Math 1200 A Class Exercises (Logic and Proof)

An implication is a statement of the form, "If A then B" or equivalently "A implies B" or "$A \Rightarrow B$". To assert that an implication is true is to assert that whenever A is true, B *must* be true.

1. You know that the statement, "$A \Rightarrow B$" is true. Which of the following are necessarily true as well? You should be able to justify your answers using ordinary language, i.e., without truth tables or other use of formal logic.

   (a) A is true.

   (b) B is true.

   (c) A is true and B is true.

   (d) A is true or B is true.

   (e) A is false or B is true.

   (f) Whenever A is true then B is true.

   (g) Whenever A is false then B is false.

   (h) Whenever B is false then A is false.

   (i) Whenever B is true then A is true.

2. Consider each of the following pairs of statements. Decide whether each statement is true or false. If it is true, prove it. If it is false, explain why. Let m and n be integers.

   (a)   i. If m is even then mn is even.

         ii. If mn is even then m is even.

   (b)   i. If $m - n$ is even then both m and n are even or both m and n are odd.

         ii. If $5m - 2n$ is even then both m and n are even or both m and n are odd.

   (c)   i. If mn is odd then both m and n are odd.

         ii. If mn is odd then m is odd or n is odd.