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CMS NOTES de la SMC

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CMS NOTES

It's You, Taylor, You're the Problem, It's You

Cover Article

December 2024 (Vol. 56, No. 6)

Termeh Kousha

Executive Director, CMS

In the past year, the CMS has made significant progress in supporting and promoting mathematics in Canada, as well as Canadian mathematics on the international stage. Through the dedication of our volunteers, members, and staff, as well as the support of our partners and sponsors, the CMS has achieved numerous milestones. These include hosting semi-annual meetings, organizing three open competitions and two invitational competitions, supporting two international competitions, hosting a highly successful summer camp in Ottawa and providing support to numerous other camps across the country, among other accomplishments.

In spring 2024, the CMS invited 91 of Canada's top math students to participate in the Canadian Mathematical Olympiad (CMO). Following this, Girls' Math Team Canada was formed to compete in the 2024 European Girls' Mathematical Olympiad (EGMO), and Math Team Canada was assembled for the 2024 International Mathematics Olympiad (IMO). The CMS is incredibly proud of the Canadian students who represented our country at these events, earning a combined total of 7 silver medals, 2 bronze medals, and 1 honourable mention. The CMS sincerely appreciates the support of Jane Street, which made it possible for Math Team Canada to participate in these prestigious international math competitions.

At the end of May 2024, the CMS traveled to Saskatoon (SK) for the 2024 CMS Summer Meeting. There, attendees were offered 19 scientific sessions, three mini-courses, three plenary lectures, three prize lectures, and a Quantum Computing Summer School. The CMS is grateful to the Scientific Directors, Elana Kalashnikov (University of Waterloo), Steven Rayan (University of Saskatchewan) and Jacek Szmigielski (University of Saskatchewan), for the success of this meeting.

In the summer, the CMS hosted its second annual Summer Math Camp in Ottawa, which generated a lot of excitement and quickly filled to capacity, even maintaining a full waitlist. Participants explored fun topics such as number theory, mathematical modeling, geometry, and fractals, among others. The camp's success has inspired plans for an expanded event in summer 2025, with more participants and additional weeks of programming. The CMS also plans to extend the camp to other Canadian cities beyond Ottawa in the near future.

The CMS was also proud to be able to sponsor various other camps in Canada, in cities such as Calgary, Surrey, Burnaby, Abbotsford, Fredericton, Moncton, St. John's, Halifax, Antigonish, Ottawa, London, Toronto, Sarnia, Sherbrooke, Québec, and Whitehorse.

In the fall, the CMS hosted its three annual open competitions: the Canada Lynx Mathematical Competition (CLMC), the Canadian Open Mathematics Challenge (COMC), and the Canada Jay Mathematical Competition (CJMC). Together, these three competitions attracted a record-breaking total of nearly 15,000 students worldwide, including a 34% increase in registrations for the CLMC.

More recently, building on the success of its inaugural event last year, the CMS was thrilled to host the CMS MathEd (Online) Meeting in late November. This meeting featured plenary lectures by Brent Davis (University of Calgary) and Robert Dawson (Saint Mary's University), along with a variety of engaging talks and presentations on diverse topics in mathematics education.

As I am writing these lines, the 2024 CMS Winter Meeting in Vancouver (BC) has just wrapped up. Though we encountered a few hiccups — including schedule conflicts, strikes, shipping delays, and the unexpected chaos brought on by the Taylor Swift concerts in Vancouver that same weekend, which forced us to change the dates and location of the meeting and even consider canceling it altogether — the meeting was a success. The CMS welcomed 600 attendees (the highest attendance since the start of the COVID-19 pandemic!) at the beautiful Richmond campus of the Kwantlen Polytechnic University, and the Sheraton Vancouver Airport Hotel. The Scientific Directors, Julia Gordon (UBC), Melissa Huggan (VIU) and Weiran Sun (SFU), organized a wonderful program containing 36 scientific sessions, four education sessions, three mini-courses, three plenary lectures and three prize lectures. The CMS was also very pleased to welcome some junior attendees, made possible by the childcare service provided during the meeting. In the end, despite the challenges brought on by the Taylor Swift concerts, the CMS is thrilled that the meeting not only happened but flourished thanks to the collaboration and determination of everyone involved. The CMS is very grateful for the support and participation of all attendees, organizers, speakers, volunteers, and staff.

This conference also featured a townhall with NSERC, offering members of the Canadian mathematical community valuable insights into NSERC's current funding guidelines. Furthermore, the revival of the Math Chair Meeting brought together numerous chairs from mathematics departments across Canada. This gathering provided an opportunity for meaningful discussions, networking, and collaboration. Building on its success, the CMS plans to host this meeting again at the 2025 CMS Winter Meeting in Toronto.

This year, the CMS also initiated the development of a new publication, Canadian Mathematical Communications (CMC), an open access journal. This publication format will enable the CMS to make cutting-edge mathematical research accessible to a wider audience, and more quickly. The CMS plans to officially launch a first call for submissions in early 2025.

None of this would have been possible without the outstanding dedication and support of the incredible staff at the CMS Executive Office. Julia, Maria, Minh, Sarah, Steve, Trevor, and Yvette, your tireless efforts and commitment to excellence have been instrumental in making all of this happen. I am deeply grateful for your hard work, attention to detail, and enthusiasm. Working alongside such hard-working and passionate people is a privilege, and I feel lucky to have you as my colleagues. Thank you for all that you do to ensure the success of the CMS.

Over the past couple of weeks, I have been truly overwhelmed by the incredible support I have received from the members of the Board of Directors and the math community. Your encouragement, kind words, and appreciation for my work have deeply motivated me and strengthened my dedication to this role. I would like to express my deepest gratitude to my dedicated colleagues, board members and volunteers included, who have shown unwavering support, insightful advice, and ongoing encouragements. The CMS is also very fortunate to count on the work and commitment of the volunteers who form the Executive Committee, the Board of Directors, and all of the Committees and Editorial Boards of the CMS. Your efforts do not go unnoticed and are sincerely appreciated.

Finally, the CMS is fortunate to count on the support of various math institutes, including AARMS, BIRS, CRM, Fields and PIMS; of all of its sponsors and partners, including Jane Street, Intact Insurance, The Actuarial Foundation of Canada, BC Hydro, Crowdmark, Empire Life, Honda Canada, MapleSoft, The McLean Foundation, S. M. Blair Family Foundation, Samuel Beatty Fund, QuanTA, SandboxAQ, Panago Pizza (Old Ottawa South), Giant Tiger (Merivale), the city of Saskatoon, and Pacific Authentic (Richmond, BC); of all its government partners, including Nova Scotia, Manitoba, New Brunswick, Ontario and Yukon; of its university partners, including ASDAN, Dalhousie University, McMaster University, University of British Columbia, University of Calgary, University of Manitoba, University of New Brunswick, University of Ottawa, University of Prince Edward Island, University of Regina, the Department of Mathematics & Statistics at the University of Saskatchewan, University of Toronto, University of Toronto Schools, the Faculty of Mathematics at the University of Waterloo, the Centre for Education in Mathematics and Computing, the Department of Science at York University, and Kwantlen Polytechnic University – Richmond campus. Last but certainly not least, the CMS relies on the generosity of benefactors and community members who donate to the Society. The CMS deeply appreciates your support and looks forward to continuing its collaboration with you.

In conclusion, I am incredibly proud of what the CMS accomplished in 2024, and I look forward with enthusiasm and optimism to seeing the CMS build on this momentum and embrace the opportunities that 2025 will bring.

Termeh Kousha
Executive Director, CMS

Au cours de la dernière année, la SMC a réalisé d'importants progrès dans le soutien et la promotion des mathématiques au Canada, ainsi que des mathématiques canadiennes sur la scène internationale. Grâce au dévouement de nos bénévoles, de nos membres et de notre personnel, ainsi qu'au soutien de nos partenaires et de nos commanditaires, nous avons franchi de nombreuses étapes. Nous avons notamment organisé deux réunions semestrielles, trois concours ouverts et un concours sur invitation, soutenu deux concours internationaux, organisé un camp d'été très réussi à Ottawa et apporté notre soutien à de nombreux autres camps à travers le pays, parmi d'autres réalisations.

Au printemps 2024, le SMC a invité 91 des meilleurs élèves en mathématiques du Canada à participer à l'Olympiade mathématique du Canada (OMC). Par la suite, l'équipe de mathématiques des filles du Canada a été formée pour participer à l'Olympiade européenne de mathématiques des filles (OEMF) de 2024, et l'équipe de mathématiques du Canada a été constituée en vue de l'Olympiade internationale de mathématiques (OIM) de 2024. Nous sommes incroyablement fiers des élèves canadiens qui ont représenté notre pays lors de ces événements, remportant un total combiné de 7 médailles d'argent, 2 médailles de bronze et 1 mention honorable. La SMC apprécie sincèrement le soutien de Jane Street, qui a rendu possible la participation de l'Équipe de mathématiques du Canada à ces prestigieuses compétitions internationales de mathématiques.

À la fin du mois de mai 2024, nous nous sommes rendus à Saskatoon (SK) pour la réunion d'été 2024 de la SMC. Là-bas, les participants se sont vu proposer 19 sessions scientifiques, trois mini-cours, trois conférences plénières, trois conférences de prix et une école d'été d'informatique quantique. Nous remercions les directeurs scientifiques, Elana Kalashnikov (Université de Waterloo), Steven Rayan (Université de Saskatchewan) et Jacek Szmigielski (Université de Saskatchewan), pour le succès de cette réunion.

Au cours de l'été, la SMC a organisé son deuxième camp d'été annuel de mathématiques à Ottawa, qui a suscité beaucoup d'enthousiasme et s'est rapidement rempli, avec même une liste d'attente complète. Les participants ont exploré des sujets amusants tels que la théorie des nombres, la modélisation mathématique, la géométrie et les fractales, entre autres. Le succès du camp a inspiré des plans pour un événement élargi à l'été 2025, avec plus de participants et des semaines supplémentaires de programmation. Nous prévoyons également d'étendre le camp à d'autres villes canadiennes qu'Ottawa dans un avenir proche.

La SMC est également fière d'avoir pu parrainer divers autres camps au Canada, dans des villes telles que Calgary, Surrey, Burnaby, Abbotsford, Fredericton, Moncton, St. John's, Halifax, Antigonish, Ottawa, London, Toronto, Sarnia, Sherbrooke, Québec et Whitehorse.

À l'automne, nous avons organisé nos trois concours ouverts annuels : le Concours mathématique du lynx du Canada (CMLC), le Défi ouvert canadien de mathématiques (DOCM) et le Concours mathématique du mésangeai du Canada (CMMC). Ensemble, ces trois concours ont attiré un nombre record de près de 15 000 élèves dans le monde entier, avec notamment une augmentation de 34 % des inscriptions au CMLC.

Plus récemment, après la réussite de l'événement inaugural de l'année dernière, nous avons eu le plaisir d'accueillir la réunion MathEd (en ligne) de la SMC à la fin du mois de novembre. Cette réunion comprenait des conférences plénières de Brent Davis (Université de Calgary) et de Robert Dawson (Université Saint Mary's), ainsi qu'une série d'exposés et de présentations sur divers sujets liés à l'enseignement des mathématiques.

Au moment où j'écris ces lignes, la réunion d'hiver 2024 de la SMC à Vancouver (BC) vient de se conclure. Bien que nous ayons rencontré quelques difficultés – notamment des conflits d'horaire, des grèves, des retards d'expédition et le chaos inattendu provoqué par les concerts de Taylor Swift à Vancouver le même week-end, qui nous ont obligés à changer les dates et le lieu de la réunion et même à envisager de l'annuler complètement – la réunion a été un succès. Nous avons accueilli 600 participants (la plus forte participation depuis le début de la pandémie de COVID-19 !) sur le magnifique campus de Richmond de l'université polytechnique Kwantlen et à l'hôtel Sheraton Vancouver Airport. Les directrices scientifiques, Julia Gordon (UBC), Melissa Huggan (VIU) et Weiran Sun (SFU), ont organisé un merveilleux programme comprenant 36 sessions scientifiques, quatre sessions d'éducation, trois mini-cours, trois conférences plénières et trois conférences de prix. Nous avons également eu le plaisir d'accueillir quelques participants juniors, grâce au service de garde d'enfants proposé pendant la réunion. En fin de compte, malgré les défis posés par les concerts de Taylor Swift, nous sommes ravis que la réunion n'ait pas seulement eu lieu, mais qu'elle ait prospéré grâce à

la collaboration et à la détermination de toutes les personnes impliquées. La SMC est très reconnaissante du soutien et de la participation de tous les participants, organisateurs, orateurs, bénévoles et membres du personnel.

Cette conférence a également été l'occasion d'une réunion publique avec le CRSNG, qui a permis aux membres de la communauté mathématique canadienne d'obtenir des informations précieuses sur les lignes directrices actuelles du CRSNG en matière de financement. De plus, la renaissance de la réunion des chaires de mathématiques a rassemblé de nombreux directeurs de départements de mathématiques à travers le Canada. Cette réunion a été l'occasion d'échanges fructueux, de réseautage et de collaboration. En raison de son succès, nous prévoyons d'organiser à nouveau cette réunion lors de la réunion d'hiver de la SMC de 2025 à Toronto.

Cette année, la SMC a également amorcé le développement d'une nouvelle publication, Communications mathématiques canadiennes (CMC), une revue à libre accès. Ce format de publication permettra à la SMC de rendre la recherche mathématique de pointe accessible à un public plus large, et plus rapidement. Nous prévoyons de lancer officiellement un premier appel à soumission au début de l'année 2025.

Rien de tout cela n'aurait été possible sans le dévouement et le soutien exceptionnels de l'incroyable personnel du bureau exécutif de la SMC. Julia, Maria, Minh, Sarah, Steve, Trevor et Yvette, vos efforts inlassables et votre engagement en faveur de l'excellence ont joué un rôle déterminant dans la réalisation de tout cela. Je vous suis profondément reconnaissante pour votre travail acharné, votre souci du détail et votre enthousiasme. Travailler aux côtés de gens aussi travailleurs et passionnés est un privilège, et je me sens chanceuse de vous compter parmi mes collègues. Merci pour tout ce que vous faites pour assurer le succès de la SMC.

Au cours des deux dernières semaines, j'ai été véritablement bouleversée par l'incroyable soutien que j'ai reçu de la part des membres du conseil d'administration et de la communauté mathématique. Vos encouragements, vos mots gentils et votre appréciation de mon travail m'ont profondément motivée et ont renforcé mon dévouement à ce rôle. Je tiens à exprimer ma profonde gratitude à de nombreux collègues dévoués, y compris les membres du conseil d'administration et les bénévoles, qui ont fait preuve d'un soutien indéfectible, de conseils judicieux et d'encouragements constants. La SMC a également la chance de pouvoir compter sur le travail et l'engagement des bénévoles qui forment le Comité exécutif, le Conseil d'administration et tous les comités et comités de rédaction de la SMC. Vos efforts ne passent pas inaperçus et sont sincèrement appréciés.

Enfin, la SMC a la chance de compter sur l'appui de divers instituts de mathématiques, dont AARMS, BIRS, CRM, Fields et PIMS ; de tous ses commanditaires et partenaires, dont Jane Street, Intact Insurance, la Fondation actuarielle du Canada, BC Hydro, Crowdmark, Empire Life, Honda Canada, MapleSoft, la Fondation McLean, la Fondation de la famille S. M. Blair, Samuel Beatty Fund, QuanTA, SandboxAQ, Panago Pizza (Old Ottawa South), Giant Tiger (Merivale), la ville de Saskatoon, Pacific Authentic (Richmond, BC) ; de tous ses partenaires gouvernementaux, y compris la Nouvelle-Écosse, le Manitoba, le Nouveau-Brunswick, l'Ontario et le Yukon ; de ses partenaires universitaires, notamment l'ASDAN, l'université Dalhousie, l'université McMaster, l'université de Colombie-Britannique, l'université de Calgary, l'université du Manitoba, l'université du Nouveau-Brunswick, l'université d'Ottawa, l'université de l'Île-du-Prince-Édouard, l'université de Regina, le département de mathématiques et de statistiques de l'université de Saskatchewan, l'université de Toronto, les écoles de l'université de Toronto, la faculté de mathématiques de l'université de Waterloo, le centre d'enseignement des mathématiques et de l'informatique, le département des sciences de l'université de York, et l'université polytechnique Kwantlen – campus de Richmond. Finalement, la SMC dépend de la générosité des bienfaiteurs et des membres de la communauté qui font des dons à la Société. Nous apprécions profondément votre soutien et nous avons hâte de poursuivre notre collaboration avec vous.

En conclusion, je suis incroyablement fière de ce que la SMC a accompli en 2024, et j'attends avec enthousiasme et optimisme de voir la SMC tirer parti de cet élan et saisir les opportunités que 2025 apportera.

Société mathématique du Canada — 616 Cooper St., Ottawa (ON) K1R 5J2, Canada

Robert Dawson (Saint Mary's University)
Editor, CMS Notes

You are probably familiar with Sylvanus Thompson's story about Lord Kelvin telling his physics class that Liouville was a ***mathematician***, that is to say, one to whom $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ was as obvious as 2+2 would be to them. If you are also familiar with the devilishly clever trick with polar coordinates by which this integral is customarily evaluated, you will see that Kelvin was clearly using "obvious" in an after-the-fact sense: this is a fact that ***becomes*** obvious after exposition and a little thought. I fancy that not even the great Liouville would have "on-sighted" that problem! But perhaps Kelvin's use of the word is the more interesting one, applying as it does to more of a mathematician's interaction with the theorem in question.

I was at a very interesting colloquium talk a few weeks ago. The speaker, a member of our department, needed the Quadratic Reciprocity Theorem for something, and commented that while it was certainly true, and had been proved in a multitude of ways, none of them made the theorem obvious. I was glad to hear that — all these years, since I'd first met the QRT at Cambridge, I'd thought it was just me! But it got me thinking about what is and isn't obvious.

Sometimes it's just a matter of waiting for the right argument. The Sylvester-Gallai theorem, about configurations of lines and points in the Euclidean plane, was just "Sylvester's Problem" for almost fifty years after he posed it in 1893. In the middle of the twentieth century, proofs began to appear, culminating in Kelly's mininum-distance proof, which renders it truly obvious.

Some things in mathematics seem more obvious, on first acquaintance, than they really are. The four-color theorem seems obvious to many after a half-hour's doodling, but there's still no human-comprehensible proof. There are hard theorems about infinite sets whose finite counterparts are trivial. And what could be more obvious than the Jordan Curve Theorem, stating that every simple closed curve has an inside and an outside? But it's very, very difficult to prove, to the point that practically every undergraduate textbook that needs it punts on the proof. Why this discrepancy? I think it's because when you throw in a few extra "nice" properties the proof becomes very simple… and your imagination (or at least my imagination) tends to throw those nice properties in for free when you set yourself to "imagine a simple closed curve." Conversely, if we go up a few dimensions, n-spheres develop weird properties that are hard for our three-dimensional intuitions to imagine.

Then there are the different forms of the axiom of choice. What could be more obvious than that the product of a collection of nonempty sets is nonempty? But this is logically equivalent to the very nonobvious theorem of Tychonoff, and implies the bizarre Banach-Tarski dissection paradox, surely obvious to nobody.

Clearly we mean and understand something by "obvious" in mathematics… but it's not alway obvious what!

Tout simplement pas évident

Éditorial

Décembre 2024 (tome 56, no. 6)

Robert Dawson (Saint Mary's University)
Editor, CMS Notes

Vous connaissez sans doute l'histoire de Sylvanus Thompson selon laquelle Lord Kelvin aurait dit à sa classe de physique que Liouville était un ***mathématicien***, à savoir un mathématicien pour qui $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$ était aussi évident que 2+2 l'était pour eux. Si vous connaissez également l'astuce diabolique avec les coordonnées polaires par laquelle cette intégrale est habituellement évaluée, vous verrez que Kelvin utilisait clairement « évident » dans un sens a posteriori : c'est un fait qui ***devient*** évident après un exposé et un peu de réflexion. J'imagine que même le grand Liouville n'aurait pas « vu » ce problème ! Mais c'est peut-être l'usage que Kelvin fait de ce mot qui est le plus intéressant, car il s'applique davantage à l'interaction d'un mathématicien avec le théorème en question.

Il y a quelques semaines, j'ai assisté à un colloque très intéressant. L'orateur, un membre de notre département, avait besoin du théorème de la réciprocité quadratique pour quelque chose, et a fait remarquer que bien qu'il soit certainement vrai, et qu'il ait été prouvé d'une multitude de façons, aucune d'entre elles ne rendait le théorème évident. J'étais heureux d'entendre cela – toutes ces années, depuis que j'ai rencontré le TRQ pour la première fois à Cambridge, j'ai pensé que c'était juste moi ! Mais cela m'a fait réfléchir à ce qui est évident et à ce qui ne l'est pas.

Parfois, il suffit d'attendre le bon argument. Le théorème de Sylvester-Gallai, qui concerne les configurations de lignes et de points dans le plan euclidien, est resté le « problème de Sylvester » pendant près de cinquante ans après qu'il a été posé en 1893. Au milieu du vingtième siècle, des preuves ont commencé à apparaître, culminant avec la preuve de la distance minimale de Kelly, qui rend le théorème vraiment évident.

Certaines choses en mathématiques semblent plus évidentes, au premier abord, qu'elles ne le sont réellement. Le théorème des quatre couleurs semble évident pour beaucoup après une demi-heure de gribouillage, mais il n'existe toujours pas de preuve compréhensible par les humains. Il existe des théorèmes difficiles sur les ensembles infinis dont les équivalents finis sont triviaux. Et quoi de plus évident que le théorème de la courbe de Jordan, qui affirme que toute courbe fermée simple possède un intérieur et un extérieur ? Mais il est très, très difficile à prouver, au point que pratiquement tous les manuels de premier cycle qui en ont besoin s'abstiennent d'en donner la preuve. Pourquoi cette divergence ? Je pense que c'est parce que lorsque vous ajoutez quelques « belles » propriétés supplémentaires, la preuve devient très simple… et votre imagination (ou du moins mon imagination) a tendance à ajouter ces belles propriétés gratuitement lorsque vous vous mettez en tête d'« imaginer une courbe fermée simple ». Inversement, si l'on monte de quelques dimensions, les n-sphères développent des propriétés bizarres que nos intuitions tridimensionnelles ont du mal à imaginer.

Il y a ensuite les différentes formes de l'axiome du choix. Quoi de plus évident que le produit d'une collection d'ensembles non vides est non vide ? Mais cela est logiquement équivalent au théorème très peu évident de Tychonoff, et implique le bizarre paradoxe de la dissection de Banach-Tarski, qui n'est sûrement évident pour personne.

Il est clair que nous entendons et comprenons quelque chose par « évident » en mathématiques… mais il n'est pas toujours évident de savoir quoi !

Société mathématique du Canada — 616 Cooper St., Ottawa (ON) K1R 5J2, Canada

Education Notes December 2024 (Vol. 56, No. 6)

This is the last month in which John Grant McLoughlin will be co-editing the *Education Notes*. John has been filling this role with different co-editors for fifteen years. That's a long time in any discipline, and in some ways, maybe more so in math education than in mathematics itself. Most results in mathematics are definite: they may not be the last word, but each level is usually strong enough to support the next. Math education works with the "gnarled timber of humanity" and one decade's new idea may be another decade's deprecated approach or failed experiment. This, of course, makes timely reporting even more important! For many decades, the column has kept us up to date with a huge range of useful and thought-provoking articles on various aspects of math education, most recently under John's guidance. Thanks, John! Your work has been appreciated.

In the January issue, Egan Chernoff will begin working with Kseniya Garaschuk on the column. Welcome, Egan!

Sharing Mathematics: An Unfinished Tapestry

Education Notes December 2024 (Vol. 56, No. 6)

John Grant McLoughlin (University of New Brunswick)

Education Notes bring mathematical and educational ideas forth to the CMS readership in a manner that promotes discussion of relevant topics including research, activities, issues, and noteworthy news items. Comments, suggestions, and submissions are welcome.

John Grant McLoughlin, *University of New Brunswick (johngm@unb.ca)*
Kseniya Garaschuk, *University of the Fraser Valley (kseniya.garaschuk@ufv.ca)*

It was the summer of 2009 when I agreed to take on the editorial role for *Education Notes* beginning in 2010. Ed Barbeau had been a mentor to me particularly with respect to problem solving, a relationship that began as a graduate student of Ed's in the *Master's of Science in Teaching* at University of Toronto in the late 1980's. Ed's strong support and recommendation in succeeding him as an editor was appreciated. The opportunity would build upon two of my interests, namely, writing and networking. Meanwhile, my academic career continues to navigate that teetering span between mathematics and mathematics education.

My familiarity with the Canadian mathematical community at large would be valuable in drawing forth potential authors of pieces for *Education Notes*. However, I felt that two perspectives would be better than one and gave thought to the idea of a co-editor. Fifteen years and 90 issues of *CMS Notes* later, I write this piece in my final issue as a co-editor. Both Kseniya Garaschuk (2018 – present) and Jennifer Hyndman (2010 – 2017) have been delightful to work with as co-editors. Mathematicians are people first and foremost. Life goes on around us and through losses and challenges, whether health of family members or difficult times, the sharing of responsibilities and the give and take of picking up the reins have served us well. As a faculty member primarily in education with a keen interest in mathematics, it has been beneficial to collaborate with Kseniya and Jennifer as faculty in mathematics with keen interests in education. The blends of local knowledge, awareness of activities, along with generally being able to share writing and editorial responsibilities as needed, have enriched the experience. Thank you Kseniya and Jennifer!

Community Connection

Reflecting upon years in the role it is community that comes to the forefront. Whether through conversations, written exchanges, or personal encounters, it is about building relationships. *Education Notes* offers a window into the works of others in that broader mathematical circle. Many contributions have grown out of authors sending articles about endeavours that appear as one-off issues. Yet collectively these single pieces add to a picture of the Canadian mathematical community with glimpses into other places. Various articles have responded to calls on themes such as outreach or invitations to individuals to write about projects. *Education Notes* has also provided a forum for initiatives such as *First Year Math and Stats in Canada (FYMSIC)* or specific education sessions or working groups to share ideas, summaries, or invitations for participation. The range of contributions and contributors adds to the richness.

People's stories fascinate me. My finest honour as a mathematician came through the CMS during this editorial period. I was named the 2013 recipient of the *Adrien Pouliot Award*. The recognition reinforced a personal commitment to outreach that extended into the work around the *Notes*. The title of my prize lecture, *An Unfinished Tapestry,* was inspired by reviewing the list of prior recipients. All but two of these people had played pivotal roles along my mathematical path whether as collaborators, mentors, teachers, colleagues, or some blend of these. The initial recipient, Ed Barbeau, was mentioned already. My respect for the second recipient, Bruce Shawyer, was articulated in the February 2022 *CMS Notes* piece https://notes.math.ca/en/article/remembering-bruce-shawyer-mathematician-musician-educator-gardener-loving-husband-father-and-grandfather/. I could go on. Rather I will go back to 2009 to share a story that depicts the intertwining of paths and impacts. The connection to *Education Notes* will become apparent.

Sharing Mathematics

Spring 2009 had featured a personally significant mathematical event as a co-organizer (with Rick Brewster and Fae DeBeck) of *Sharing Mathematics: A Tribute to Jim Totten*, a conference held in Kamloops to honour Jim's legacy. Three plenary talks along with a range of presentations and social time figured into this event. The plenary talks addressed mathematical problem solving, teaching and outreach respectively. The complete proceedings including the plenary talks, numerous presentations, and anecdotes about Jim are accessible at https://unbscholar.lib.unb.ca/handle/1882/22500.

This event took place in May 2009. It was shortly after that I agreed to edit *Education Notes* and when thoughts went to a possible co-editor, the reacquainting with

Jennifer Hyndman at that gathering led me to invite Jennifer to be a co-editor. About ten years earlier we had been on the CMS Education Committee together and in the early 80's undergraduates at Waterloo. Paths cross in curious ways. We collaborated on *Education Notes* for eight years. The initial issue under our editorship (Volume 42, Issue 1) featured Harley Weston's plenary talk on outreach entitled *Some New Initiatives at Math Central*. Jennifer's fascinating plenary talk, *Hands-free Teaching*, had appeared shortly beforehand in October 2009 (Volume 41, Issue 6).

My initial collaborations with Jim Totten came through work on a BC math contest while I was at the former Okanagan University College in the mid-1990's. However, it was through the CMS that we collaborated closely as Jim succeeded the forementioned Bruce Shawyer as the editor-in-chief of *Crux Mathematicorum with Mayhem*. The torch was passed along via Vazz Linek and Shawn Godin before landing in the hands of Kseniya Garaschuk who has edited *Crux Mathematicorum* since 2014. As a long-time *Crux* editorial board member and more recently as Associate Editor of *Mathemattic*, the secondary school-oriented portion of the journal, I have worked closely with Kseniya over many years. Kseniya graciously accepted an invitation to take over from Jennifer as a co-editor of *Education Notes*. That takes us to today and the connection with *Sharing Mathematics* holds another curious tie.

I take this opportunity to welcome the incoming editor who will take my place and work alongside Kseniya in the new year. Responses to a request for stories about Jim for the proceedings included several contributions from people not present at the event. One such contribution came from a person who knew Jim from the community and the classroom as both a student of Jim's and a fellow golfer, and that person is Egan Chernoff.

Going forward: A call to reach out

One of the foci during my tenure has been to highlight mathematical outreach. It is in that spirit of reaching out that *Education Notes* can effectively facilitate heightened awareness of mathematical activities. As far as outreach goes, I'll sign off with a request to all readers to tap a broader audience by encouraging a colleague down the hall, your child's math teacher, or perhaps a mathematically curious friend to drop in online and read a few issues of *Education Notes*. If you have not done so, reach out and take a look at the *ATOM* (*A Taste of Mathematics*) series https://cms.math.ca/publications/atom-booklets/ with a range of enriching mathematical ideas. I draw attention to the most recent addition (*Volume XVII*), namely, *Mathematical Logic Puzzles on a Grid* authored by Susan Milner. The fully downloadable collection has been enjoyed by elementary and high school teachers as well as so-inclined armchair mathematical folk of many ages. While *Crux Mathematicorum* is an international problem-solving journal, the section *Mathemattic* offers challenging problems and articles well suited to the development of youthful mathematical minds. Many problem solvers will find one of the levels appealing to consider. The CMS is working to enhance outreach in mathematics and education. Your support in the reaching out process is welcomed.

Gratitude

Thank you all for the privilege of serving in an editorial role with *Education Notes* for the past 15 years. I appreciate the many helping hands at CMS over the years, particularly Sarah Watson, who has facilitated the recent transitions and directions. Kseniya, thank you so much for working together these past seven years and staying aboard to facilitate the transition. Egan, I wish you well in the role and look forward to seeing how the shape of *Education Notes* unfolds in the future. Thanks everyone.

– John Grant McLoughlin

The Surprising Role of the Pendulum in Stokes's Discovery of New Physical Theory and Use of Divergent Series

CSHPM Notes December 2024 (Vol. 56, No. 6)

Brenda Davison (Simon Fraser University)

CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to the column's editors:
Amy Ackerberg-Hastings, *independent scholar (aackerbe@verizon.net)*
Nicolas Fillion, *Simon Fraser University (nfillion@sfu.ca)*

Consider a function, represented in its infinite series Taylor expansion. For example:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

It is possible to approximate the value of e^x for any value of x by using the first n terms of the series and then truncating and ignoring the rest. This procedure produces an approximation with which an error is associated.

In the case of this example, the infinite series for e^x converges for all values of x, resulting in an approximation with increasingly less error as n increases. However, depending on the function and the representative infinite series, the series may converge for only limited values of the argument x. Convergent series are well understood, with well-defined methods of bounding the error of the approximations made by using truncated infinite series.

Divergent series, on the other hand, are more difficult and more interesting. A canonical example of a function and its associated divergent infinite series representation is the Stirling series for the factorial function,

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left(1 + \frac{1}{12n} + \frac{1}{288n^2} - \cdots\right).$$

The infinite series on the right diverges for all values of n. However, a good approximation for $n!$ can be obtained by using a carefully selected number of terms of the divergent series. Furthermore, even though the series is divergent, it is possible to bound, but not to make arbitrarily small, the error in the approximation.

The Stirling series representation for $n!$ was formulated by Abraham De Moivre (1667–1754) and improved upon by James Stirling (1672–1770). The series was shown to be divergent by Thomas Bayes (ca 1702–1761) in a letter published in 1763. This type of series, when truncated after a given number of terms, is now called an asymptotic expansion of the function, or an asymptotic approximation to the function. Nearly all mathematicians by 1800 were familiar with this example, and it caused confusion and consternation along with appreciation for its obvious utility.

The utility of asymptotic approximations during the 18th and early 19th centuries—for efficient computation, in particular—could not be ignored, particularly in an era when computations were done by hand. It was, however, not clear at this time why these approximations worked so well, or what the ultimate divergence of the series meant for the approximation.

Figure 1. George Gabriel Stokes (1819–1903). *Wikipedia*.

During the mid-19th century, divergent series were successfully used by George Gabriel Stokes (Figure 1) for two purposes. First, divergent series allowed Stokes to generate numbers from existing theory and then, second, he used those numbers to verify new physical theory. In trying to understand how divergent series were used in the mid-19th century and how the numbers generated by using them were verified, I became aware of the importance of the role of the pendulum.

In the early 19th century, in Britain and elsewhere, it was important to determine as accurately as possible the period of the pendulum, largely because pendulum measurements were extensively used in surveying, in navigation, and in the determination of physical constants, including the gravitational constant, the ellipticity of the Earth, and the mean density of the Earth.

The level of precision required meant that pendulum theory needed to move well beyond the idealized pendulum shown in Figure 2, in which simplifying assumptions causes significant differences between the calculated period of a pendulum and the actual period of a pendulum as measured in a laboratory.

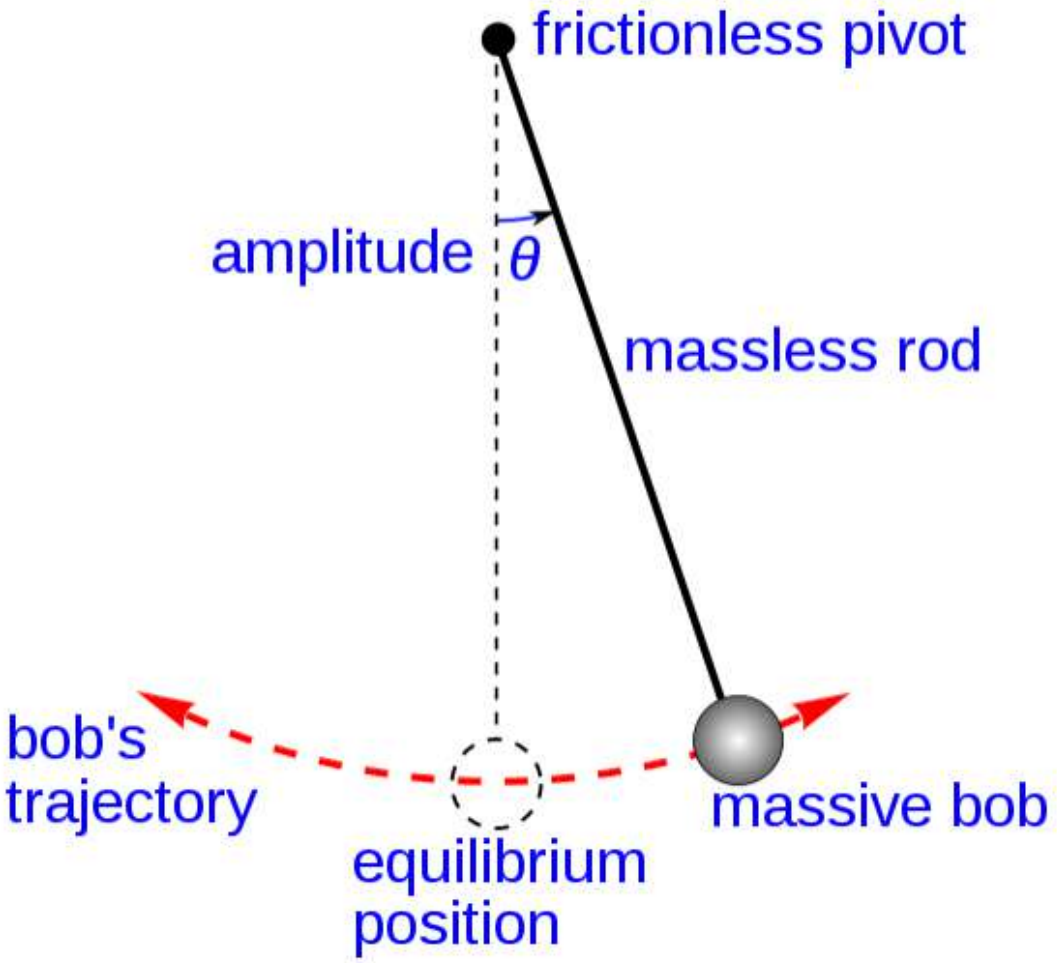

Figure 2. An idealized, mathematical model of the pendulum. *Wikipedia*.

An ideal pendulum consists of a pendulum bob swinging from a pivot on a rod. The pivot is considered to be frictionless, the rod is considered to be massless, and the pendulum bob is a massive point mass. Analyzing the forces on this idealized pendulum is a standard application of basic mechanics and yields an exact solution for the period of the pendulum in the form of an infinite series,

$$T = 2\pi\sqrt{\frac{L}{g}}\left(\sum_{n=0}^{\infty}\frac{(2n)!}{2^{2n}(n!)^2}\sin^{2n}\left(\frac{\theta_0}{2}\right)\right)$$

where the first few terms of the series are

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \dots\right).$$

If the assumption is made that the initial displacement angle, θ, is small, then the period of the pendulum is well approximated by the formula

$$T \approx 2\pi\sqrt{\frac{L}{g}}.$$

Error results from not taking into account the actual physical situation: e.g., there is friction at the pivot; the rod is not massless and its length may change with temperature or humidity; the pendulum bob is not a point mass and may not have uniform density, and it most certainly will be slowed by drag in the medium through which it is swung. In addition, temperature, altitude, and humidity change the drag characteristics of the medium through which the bob is swung.

A further source of error is the mathematical approximation made in order to make computation of the period possible. There is a loss of accuracy due to the truncation of the infinite series, and that error is a function of the initial displacement angle, which itself may be difficult to set repeatedly.

For the pendulum, as was typical practice, very precise empirical measurements were compared to predictions made from theory. Additionally, it was not only that theory was being validated by comparison to measured data. There was a further mathematical difficulty because it was a non-trivial task to produce numerical values from the theory for comparison in the first place, due to the prohibitively large number of calculations required.

The large number of computations resulted from the necessity of evaluating definite integrals that were not integrable in finite terms. Therefore, numerical results were being obtained via partial sums of convergent infinite series after integration of the power series of the integrand. However, the number of terms required to obtain the desired precision was too large for the human calculator.

By 1832, a large volume of very careful measurements of pendulum motion was available, and it was apparent to many natural philosophers that there was no theory available to explain the experimental results. These precise pendulum measurements led Stokes to new physical theory and, in order to generate the numbers from his theory, Stokes also developed asymptotic expansions in order to make possible the computations required.

These were both rather large breakthroughs. The first was the physical realization that there exists in fluid motion a previously unknown phenomenon—that which results in the boundary layer—which Stokes called the index of friction of the fluid. The second breakthrough was the ability to use asymptotic approximations to divergent series in order to obtain numbers from this new theory.

The new physical theory, in the case of the cylindrical bob, resulted in a convergent ascending series (meaning in increasing positive powers of the variable) involving the derivative of the gamma function. As noted above, the Stirling series, an asymptotic approximation to the gamma function, was stated long before Stokes's lifetime. It is possible that seeing the derivative of the gamma function caused Stokes to consider trying to find an analogous approximation method.

In the case of the cylinder, Stokes applied his new mathematical tool that consisted of converting the convergent series to a descending divergent series, from which he used the first few terms as an approximation. Stokes used the contemporaneous term "descending series" to refer to series where the power of the variable increases in the denominator as the terms progress. This simplified, in fact made possible, the numerical calculations. Stokes noted:

> The author has also obtained a descending series, which is much more convenient for numerical calculation when the diameter of the cylinder is large [1, p. 7].

In 1848 Stokes summarized the important formulae he had obtained, but he did not indicate how the formulae were obtained. He also provided three numerical calculations for three differently-sized cylindrical pendulum bobs. Stokes compared those calculations against previously-obtained standard theoretical results as well as with the experimental results. His new theory agreed much better with the experimental results than the earlier theoretical predictions, which did not account for internal friction, did. Here for the first time, as far as I know, Stokes announced his use of a divergent series for computation with respect to pendulums.

Pendulums were important for deducing results about physical phenomena and thus experimentalists such as Francis Baily (1774–1844) and Henry Kater (1777–1835) had spent a lot of time and energy in the early part of the 19th century on making accurate pendulum measurements, as Stokes was well aware:

> The great importance of the results obtained by means of the pendulum has induced philosophers to devote so much attention to the subject, and to perform the experiments with such a scrupulous regard to accuracy in every particular, that pendulum observations may be justly ranked among those most distinguished by modern exactness [3, p. 8].

A few pages later, as he prepared to discuss his investigations into the pendulum correction factors, Stokes described how the previous work had fallen short:

> The preceding [as I have summarized above] are all the investigations that have fallen under my notice, of which the object was to calculate from hydrodynamics the resistance to a body of given form oscillating as a pendulum. They all proceed on the ordinary equations of the motion of fluids. They all fail to account for one leading feature of the experimental results, namely, the increase of the factor n with a decrease in the dimensions of the body. They recognize no distinction between the action of different fluids, except what arises from their difference of density [3, p. 12].

None of the previous theories accounted for what is now termed viscosity, and it was evident from the experiments of several people that theory and experiment were not in agreement.

In contrast, Stokes produced a refined hydrodynamical theory that took into account viscosity, and that theory agreed with experimental evidence in the spherical pendulum case. No new mathematical techniques had been required for the computations thus far; those would be required for the cylindrical pendulum bobs.

Stokes then temporarily put aside the pendulum computations and attempted to compute zeros of the Airy integral. The theory for this was well-known—the difficulty lay entirely in being able to obtain numbers from the theory. After he devised a new mathematical technique that he used to make the Airy integral computations (which he verified against empirical data), Stokes returned to the pendulum computations in the case of the cylindrical bob and was able to use his new technique to obtain results:

> I found the method which I had employed in the case of this integral [the Airy integral] would apply to the problem of the resistance to a cylinder and it enabled me to get over the difficulty with which I had before beeen [*sic*] baffled. I immediately completed the numerical calculation so far as was requisite to compare the formulae with Baily's experiment on cylindrical rods, and found a remarkably close agreement between theory and observation [3, p. 13]

Stokes further verified his pendulum results by using an index of friction for water taken from the results of Coulomb's experiment on a spinning disk in water. (This value for the index of friction of water was not computed using a pendulum.) By using Coulomb's value for the index of friction to compute the pendulum vibration period of a pendulum swinging in water, Stokes found his theory agreed with experiments.

In addition to pendulum calculations, Stokes's discovery of the index of friction provided an explanation for the formation of clouds. In simplified terms, the explanation was as follows:

1. A sphere (water droplet) traveling uniformly in a fluid was considered as a limiting case of a ball pendulum as the length of the wire became arbitrarily large.
2. Stokes's theory showed that the resistance due to internal friction of a sphere moving through a fluid was proportional to the radius of the sphere rather than to the surface area of the sphere.
3. The index of friction for air was known from pendulum experiments.
4. The terminal velocity of the water droplets was calculated using the index of friction for air (the other sources of friction, proportional to the square of the velocity, were much less significant and were ignored) and was so small that the suspension of water droplets to form clouds was explained.

This was, perhaps, another verification of the new theory of the index of friction.

Stokes had developed a method, using divergent series, to obtain numbers from theory that were not previously practically computable, which he could then compare to something that was measurable in a laboratory. The pendulum had been a vital component in the advancement of theory and spurred the development of new mathematics. For further detail and contextualization, please see my recent thesis [1].

References

[1] Davison, Brenda. (2023) Divergent series and asymptotic expansions, 1850–1900. PhD diss., Simon Fraser University.

[2] Stokes, George Gabriel (1848) On the resistance of the air to pendulums. *Notices and Abstracts of Miscellaneous Communications to the Sections*, 7–8. Appended to *Report of the Eighteenth Meeting of the British Association for the Advancement of Science; Held at Swansea in August 1848*, London: John Murray, 1849.

[3] Stokes, George Gabriel. (1851) On the effect of the internal friction of fluids on the motion of pendulums. *Transactions of the Cambridge Philosophical Society* 9(2), 8–14.

Brenda Davison is a senior lecturer in mathematics at Simon Fraser University. Her academic interests include the history of mathematics and mathematics education; she is particularly interested in using the history of mathematics to enliven and inform the teaching of mathematics. Brenda obtained her PhD in pure mathematics in 2023 from Simon Fraser University with a dissertation titled "Divergent Series and Asymptotic Expansions, 1850–1900". Her avocations include reading literature, climbing, biking, and trail running.

Call for Nominations for the 2025 CMS Blair Spearman Doctoral Prize | Deadline January 31, 2025

The CMS Doctoral Prize Selection Committee invites nominations for the 2025 CMS Blair Spearman Doctoral Prize.

The prize is awarded to one recipient of a Ph.D. from a Canadian university whose overall performance in graduate school is judged to be the most outstanding. Although the dissertation will be the most important criterion (the impact of the results, the creativity of the work, the quality of exposition, etc.) it will not be the only one. Other publications, activities in support of students and other accomplishments will also be considered.

CMS aims to promote and celebrate diversity in the broadest sense. We strongly encourage department chairs and nominating committees to put forward nominations for outstanding candidates for research in the mathematical sciences regardless of race, gender, ethnicity or sexual orientation.

Candidates must be nominated by their university and the nominator is responsible for preparing the documentation described below, and submitting the nomination to the address below. The deadline for the receipt of nominations is indicated above.

The documentation shall consist of:

- A curriculum vitae prepared by the student.
- A resumé of the student's work written by the student and which must not exceed ten pages. The resumé should include a brief description of the thesis and why it is important, as well as of any other contributions made by the student while a doctoral student.
- Three letters of recommendation of which one should be from the thesis advisor and one from an external reviewer. A copy of the external examiner's report may be substituted for the latter. More than three letters of recommendation are not accepted.

All documentation, including letters of recommendation, should be submitted electronically, preferably in PDF format, by the deadline of **January 31, 2025**, to **docprize@cms.math.ca**.

For further details, please view the call here.

Appel de mises en candidature Prix de doctorat Blair-Spearman de la SMC 2025 | date limite 31 janvier

Le Comité de sélection du Prix de doctorat invite un appel de mises en candidatures pour le Prix de doctorat Blair-Spearman de la SMC 2025.

Le prix sera décerné à une personne qui aura reçu son diplôme de troisième cycle d'une université canadienne l'année précédant sa mise en candidature (entre le 1er janvier et le 31 décembre) et dont les résultats pour l'ensemble des études supérieures seront jugés les meilleurs. La dissertation constituera le principal critère de sélection (impact des résultats, créativité, qualité de l'exposition, etc.), mais ne sera pas le seul aspect évalué. On tiendra également compte des publications de l'étudiant.e, de son engagement dans la vie étudiante et de ses autres réalisations.

La SMC a pour but de promouvoir et de célébrer la diversité au sens le plus large. Nous encourageons fortement les directrices et les directeurs de départements et les comités de mise en candidature à proposer des candidat.e.s exceptionnel.le.s sans distinction de race, de genre, d'appartenance ethnique ou d'orientation sexuelle.

Les candidat.e.s doivent être nommé.e.s par leur université; la personne qui propose un.e candidat.e doit se charger de regrouper les documents décrits dans les paragraphes suivants et de faire parvenir la candidature à l'adresse ci-dessous. La date limite pour recevoir la candidature est indiqué ci-dessus.

Le dossier sera constitué des documents suivants :

- Un curriculum vitae rédigé par l'étudiant.e.
- Un résumé du travail du candidat.e d'au plus dix pages, rédigé par l'étudiant.e, où celui-ci décrira brièvement sa thèse et en expliquera l'importance, et énumérera toutes ses autres réalisations pendant ses études de doctorat.
- Trois lettres de recommandation, dont une du direct.rice.eur de thèse et une d'un.e examinat.rice.eur de l'extérieur (une copie de son rapport serait aussi acceptable). Le comité n'acceptera pas plus de trois lettres de recommandation.

Veuillez faire parvenir tous les documents par voie électronique, de préférence en format PDF, avant la date limite du **31 janvier 2025**, à **prixdoc@smc.math.ca**.

Pour de plus amples renseignements, veuillez consulter l'appel ici.

Société mathématique du Canada — 616 Cooper St., Ottawa (ON) K1R 5J2, Canada

Call for Nominations: Editor-in-Chief (EIC), Crux Mathematicorum

The CMS invites expressions of interest for the **Editor-In-Chief (EIC) of Crux**; one EIC is being solicited, with a five-year term scheduled to commence **January 1, 2026**. Funding from the CMS is available for this EIC position.

Since 1975, Crux Mathematicorum has published problems and solutions, aimed primarily at secondary and undergraduate students. First by subscription, and now as a free online publication, Crux provides a valuable resource to students and educators around the world. For more on the publication, see the website: https://cms.math.ca/publications/crux/

Expressions of interest should include a cover letter, your curriculum vitae, and an expression of views regarding the publication including any proposed changes or new directions in policy. Please also include an indication of support from your home institution.

Please submit your expression of interest electronically to communications@cms.math.ca before July 15, 2025.

To view the terms of reference for this position, please visit: https://cms.math.ca/about-the-cms/governance/terms-of-reference/

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La SMC invite les manifestations d'intérêt pour le poste de **rédacteur en chef (REC) de Crux** ; un REC est recherché, avec un mandat de cinq ans prévu pour débuter le **1er janvier 2026**. Un financement de la SMC est disponible pour ce poste de rédacteur en chef.

Depuis 1975, Crux Mathematicorum publie des problèmes et des solutions, destinés principalement aux étudiants du secondaire et du premier cycle. D'abord par abonnement, et maintenant en tant que publication en ligne gratuite, Crux fournit une ressource précieuse aux étudiants et aux éducateurs du monde entier. Pour en savoir plus sur la publication, consultez le site Web : https://smc.math.ca/publications/crux-fr/ Les manifestations d'intérêt doivent comprendre une lettre de motivation, votre curriculum vitae et une expression de votre point de vue sur la publication, y compris toute proposition de changement ou de nouvelle orientation des politiques. Veuillez également inclure une indication du soutien de votre institution d'origine.

Veuillez soumettre votre manifestation d'intérêt par voie électronique à communications@cms.math.ca avant le **15 juillet 2025.**

Pour consulter les termes de mandat de ce poste, veuillez vous rendre sur le site Web : https://smc.math.ca/apropos-de-la-smc/la-gouvernance/termes-de-mandat/

The Canadian Mathematical Society (CMS) welcomes and invites scientific session proposals for the 2025 CMS Summer Meeting in Quebec City, Quebec from June 6 to June 9, 2025.

- The purpose of the scientific sessions is to share cutting edge research on a given mathematical topic, as suggested by the organizers.
- Sessions are scheduled in 2-hour blocks and take place from June 1-3. Typical scientific sessions have between 10 and 20 talks of 20 minutes each, with 10 minutes between talks, but 50-min talks are possible. Indeed, the organizers are welcome to suggest non-traditional usage of the block times and format.
- In accordance with the CMS mandate to propose conferences which are accessible and welcoming to all groups, diversity amongst organizers and speakers is strongly encouraged. To support organizers in their important work and in their efforts towards inclusivity and diversity, the CMS will host an open call for abstracts for all sessions, and asks organizers to consider all eligible abstract submissions for their session.
- Diversity includes topics of interest, career stages, geographic location, and demographics; designated underrepresented groups include, but are not limited to, women, Indigenous Peoples, persons with disabilities, members of visible minority/racialized groups, and members of LGBTQ2+ communities.
- Note that there will be a separate follow-up call for Education Sessions.
- All proposed sessions should be in line with the CMS Code of Conduct.

Proposals should be submitted online, and will require the following:

1. Names, affiliations, and contact information for two or three organizers: A lead organizer and one or two co-organizer(s).
2. A title and a two to three-sentence summary that will be posted on the website for potential speakers.
3. The number of 2-2.5 hour blocks requested.
4. A pdf file including a description of the topic and purpose of the session (1-2 paragraphs), for consideration by the Scientific Committee, not to be posted.
5. A spreadsheet including list of possible speakers. Please have columns "Last Name", "First Name", "Affiliation", "Career Stage", and "Webpage", with as much information filled out for potential speakers as possible.

Proposals will be selected by the Scientific Organizing Committee, limited by available classroom space, with priority for sessions that show intention to include a mix of senior and junior researchers, to make parts of their session accessible to graduate students, and to include speakers from designated underrepresented groups.

A note on Organizers

The lead organizer should hold a PhD or equivalent in the area of expertise relevant to the session's subject. Having a senior researcher (e.g. Professor or tenured Associate Professor) paired with someone earlier in their career (e.g. tenure track Assistant Professor or Postdoctoral Fellow) would be ideal.

We place great value on your contributions as organizers. Our primary goal is to provide the best possible experience for all attendees, and this is often better achieved by dedicating your efforts to a single, impactful session. As such, we recommend that each potential organizer only propose a single session.

Submission Form and Deadlines:

Please submit proposals by filling out this form. There will be two rounds of submissions. Proposals submitted by January 31, 2025, will be considered in the first round, with responses ongoing. The deadline for the second round will be March 14, 2025.

La Société mathématique du Canada (SMC) accueille et invite les propositions de sessions scientifiques pour la réunion d'été 2025 de la SMC qui se tiendra à Québec (Québec) du 6 au 9 juin 2025.

- L'objectif des sessions scientifiques est de partager la recherche de pointe sur un sujet mathématique donné, tel que suggéré par les organisateurs.
- Les sessions sont programmées par blocs de 2 heures et se déroulent du 1er au 3 juin. Les sessions scientifiques typiques comportent entre 10 et 20 exposés de 20 minutes chacun, avec 10 minutes entre les exposés, mais des exposés de 50 minutes sont possibles. En effet, les organisateurs sont invités à suggérer une utilisation non traditionnelle des blocs de temps et du format.
- Conformément au mandat de la SMC de proposer des conférences accessibles et accueillantes pour tous les groupes, la diversité parmi les organisateurs et les orateurs est fortement encouragée. Afin de soutenir les organisateurs dans leur travail important et dans leurs efforts en faveur de l'inclusion et de la diversité, la SMC lancera un appel à résumés ouvert pour toutes les sessions, et demande aux organisateurs de prendre en considération toutes les soumissions de résumés éligibles pour leur session.
- La diversité comprend les sujets d'intérêt, les étapes de la carrière, la situation géographique et les données démographiques ; les groupes sous-représentés désignés comprennent, sans s'y limiter, les femmes, les peuples autochtones, les personnes handicapées, les membres des minorités visibles et des groupes raciaux, et les membres des communautés LGBTQ2+.
- Veuillez noter qu'il y aura un appel séparé pour les sessions d'éducation.
- Toutes les sessions proposées doivent être conformes au Code de conduite de la SMC.

Les propositions doivent être soumises en ligne et doivent comporter les éléments suivants :

1. Les noms, affiliations et coordonnées de deux ou trois organisateurs : Un organisateur principal et un ou deux co-organisateurs.
2. Un titre et un résumé de deux à trois phrases qui seront affichés sur le site Web à l'intention des orateurs potentiels.
3. Le nombre de blocs de 2 à 2,5 heures demandés.
4. Un fichier pdf comprenant une description du sujet et de l'objectif de la session (1 à 2 paragraphes), pour examen par le comité scientifique, qui ne sera pas affiché.
5. Un tableur excel comprenant la liste des orateurs possibles. Les colonnes « Nom », « Prénom », « Affiliation », « Stade de carrière » et « Page Web » doivent contenir autant d'informations que possible sur les orateurs potentiels.

Les propositions seront sélectionnées par le comité d'organisation scientifique, dans la limite des places disponibles dans les salles de classe, avec une priorité pour les sessions qui montrent l'intention d'inclure un mélange de chercheurs seniors et juniors, de rendre certaines parties de leur session accessibles aux étudiants de troisième cycle, et d'inclure des orateurs issus de groupes sousreprésentés désignés.

Note sur les organisateurs

L'organisateur principal doit être titulaire d'un doctorat ou d'un diplôme équivalent dans le domaine d'expertise correspondant au sujet de la session. L'idéal serait qu'un chercheur chevronné (par exemple, un professeur ou un professeur associé titulaire) soit associé à une personne en début de carrière (par exemple, un professeur assistant titulaire ou un boursier postdoctoral).

Nous accordons une grande importance à vos contributions en tant qu'organisateurs. Notre objectif principal est d'offrir la meilleure expérience possible à tous les participants, et il est souvent plus facile d'y parvenir en consacrant vos efforts à une seule session à fort impact. C'est pourquoi nous recommandons à chaque organisateur potentiel de ne proposer qu'une seule session.

Formulaire de soumission et délais :

Veuillez soumettre vos propositions en remplissant ce formulaire. Il y aura deux séries de soumissions. Les propositions soumises avant le **31 janvier 2025** seront examinées lors de la première série, et les réponses se poursuivront. La date limite pour le deuxième tour sera le **14 mars 2025**.

Heydar Radjavi (University of Waterloo)

Rather than giving technical details about Peter's many mathematical contributions, or listing his publications that can be easily accessed elsewhere, let me just talk a little about one central motivation for a great part of his research. Something that was the starting point for uncountable discussions, lectures, arguments, and even jokes with his close colleagues and collaborators, the group that I was so lucky to be a member of.

I am referring to the invariant subspace problem, a well known, and by now almost infamous, problem in operator-theory circles. It has been an unsolved problem about operators on a Hilbert space for many decades. (I should add here right away that Per Enflo has recently announced an affirmative solution to this problem and presented proofs, but his solution hasn't yet been verified and independently confirmed as I write this.)

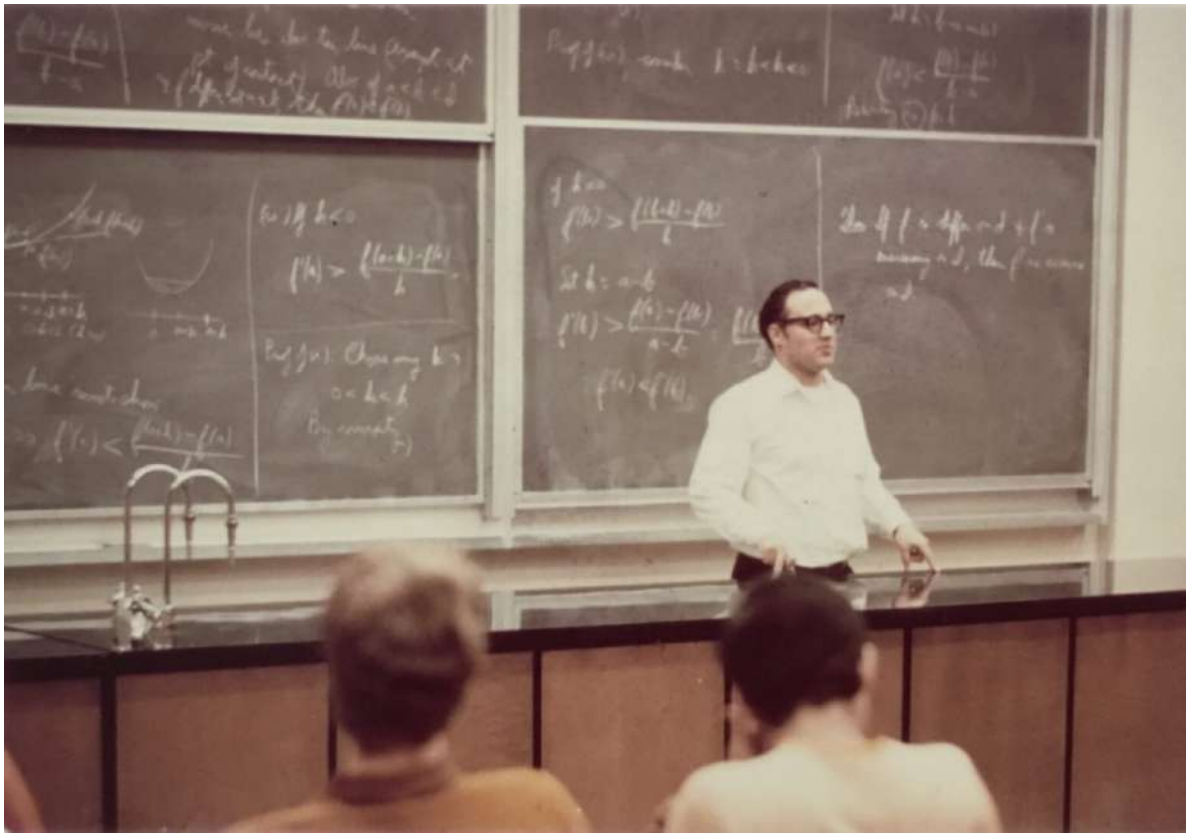
This easily stated question is about the possible extension of the following simple fact in linear algebra: Every linear operator A on a finite-dimensional space H over complex numbers has a nontrivial invariant subspace; that is, there is a subspace K of H, other than {0} and H itself, which is taken into itself by A. (Okay, if you are going to be picky, we'll assume that H has dimension greater than one!) A more general statement is Burnside's theorem: the only transitive algebra of linear operators on a finite-dimensional complex space H is the algebra of all operators. "Transitive" means, of course, that the members of the algebra do not have a common nontrivial invariant subspace. (To see the existence of an invariant subspace for a single operator A, just note that the algebra generated by A is commutative, and if it were transitive, then, by Burnside, all linear operators would commute with each other; a contradiction.)

It was natural to ask whether these results could be generalized to infinite-dimensional settings. Continuous linear operators on Banach spaces, and particularly on Hilbert spaces, were considered. Let us emphasize here that when we say "subspace" now, we mean a closed subspace (which is automatic in finite dimensions, as is the continuity of our operators). Otherwise, the answer to the existence question is easily seen to be affirmative: Let A be any operator on the Hilbert space H. Pick a nonzero x in H and apply consecutive powers of A to x, take all linear combinations to form a subspace L of H, which is clearly of countable dimension, and thus a proper subspace of H, whose dimension is uncountable if infinite.

Negative results on a Banach space were obtained by Per Enflo and then C. J. Read, but the case of Hilbert spaces kept resisting a resolution. Various affirmative results were obtained, though always with additional hypotheses. For example, as every student of functional analysis learns early on, normal operators and compact operators have plenty of invariant subspaces.

A generalization of the Burnside theorem to the infinite-dimensional setting would be the following statement: an algebra of operators on H that don't have any common nontrivial invariant subspaces is dense in the whole algebra of operators on H. (The density here is in the appropriate sense of weak operator topology.) Again, all the affirmative results proven so far have used additional hypotheses. For example, W. B. Arveson proved it under the condition that the algebra contained a "substantial enough" normal operator. This is a natural generalization, to infinite dimensions, of what is a normal operator with distinct eigenvalues on a finite-

dimensional space. Peter was instrumental in the wide ranging work that extended these results, by him and his students and collaborators. A sample result concerns algebras he called "Hermitian" first, and then "reductive" as suggested by P. R. Halmos. This means an algebra of operators on H with the property that the orthogonal complement of every (common) invariant subspace is also invariant. The extension was that a reductive algebra containing a substantial normal operator was dense in a selfadjoint algebra.



Just as Peter Rosenthal was writing his Ph.D. dissertation under Halmos's supervision at the University of Michigan, quite a few operator theorists besides Halmos were already busy with the above questions. Peter did very consequential work on the structure of lattices of invariant subspaces, and moved to the University of Toronto right after he got his degree. That is where I met him for the first time at the joint meeting of AMS and CMS in 1967. We were both admirers of Halmos and his approach to research and teaching. Our meeting started a long-lasting friendship and cooperation. The best and simplest way of describing his work with colleagues was that it was fun. For quite a few years Peter and I joined our colleagues Eric Nordgren and Don Hadwin at the University of New Hampshire every summer to work together. Every summer we started by declaring, half-seriously of course, that we would solve the invariant subspace problem by the end of the summer; then gradually settled to lighter goals. We also had various bets with each other and with other people on whether the problem had an affirmative solution. I remember one summer, when Peter came up with an idea for answering a question raised by C. Pearcy and A. Shields on how general Lomonosov's theorem was. Lomonosov had proved that every nonzero compact operator has a hyper-invariant subspace, i.e., the algebra of those operators that commute with it has a common nontrivial invariant subspace. The natural question that was then asked was, did this include all operators, perhaps? Peter suggested an excellent idea for a counterexample. It took some days, but it worked. Some bets were lost and won, but Peter was right: there was an operator A such that no non-scalar operator commuted both with it and with a nonzero compact operator.

After invariant subspaces what comes naturally is triangularization of operators, and then simultaneous triangularization of groups and semigroups of operators. Peter has made extensive contributions to research in this connection. I would like to mention one simple result here that geometers may find of some interest. It is a geometric equivalent of the invariant subspace problem, which does not even mention operators: Let H = K + L be a direct-sum decomposition of the Hilbert space H into two subspaces. Let us call a given subspace M of H "admissible" with respect to this decomposition if M is the sum of a subspace of K and a subspace of L. That is the only definition we need. Here comes a question equivalent to our stubborn one: If two different decompositions of H are given, does there exist a nontrivial subspace of H which is admissible with respect to both decompositions? That summer in New Hampshire we dreamed that a geometer smarter than us would come up with an answer.

Most readers of this short account are aware that Peter was well known, not just as a mathematician, but also as a lawyer, whose free services were often used in many legal cases in defence of marginalized clients. So, I will end with a related note. It was inevitable that Peter's mathematical colleagues would make remarks about the similarity of his life to Fermat's. They did, and he answered in kind. A gift I received from him about forty years ago is a T-shirt with a message. I bet you are in the process of guessing what the message said: "I have a proof of the invariant subspace problem. Shirt too small to put it on."

Peter will be missed by all who knew him.

Heydar Radjavi,
University of Waterloo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Dear all,

The CMS Student committee is pleased to announce that the 2025 *Canadian Undergraduate Mathematics Conference* (CUMC) will be held at the **University of Waterloo**,from **June 23rd – June 27th, 2025**! Be on the lookout for more announcements in the upcoming months for further information. We hope to see you there**,**

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Bonjour à tous,

Le comité étudiant de SMC a le plaisir de vous annoncer que le *Congrès canadien des étudiant.e.s en mathématiques* 2025 (CCÉM) aura lieu à l'**Université de Waterloo**, du **23 au 27 juin** ! Soyez à l'affût d'autres annonces dans les mois à venir pour plus d'informations.

Nous avons hâte de vous y rencontrer,

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