

Merge Sort

`mergeSort(arr):`

if `len(arr) == 0:`

return `arr`

`first_half` ← first half of `arr`

`second_half` ← second half of `arr`

`sorted` ← `merge(mergeSort(first_half), mergeSort(second_half))`

return `sorted`

`merge(sorted1, sorted2):`

`i, j` ←

`newArr` ← empty array

while `i < len(arr1)` and `j < len(arr2):`

if `sorted1[i] ≤ sorted2[j]`

`newArr[i+j] ← sorted1[i]`

`i = i + 1`

else:

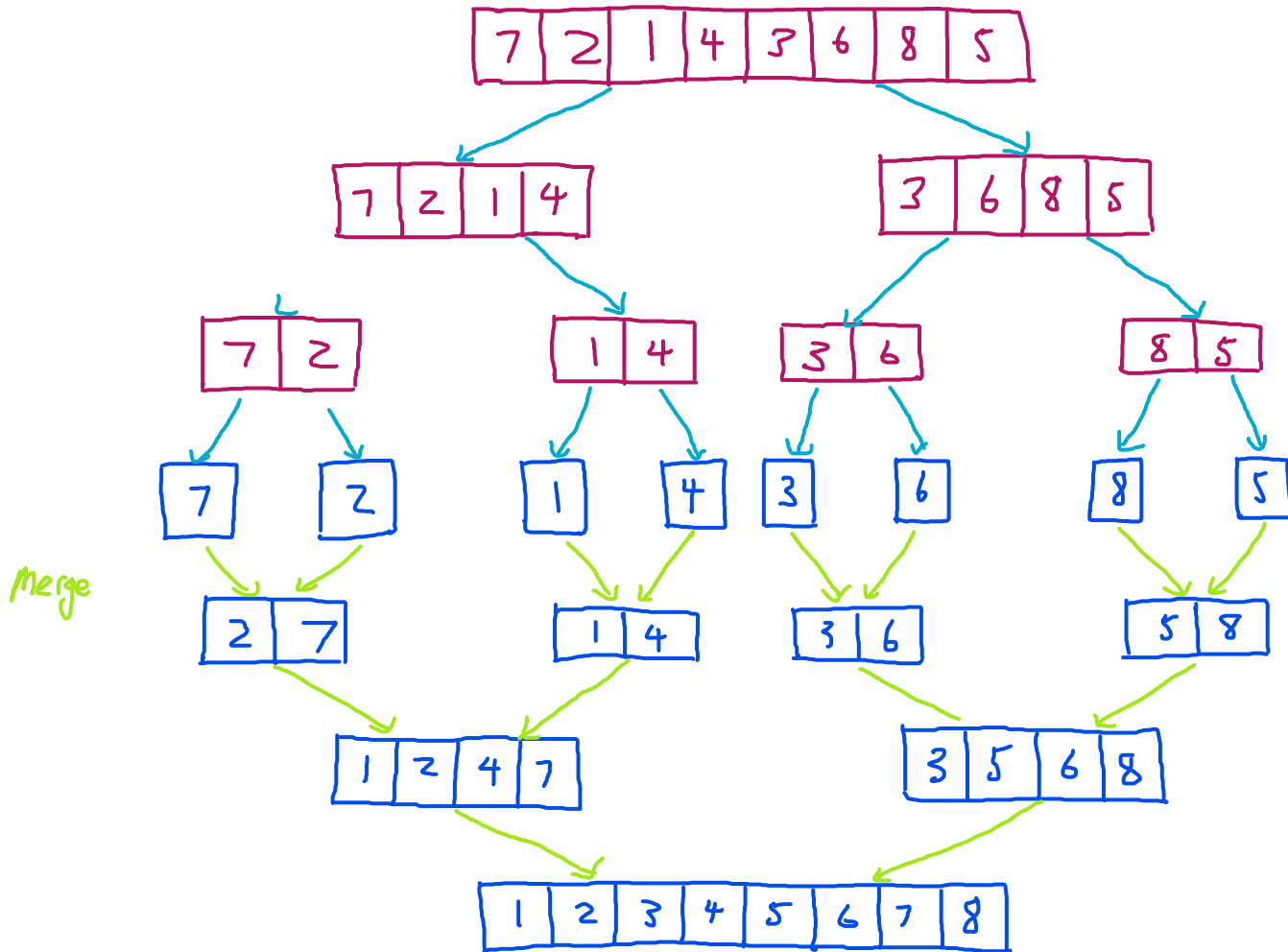
`newArr[i+j] ← sorted2[j]`

`j = j + 1`

return `newArr`

In merge sort, we recursively divide the array by half, then call the same procedure on each half. Afterwards, both halves will be sorted (recursive leap of faith). We then merge the two halves (go thru both lists, compare leftmost unadded items of each list, then add the smaller value).

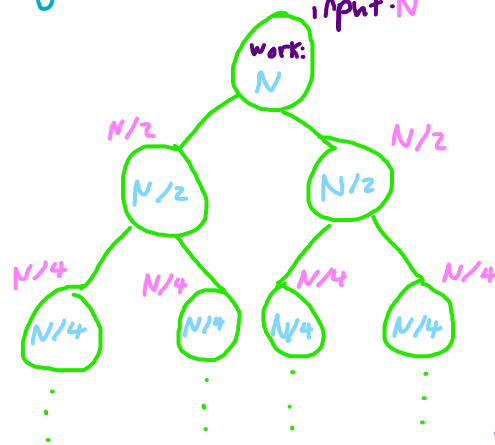
See example below on why this works.



Runtime? $\Theta(N \log N)$

① Realize that **merge** takes $\Theta(N)$ time.

② There are two recursive calls in **mergeSort**, each call with $\frac{1}{2}$ the input
mergeSort(arr), len(arr)=N
input: N



Level	Work
0	N
1	$\frac{N}{2} + \frac{N}{2} = N$
2	$\frac{N}{4} + \frac{N}{4} + \frac{N}{4} + \frac{N}{4} = N$
$\vdots$	$\vdots$
$\log N$	N

$$N \cdot \log N = \boxed{\Theta(N \log N)}$$