- Sums:

Arithmetic: $1+2+3+\dots+N$ operations $=\Theta(N^2)$

$1+4+7+\dots+N$ operations $=\Theta(N^2)$

$+c$ $+c$ (next number is $+c$ larger)

Geometric: $1+2+4+\dots+N$ operations $=\Theta(N)$

$1+8+64+\dots+N$ operations $=\Theta(N)$

$\times c$ $\times c$ (next number is $\times c$ larger)

- Analyzing recursive function runtimes:

STEPS:

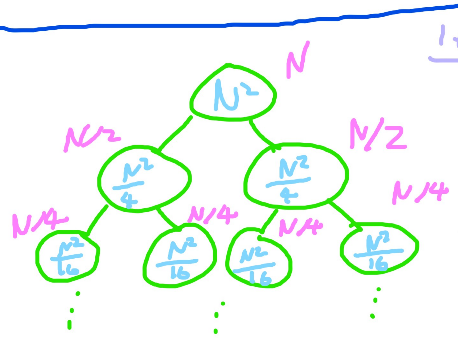
1. How many **operations** in this function? — size of input N
2. Draw a **tree**! Draw the first function call as a node like so: N^2
3. How many **recursive calls** in this function? Draw as child node as so:

4. **Repeat** 1-3 on recursive calls until notice pattern
5. Label recursive calls by **level** (distance away from first function call) - see example
6. Add up number of **operations per level**. Then add up operations at each level for each level.

EXAMPLE:

```
public static void weirdCount(N) {
    if (N <= 1) { return 1 }
    int sum = 0;
    for (int x = 0; x < N; x++) {
        for (int y = 0; y < N; y++) {
            sum += 1;
        }
    }
    return sum + weirdCount(N/2)
               + weirdCount(N/2);
}
```

1. In our first function weirdCount(N), there are $\Theta(N^2)$ operation
2. Create tree!
3. Two recursive calls: weirdCount($\frac{N}{2}$) and weirdCount($\frac{N}{2}$)
4. Notice that each call to weirdCount has two child nodes (recursive calls) with half the input.
5. We realize that there are $\log N$ levels because the input N decreases by half each level; N halves $\log N$ times until it reaches 1 (base case=leaf nodes; no more recursive calls).
6. Notice how we see a pattern in the operations at each level ℓ: $\frac{N^2}{2^\ell}$. Now we add up all the operations in the "operation" column.



level	Operation
0	$\frac{N^2}{2^0}=\frac{N^2}{1}$
1	$\frac{N^2}{4}+\frac{N^2}{4}=\frac{N^2}{2^1}$
2	$\frac{N^2}{16}+\frac{N^2}{16}+\frac{N^2}{16}+\frac{N^2}{16}=\frac{N^2}{2^2}$
⋮	⋮
$\log N$	$\frac{N^2}{2^{\log N}}=\frac{N^2}{N}=N$

$$\frac{N^2}{1}+\frac{N^2}{2}+\frac{N^2}{4}+\dots+N$$

$$=N\underbrace{\left(N+\frac{N}{2}+\frac{N}{4}+\dots+1\right)}_{\text{Geometric Sum!}}=N\cdot\Theta(N)$$

$$=\Theta(N^2)$$