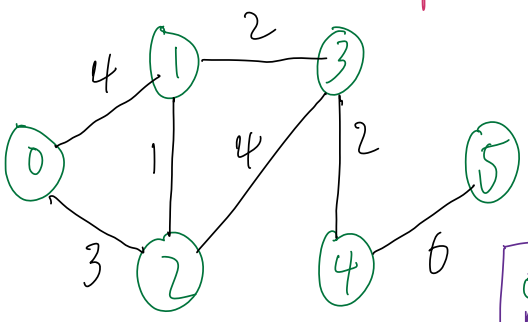
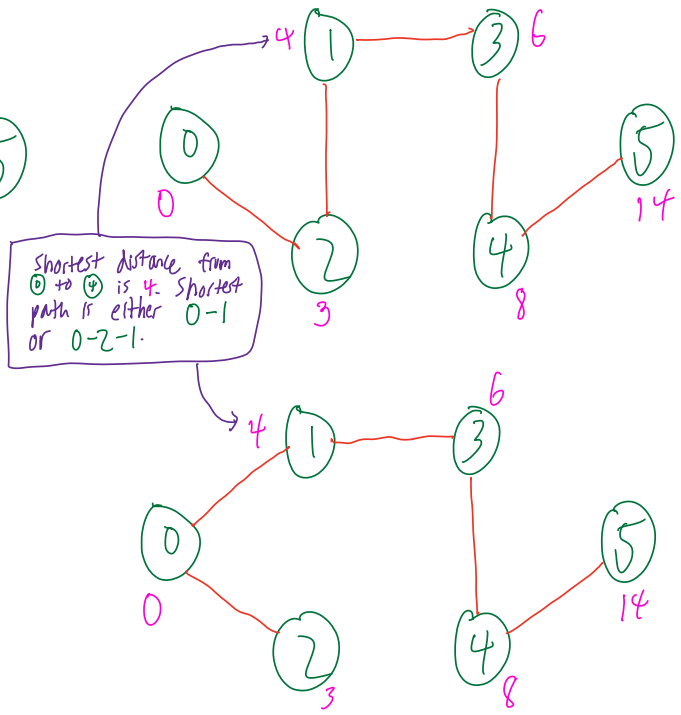
# Dijkstra's Alg

- Goal: Find shortest path tree (a tree that contains shortest paths from source s to every other node).

Undirected Graph




Possible Shortest Paths Trees from 0



# • How it works

1. Create distTo and edgeTo lists and priority queue PQ (which prioritizes nodes closest to source)
   - distTo(v): best known distance from source s to v
   - edgeTo(v): best known vertex predecessor to v
   - PQ contains all unvisited vertices in order of distTo
2. Initialize:
   - for each index i corresponding to vertex i, set distTo(i) = ∞; except vertex s → distTo(s) = 0.
   - for each index i corresponding to vertex i, set edgeTo(i) = null.
3. Repeat:
   - Visit v: remove closest vertex v from PQ
   - Relax edges

     for each outgoing edge e from v:

     if dist(s, v) + e < distTo(w):

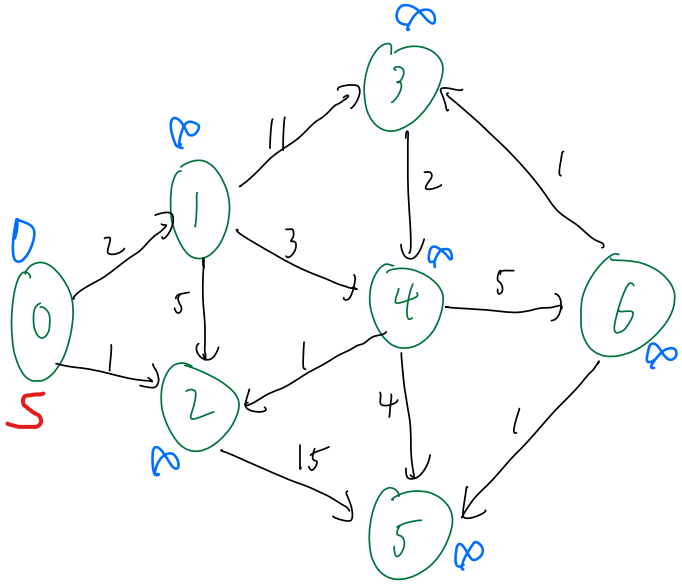
     distTo(w) = dist(s, v) + e

     edgeTo(w) = v
4. End: when PQ is empty

- Example:

Iteration 0:

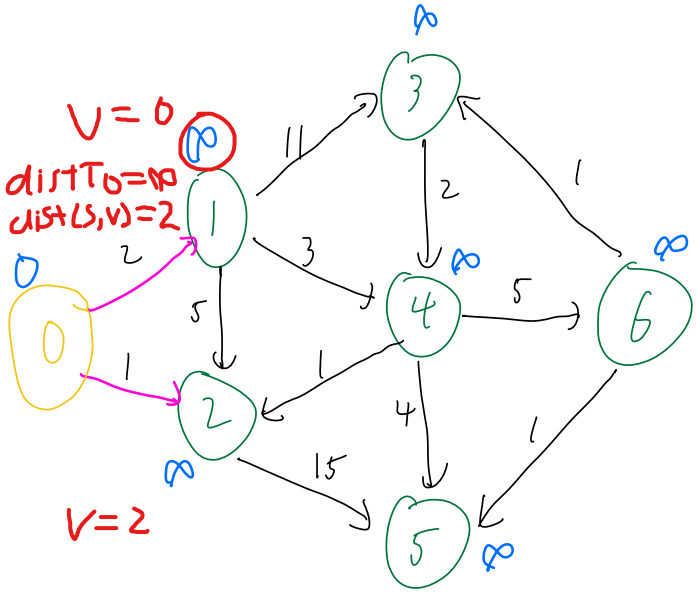
| Node # | distTO | edgeTo |
|---|---|---|
| 0 | 0 | null |
| 1 | ∞ | null |
| 2 | ∞ | null |
| 3 | ∞ | null |
| 4 | ∞ | null |
| 5 | ∞ | null |
| 6 | ∞ | null |




PQ: ~~(0 : 0)~~, (1 : ∞), (2 : ∞), (3 : ∞), (4 : ∞), (5 : ∞), (6 : ∞)

Visit vertex 0

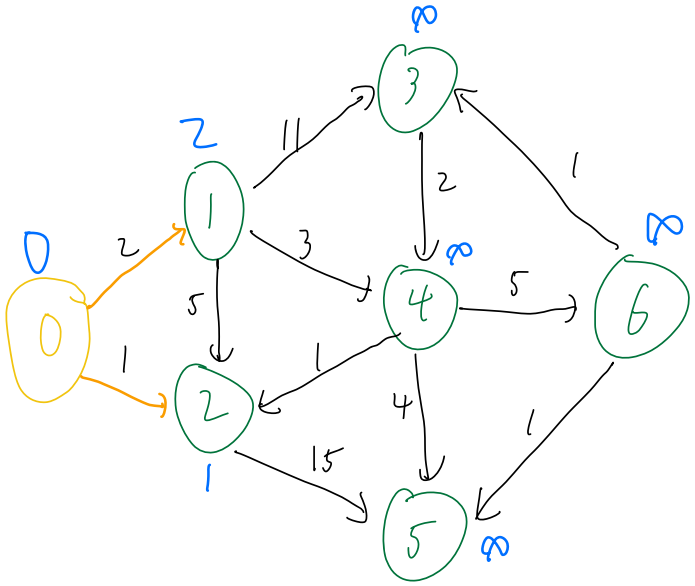
| Node # | distTO | edgeTo |
|---|---|---|
| 0 | 0 | null |
| 1 | ∞ | null |
| 2 | ∞ | null |
| 3 | ∞ | null |
| 4 | ∞ | null |
| 5 | ∞ | null |
| 6 | ∞ | null |




PQ: (1 : ∞), (2 : ∞), (3 : ∞), (4 : ∞), (5 : ∞), (6 : ∞)

# Relax outgoing edges from vertex 0

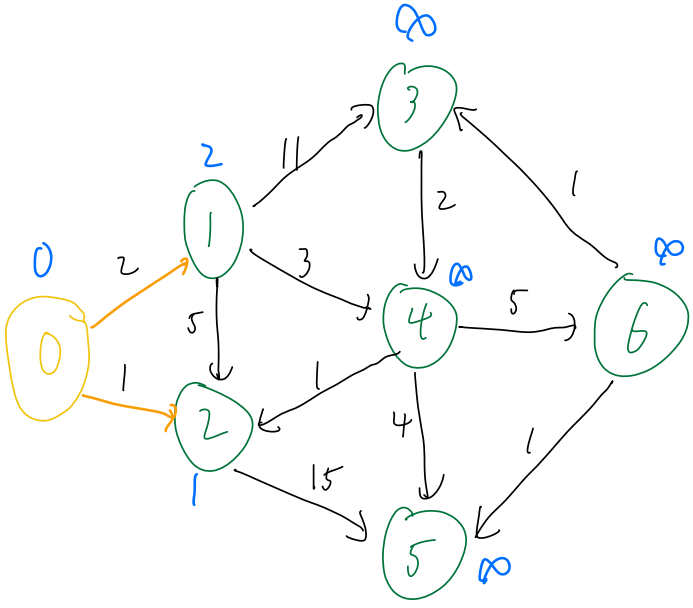
| Node # | distTO | edgeTo |
|---|---|---|
| 0 | 0 | null |
| 1 | 2 | 0 |
| 2 | 1 | 0 |
| 3 | ∞ | null |
| 4 | ∞ | null |
| 5 | ∞ | null |
| 6 | ∞ | null |




PQ: ~~(2:1)~~, (1:2), (3:∞), (4:∞), (5:∞), (6:∞)

## Iteration 2

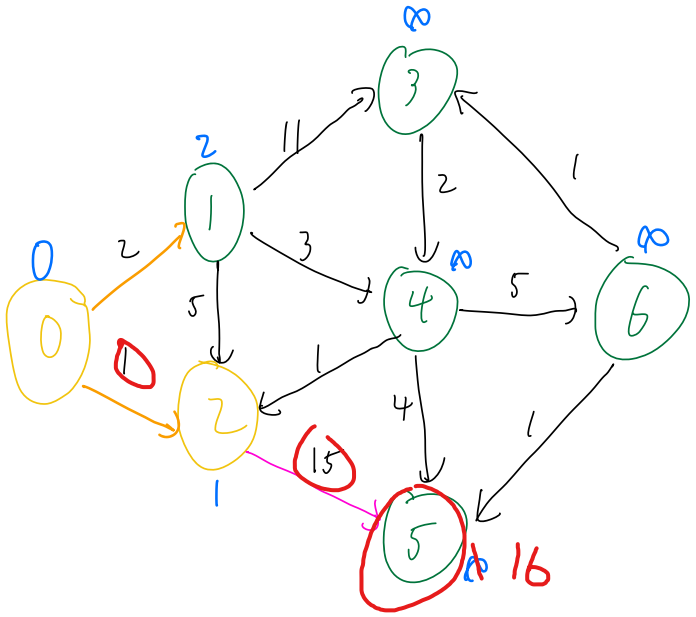
| Node # | distTO | edgeTo |
|---|---|---|
| 0 | 0 | null |
| 1 | 2 | 0 |
| 2 | 1 | 0 |
| 3 | ∞ | null |
| 4 | ∞ | null |
| 5 | ∞ | null |
| 6 | ∞ | null |




PQ: ~~(2:1)~~, (1:2), (3:∞), (4:∞), (5:∞), (6:∞)

Visit vertex 2

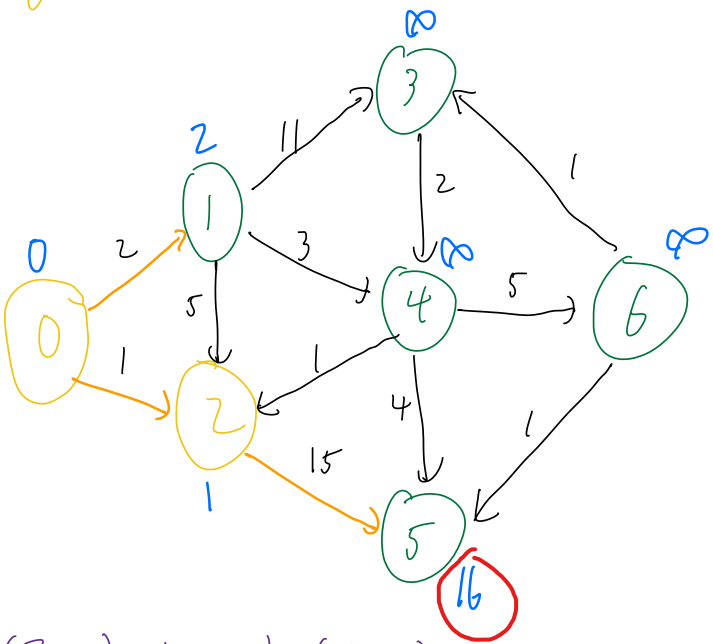
| Node # | distTO | edgeTo |
|---|---|---|
| 0 | 0 | null |
| 1 | 2 | 0 |
| 2 | 1 | 0 |
| 3 | ∞ | null |
| 4 | ∞ | null |
| 5 | ∞ | null |
| 6 | ∞ | null |




PQ: (1:2), (3:∞), (4:∞), (5:∞), (6:∞)

Relax outgoing edges from vertex 2

| Node # | distTO | edgeTo |
|---|---|---|
| 0 | 0 | null |
| 1 | 2 | 0 |
| 2 | 1 | 0 |
| 3 | ∞ | null |
| 4 | ∞ | null |
| 5 | 16 | 2 |
| 6 | ∞ | null |




PQ: (1:2), (5:16), (3:∞), (4:∞), (6:∞)

END

| Node # | distTO | edgeTo |
|---|---|---|
| 0 | 0 | null |
| 1 | 2 | 0 |
| 2 | 1 | 0 |
| 3 | 11 | 6 |
| 4 | 5 | 1 |
| 5 | 9 | 4 |
| 6 | 10 | 4 |

PQ: empty