revenues is given by

$$\hat{\tau}^I = \frac{\theta}{\theta - 1 - \varepsilon\frac{\bar{\alpha}}{1-\bar{\alpha}}\frac{\hat{C}^*_H}{\hat{C}_H}}\left[\frac{\varepsilon\bar{\alpha}\hat{C}^*_H}{\bar{\alpha}\hat{C}^*_H - \bar{b}} - \frac{\bar{b}^*}{\bar{\gamma}\hat{C}_F - \bar{b}^*}\right], \quad \text{where } \varepsilon = 1 + \frac{\frac{1}{(1-\bar{\alpha}^*)\hat{C}^*_F} + \theta\frac{\bar{\gamma}\hat{\mathcal{Q}}\hat{C}_F - \bar{\alpha}\hat{C}^*_H}{\bar{\alpha}\hat{C}^*_H - \bar{b}}}{\theta\frac{\bar{\alpha}\hat{C}^*_H}{\bar{\alpha}\hat{C}^*_H - \bar{b}} - 1}.$$

- Trade war: assuming no export taxes and $\Omega = 1$, the optimal import tariffs in Nash equilibrum are

$$\hat{\tau}^I = 1 + \frac{1}{\theta\frac{\bar{\alpha}\hat{C}^*_H}{\bar{\alpha}\hat{C}^*_H - \bar{b}} - 1}\frac{1}{\bar{\Lambda}^*\hat{\Lambda}^*}, \qquad \hat{\tau}^{I*} = 1 + \frac{1}{\theta\frac{\bar{\gamma}\hat{C}_F}{\bar{\gamma}\hat{C}_F - \bar{b}^*} - 1}\frac{1}{\bar{\Lambda}\hat{\Lambda}}.$$

A4 Production Economy and Manufacturing Jobs

Production economy We consider an extension of the baseline model with tradable and non-tradable goods, endogenous production, and a fixed supply of labor that is endogenously allocated between tradable and non-tradable sectors, where production functions exhibit decreasing returns to scale (DRS):

$$C_N = Y_N = F_N(L_N), \qquad Y = F_T(L_T), \qquad L_N + L_T = L, \tag{A7}$$

Tradable output is allocated between domestic consumption and exports, $Y = C_H + C_H^*$. Households preferences are described by a nested CES utility:

$$u = \frac{\rho}{\rho-1}\left(\kappa C_N^{\frac{\rho-1}{\rho}} + C_T^{\frac{\rho-1}{\rho}}\right), \qquad C_T = \left[(1-\gamma)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{A8}$$

with $\rho \geq 0$ and $\theta > 1$, and where C_F are imports.

Trade possibilities frontier We focus on the case with imports tariff $\tau^I = \tau$ and no export tax, $\tau^E = 1$, so that $\mathcal{Q} = \mathcal{S}$. Lemma 1 still applies and characterizes the trade possibilities frontier (TPF), $C_H^* = g(C_F; B_0, B_0^*)$, as in (12). Trade policy shifts the equilibrium along this frontier g by choosing a combination of export and import quantities (C_H^*, C_F) that satisfy it. We maintain the assumption that foreign output is exogenously and allocated between home's imports and local foreign consumption, $Y^* = C_F + C_F^*$.[44] To simplify analysis, we assume that home is a small economy in that it does not affect the price of the foreign good in the foreign market, and we capture this with a quasi-linear foreign utility, $u^* = C_F^* + \frac{\eta}{\eta-1}(\gamma^*)^{1/\eta}(C_H^*)^{\frac{\eta-1}{\eta}}$. With this utility, the equilibrium terms of trade are determined as follows: $\mathcal{S} = P_F^*/P_H^* = u_F^*/u_H^* = (C_H^*/\gamma^*)^{1/\eta}$ with $\eta > 1$. We can then write the trade possibilities frontier (12) as:

$$C_F = B_0^* + (\gamma^*)^{1/\eta}(C_H^* - B_0)/(C_H^*)^{1/\eta}, \tag{A9}$$

which defines explicitly the inverse TPF $g^{-1}(\cdot)$ such that C_F is an increasing function of C_H^*.

Labor market equilibrium Free worker mobility across sectors requires equalization of wages in the two sector which, in turn, reflect marginal products of labor in the two sectors. We have: $P_H F_T'(L_T) = W = P_N F_N'(L_N)$. Combining together with labor market clearing, $L_T + L_N = L$, this gives the following locus of the labor market equilibrium points:

$$\frac{P_H}{P_N} = \frac{F_N'(L - L_T)}{F_T'(L_T)} \tag{A10}$$

[44]The analysis generalizes to foreign output produced endogenously with a symmetric production structure to (A7), in which case the elasticity of the trade possibilities frontier $g(\cdot)$ depends not only on foreign marginal utility but also on the curvature of foreign production functions.

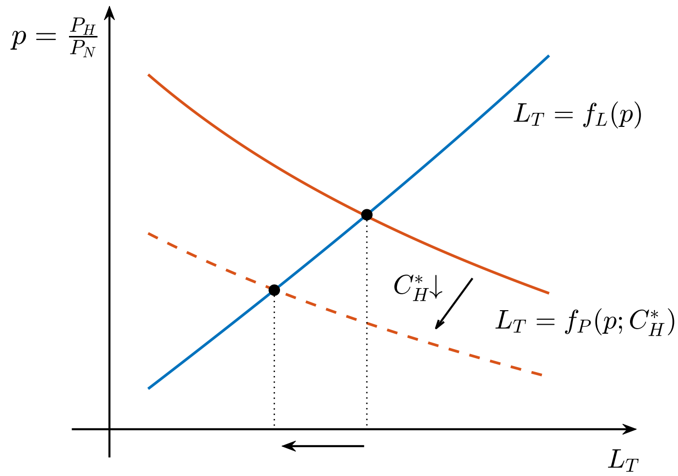

Figure A2: Tradable-sector employment

Note: The figure plots f_L, the locus of labor market equilibrium points (A10), and f_P, the locus of product market equilibrium points (A11), which together determine the equilibrium tradable-sector employment L_T and the relative price of home tradables $p = P_H/P_N$. Trade policy shifts export quantity C_H^*, which changes the locus of product market equilibrium points.

in the space (L_T, p) where $p \equiv P_H/P_N$ is the relative price of the tradable good in the domestic market. Due to DRS, the right-hand side is increasing in L_T, and therefore (A10) implies that L_T is increasing in p: $L_T = f_L(p)$ such that $f_L'(\cdot) > 0$ where f_L denotes the locus of points corresponding to the equilibrium in the labor market. The home relative price p is a sufficient statistic for L_T in the sense that trade policy does not shift the f_L schedule directly, only via its effect on the equilibrium price p.

Product market equilibrium Home consumers allocate expenditure between home tradable and non-tradable goods according to:

$$\frac{P_H}{P_N} = \frac{u_H}{u_N} = \frac{(1-\gamma)^{1/\theta}}{\kappa} \left(\frac{C_N}{C_T}\right)^{\frac{1}{\rho}} \left(\frac{C_T}{C_H}\right)^{\frac{1}{\theta}},$$

and we rewrite this condition using $C_N = Y_N$ and (A8) as:

$$\frac{P_H}{P_N} = \frac{(1-\gamma)^{1/\theta}}{\kappa} Y_N^{\frac{1}{\rho}} \left[(1-\gamma)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}\left(\frac{1}{\theta}-\frac{1}{\rho}\right)} C_H^{-\frac{1}{\theta}}, \tag{A11}$$

where $C_H = Y - C_H^*$ and $C_F = g^{-1}(C_H^*, B_0, B_0^*)$ defined in (A9), and C_H^* is determined by trade policy such that C_H^* is decreasing in τ. Replacing $Y_N = F_N(L - L_T)$ and $Y = F_T(L_T)$ and substituting C_H and C_F into (A11), we obtain a locus of equilibria points in the home product market, again in the space (L_T, p), where export quantity C_H^* is an exogenous shifter implemented by trade policy τ. We denote this locus as $L_T = f_P(p; C_H^*)$, which is decreasing in p, and below we establish condition when it is increasing in C_H^* (or, equivalently, decreasing in τ).

For a given trade policy τ and a corresponding export quantity C_H^*, the equilibrium (L_T, p) solves the system (A10)–(A11), and trade policy shifts (A11), as we illustrate in Figure A2. The next result characterizes this equilibrium comparative statics formally:

Proposition A2 *The equilibrium tradable employment L_T decreases in import tariff τ when:* (i) $\rho \geq \theta$ *or* (ii) $\rho < \theta$ *and*

$$\rho + (\theta - \rho)\tau\Lambda \frac{1 - \eta\frac{\tau-1}{\tau} - \frac{\mathcal{B}}{EX}}{\eta} > 0, \tag{A12}$$

where $\mathcal{B} = P_H^* B_0$, $EX = P_H^* C_H^*$, and $\Lambda = \frac{P_H C_H}{P_F C_F + P_H C_H}$ is the consumer tradable expenditure share on home good. Around free trade, $\tau = 1$, $\mathcal{B} < EX$ is sufficient for condition (A12) to hold.

We prove this result below. Note that when $B_0 = 0$, it is always the case that $\partial L_T / \partial \tau < 0$ around free trade. In the extreme case with $\rho = 0$ (Leontief utility over tradables and non-tradables), the necessary and sufficient condition for $\partial L_T / \partial \tau > 0$ around free trade is $\mathcal{B} > EX$ (or, equivalently, $B_0 > C_H^*$). As we discuss in the calibration Section 3.3, large external liabilities of the U.S. are still quite a bit smaller than exports when converted in flow-value terms (7% vs 12% of GDP). When $\rho > 0$, a sufficient condition for $\partial L_T / \partial \tau < 0$ is weaker than $\mathcal{B} \leq EX$.

Proof: Combining (A10) and (A11) to eliminate $p = P_H / P_N$, we obtain a condition that determines equilibrium L_T given C_H^*:

$$\frac{F_N'(L - L_T)}{F_T'(L_T)} = \frac{(1-\gamma)^{\frac{1}{\theta}}}{\kappa} \frac{Y_N^{1/\rho}}{(Y - C_H^*)^{1/\theta}} \left[(1-\gamma)^{\frac{1}{\theta}} (Y - C_H^*)^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}\left(\frac{1}{\theta} - \frac{1}{\rho}\right)}, \tag{A13}$$

where $C_F = g^{-1}(C_H^*; B_0, B_0^*)$ is given by (A9) and increasing in C_H^*.

The left-hand side is increasing in L_T. The right-hand side is decreasing in L_T. To establish this, note that the right-hand side increases in $Y_N = F_N(L - L_T)$ which is decreasing in L_T; it also decreases in $Y = F_T(L_T)$ which itself is increasing in L_T. This completes the proof. Note that, indeed, the right-hand side decreases in Y for all values of parameters.

Therefore, equilibrium L_T increases in C_H^* if and only if the right-hand side is increasing in C_H^* (taking into account its effect on C_F). We rewrite the relevant part of the right-hand side as:

$$\frac{1}{(Y - C_H^*)^{1/\rho}} \left[1 + \left(\frac{\gamma}{1-\gamma} \right)^{\frac{1}{\theta}} \left(\frac{B_0^* + (\gamma^*)^{1/\eta} \big[(C_H^*)^{\frac{\eta-1}{\eta}} - B_0 / (C_H^*)^{1/\eta} \big]}{Y - C_H^*} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}\left(\frac{1}{\theta} - \frac{1}{\rho}\right)},$$

where we used (A9) to substitute for C_F. The first term increases in C_H^*. The term inside the square parenthesis also increases in C_H^*. Therefore, the whole right-hand side increases in C_H^* when $\rho \geq \theta$.

Consider next the case with $\rho < \theta$. Then we calculate the derivative of the log of the expression above, which can be written after simplification as:

$$\frac{\partial \log(\cdot)}{\partial C_H^*/(Y - C_H^*)} = \left[\frac{\Lambda}{\rho} + \frac{1-\Lambda}{\theta}\right] + \left(\frac{1}{\theta} - \frac{1}{\rho}\right)(1-\Lambda)\frac{C_H/\mathcal{S}}{C_F}\frac{\eta - 1 + B_0/C_H^*}{\eta},$$

where we substituted $C_H = Y - C_H^*$ and $\mathcal{S} = (C_H^*/\gamma^*)^{1/\eta}$ and defined

$$\Lambda \equiv \frac{1}{1 + \left(\frac{\gamma}{1-\gamma}\right)^{1/\theta}\left(\frac{C_F}{Y - C_H^*}\right)^{1/\theta}} = \frac{(1-\gamma)^{1/\theta} C_H^{\frac{\theta-1}{\theta}}}{(1-\gamma)^{1/\theta} C_H^{\frac{\theta-1}{\theta}} + \gamma^{1/\theta} C_F^{\frac{\theta-1}{\theta}}} = \frac{P_H C_H}{P_H C_H + P_F C_F},$$

where the second equality uses $C_H = Y - C_H^*$ and the third equality is the result of consumer optimality $u_F/u_H = P_F/P_H$. Next we note that:

$$(1-\Lambda)\frac{C_H/\mathcal{S}}{C_F} = \frac{\tau P_H C_H}{P_H C_H + P_F C_F} = \tau\Lambda.$$

where we used the last expression for Λ and the facts that $\mathcal{S} = P_F^*/P_H^*$, $P_H^* = P_H$ and $P_F = \tau P_F^*$ when $\tau^E = 1$ and $\tau^I = \tau$. Therefore, for $\rho < \theta$, the right-hand side of (A13) increases in C_H^* if and only if:

$$\left[\frac{\Lambda}{\rho} + \frac{1-\Lambda}{\theta}\right] + \left(\frac{1}{\theta} - \frac{1}{\rho}\right)\tau\Lambda\frac{\eta - 1 + B_0/C_H^*}{\eta} > 0,$$

or equivalently:

$$\theta > (\theta - \rho)\left[(1-\Lambda) + \Lambda\frac{\eta - 1 + B_0/C_H^*}{\eta/\tau}\right],$$

or equivalently:

$$\rho + (\theta - \rho)\Lambda\frac{1 - B_0/C_H^* - \eta(1 - 1/\tau)}{\eta/\tau} > 0.$$

Using $B_0/C_H^* = P_H^* B_0/(P_H^* C_H^*) = \mathcal{B}/EX$ we obtain condition (A12) in the Lemma. Plugging in $\tau = 1$, the condition becomes

$$\rho + (\theta - \rho)\Lambda\frac{1 - B_0/C_H^*}{\eta} > 0,$$

and $B_0 < C_H^*$ (or $\mathcal{B} < EX$) is a weak sufficient condition for this inequality to hold. ■

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