The Cavity Shift of Electron $g - 2$

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Seoul National University Seminar — July 10, 2026

JHEP 03, 038 (2026), with Hannah Day, Roni Harnik, Yonatan Kahn, Shashin Pavaskar

The “Other” $g-2$

$g_\mu - 2$

team of 200, costs $\$10^8$

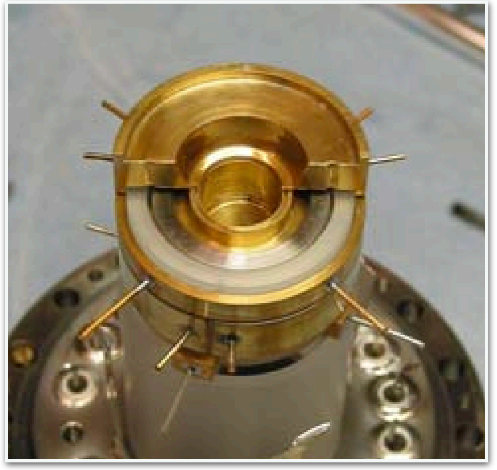
10^{11} μ^- orbit in $L \sim 10\,\mathrm{m}$ storage ring

achieves $\Delta g_\mu / g_\mu \sim 10^{-10}$

$g_e - 2$

team of 4, costs $\$10^6$

1 e^- in a trap of size $L \sim 0.01\,\mathrm{m}$

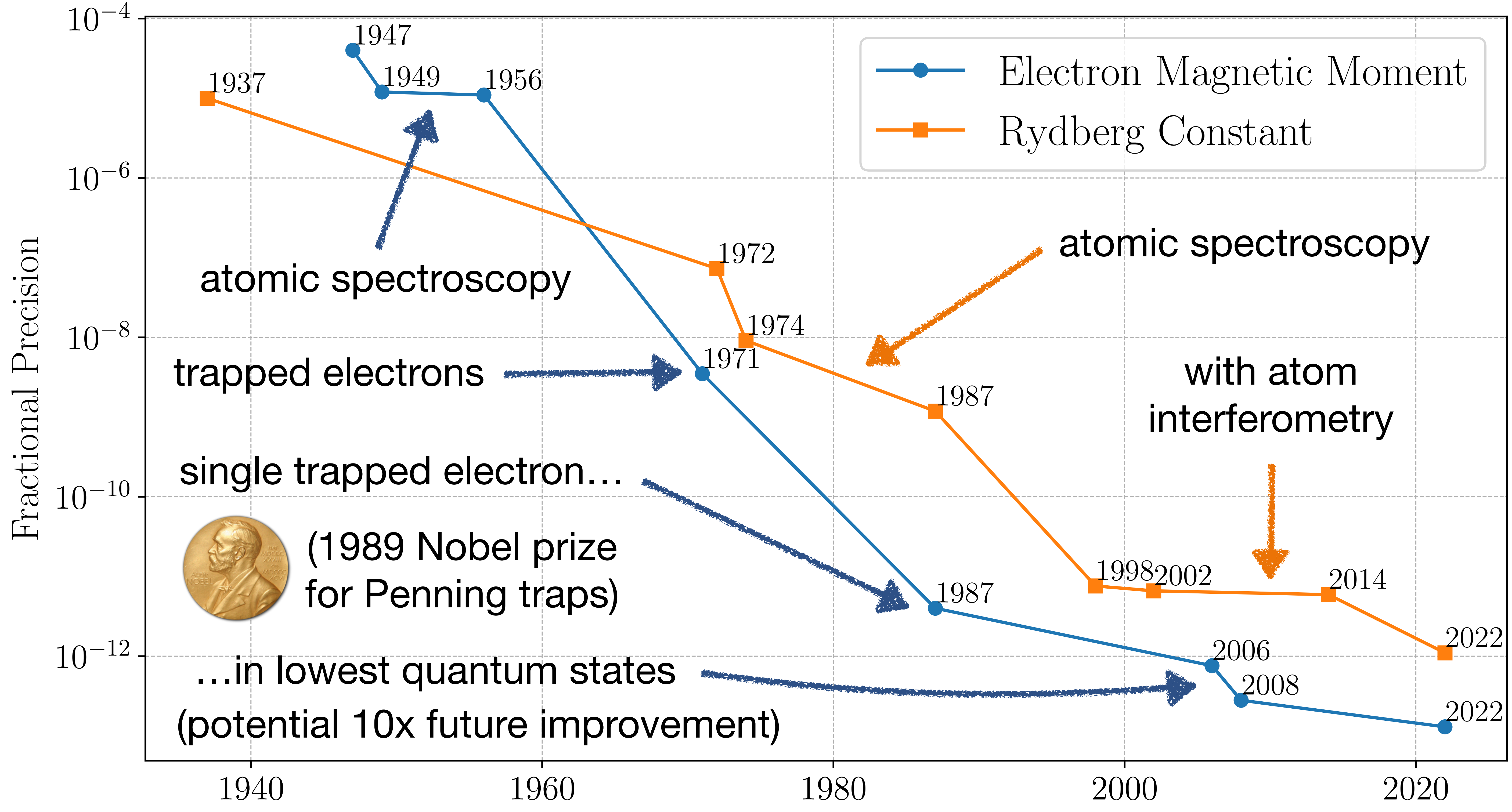


achieves $\Delta g_e / g_e \sim 10^{-13}$

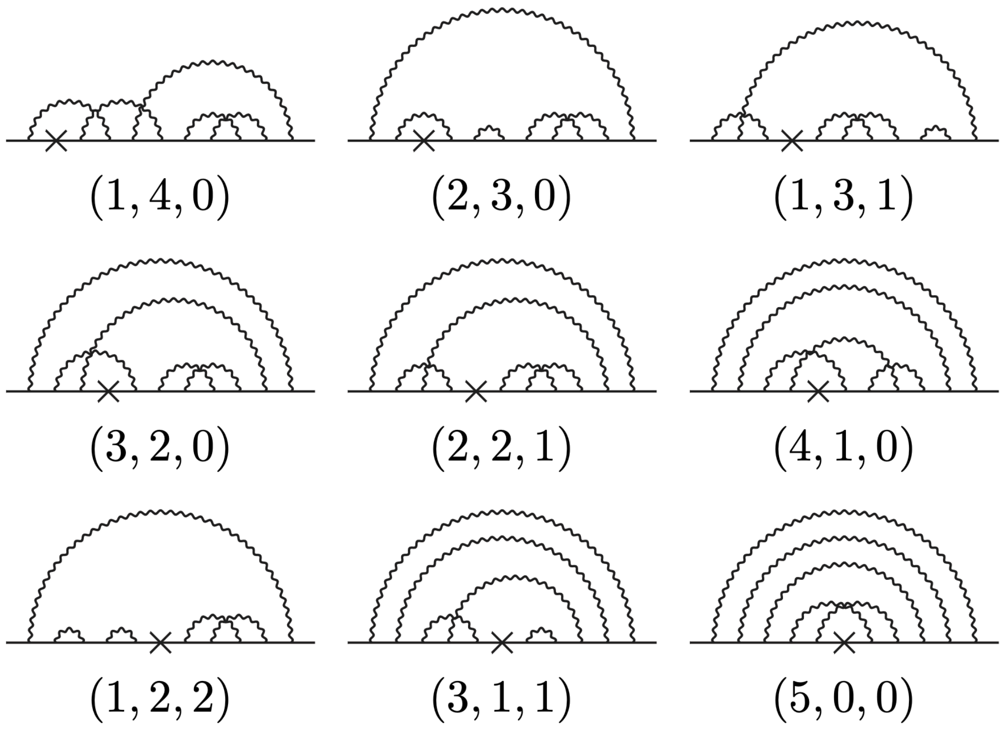
Since $(m_\mu / m_e)^2 \sim 10^4$, “generic” BSM sensitivity of $g_\mu - 2$ higher,

but $g_e - 2$ can scale up, presents different theory questions

A Brief History of Measuring QED Couplings

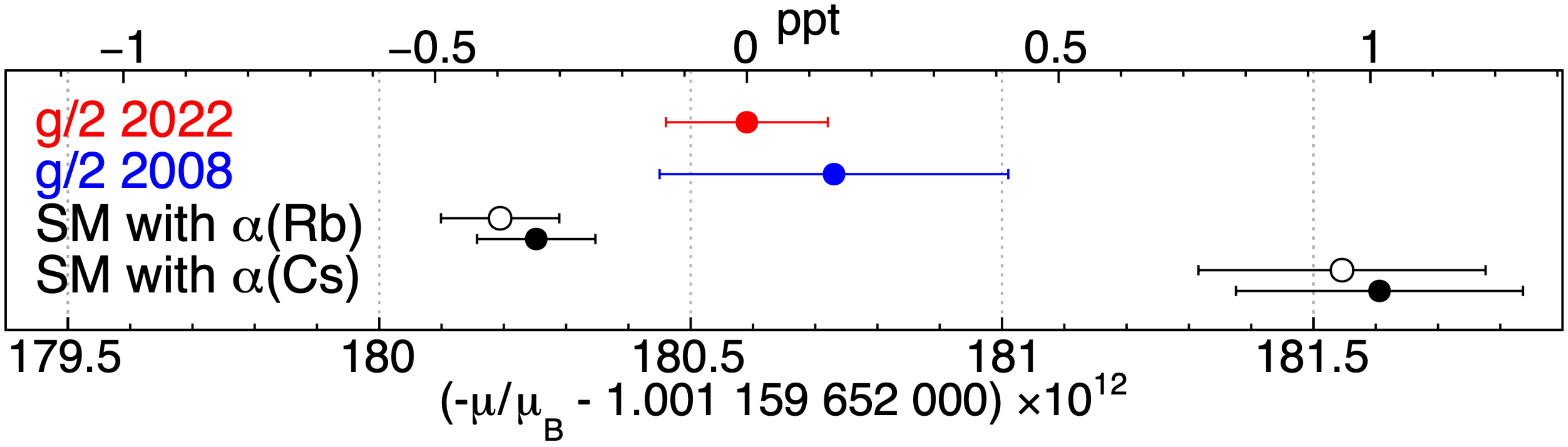


Precision QED Today



Current measurements of electron $g-2$ are sensitive to 5-loop QED corrections, and ~TeV scale electron compositeness

To compare to SM, need the value of α (currently disputed) and 5-loop theory calculation (disputed, but resolved in 2024)



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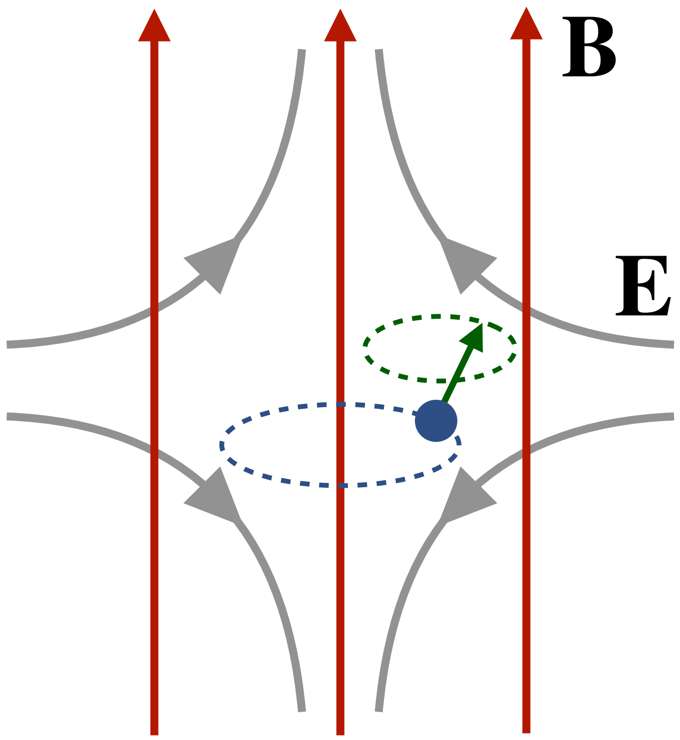
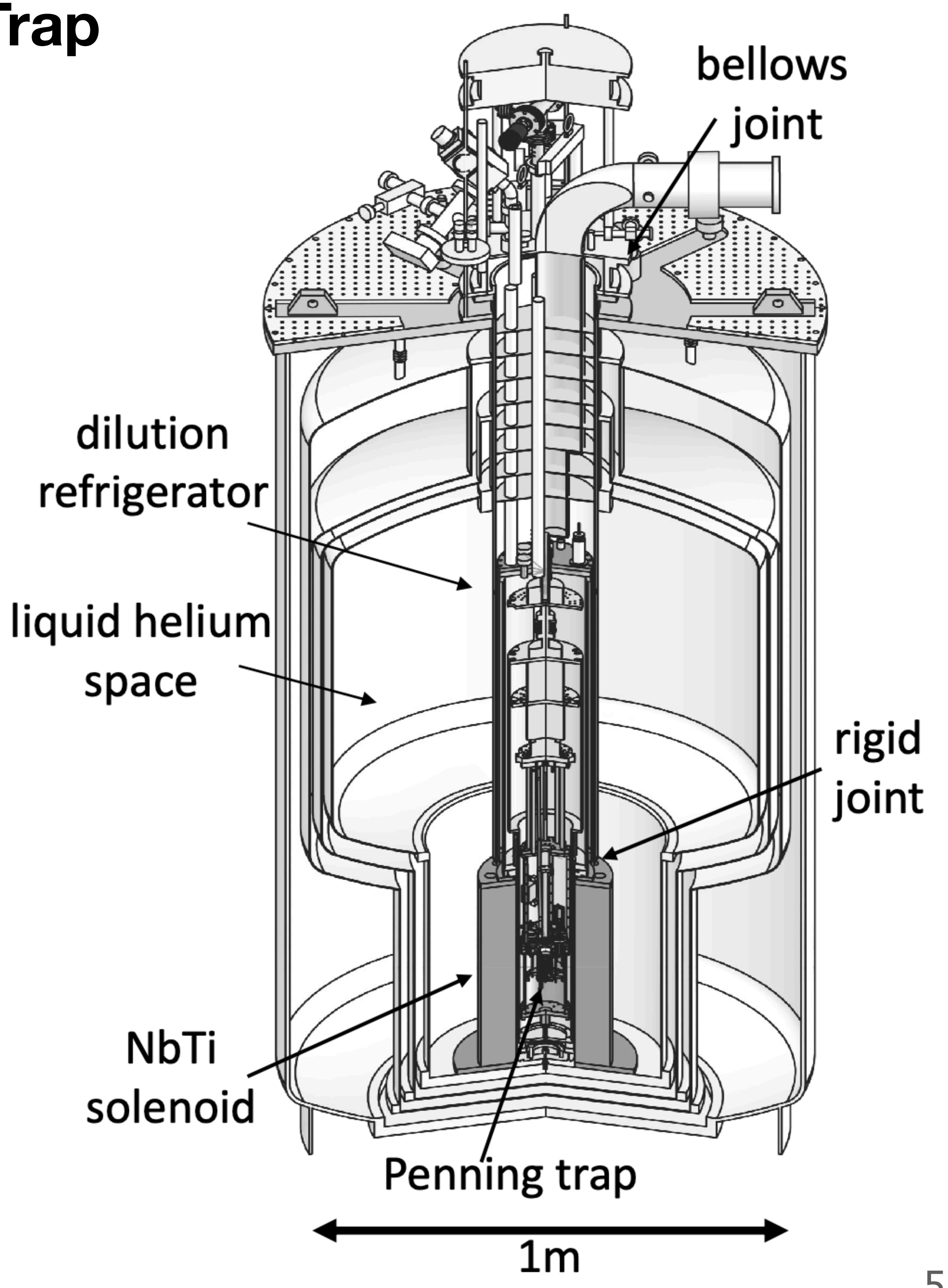
Electron in a Penning Trap





Electron placed in uniform, stable magnetic field

Confined in axial direction by electric field

Cooled to ground state with dilution fridge

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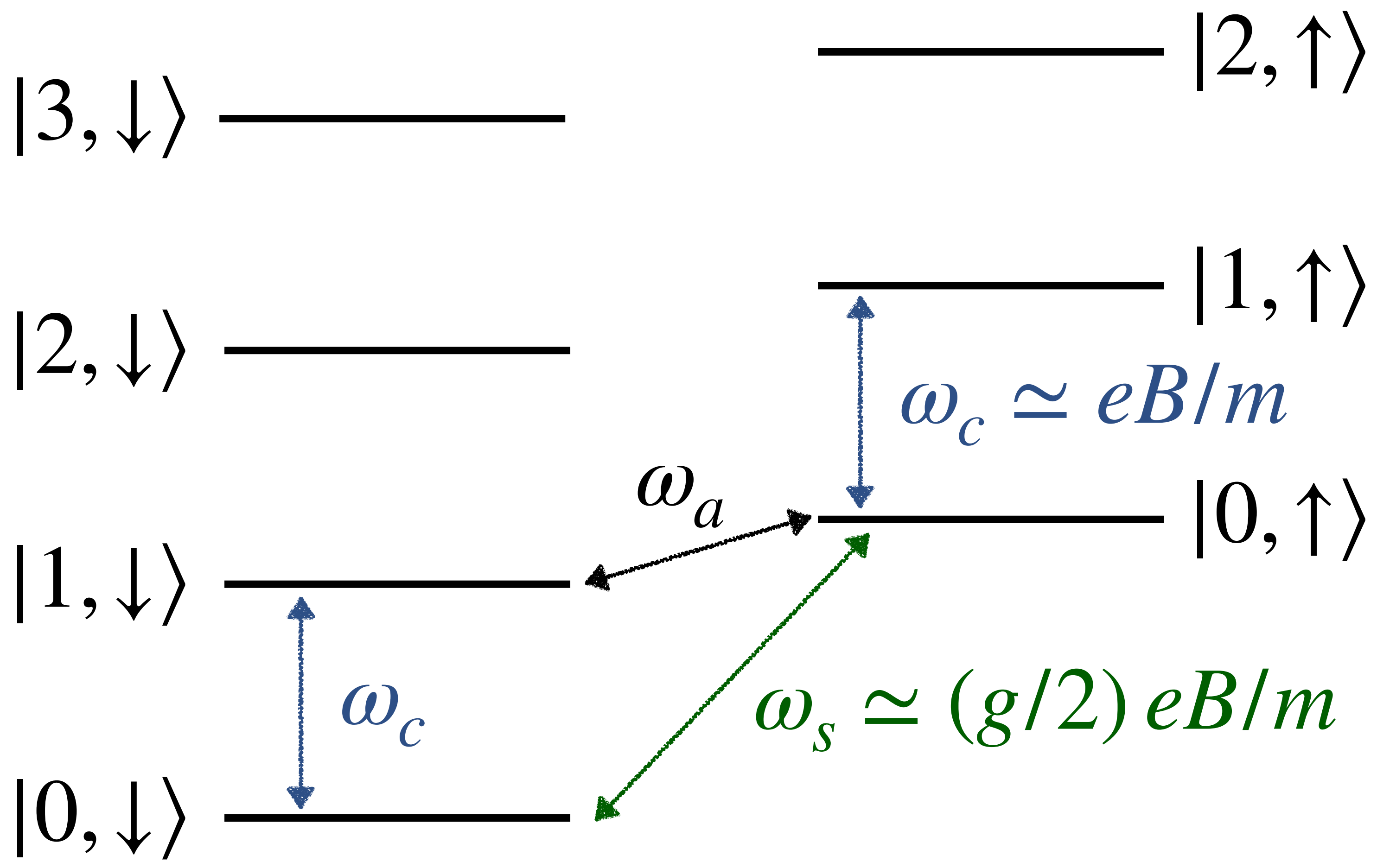
Measuring the g-factor

Electron g determines spacing ω_s between spin states

Can cause transitions by applying oscillating driving fields



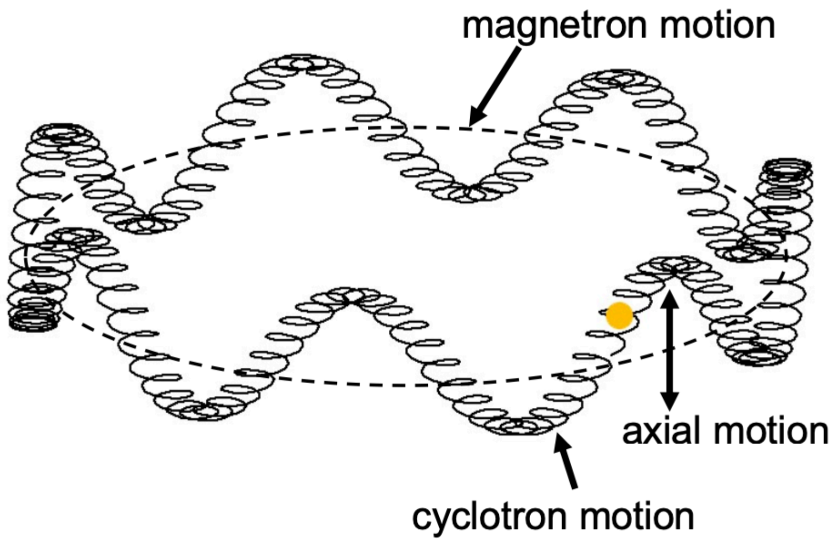
Trick: measure the "anomaly" ω_a

$$\frac{g-2}{2} = \frac{\omega_a}{\omega_c} \sim 10^{-3}$$

Cancels dependence on B and m_e,
precision on g multiplied by 10^3

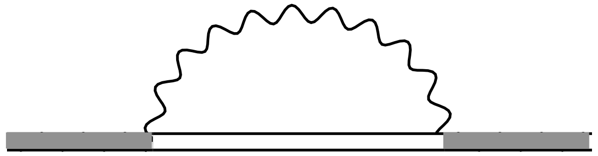
Corrections to Ideal Result

Current measurements must account for anything that shifts ω_c or ω_s at the 10^{-13} level



Slow axial motion: $\dfrac{\Delta\omega_c}{\omega_c} \sim \dfrac{\omega_z^2}{\omega_c^2} \sim 10^{-6}$

Relativistic effects: $\dfrac{\Delta\omega_c}{\omega_c} \sim \dfrac{\text{kinetic energy}}{\text{mass energy}} \sim \dfrac{\omega_c}{m} \sim \dfrac{\text{meV}}{\text{MeV}} \sim 10^{-9}$

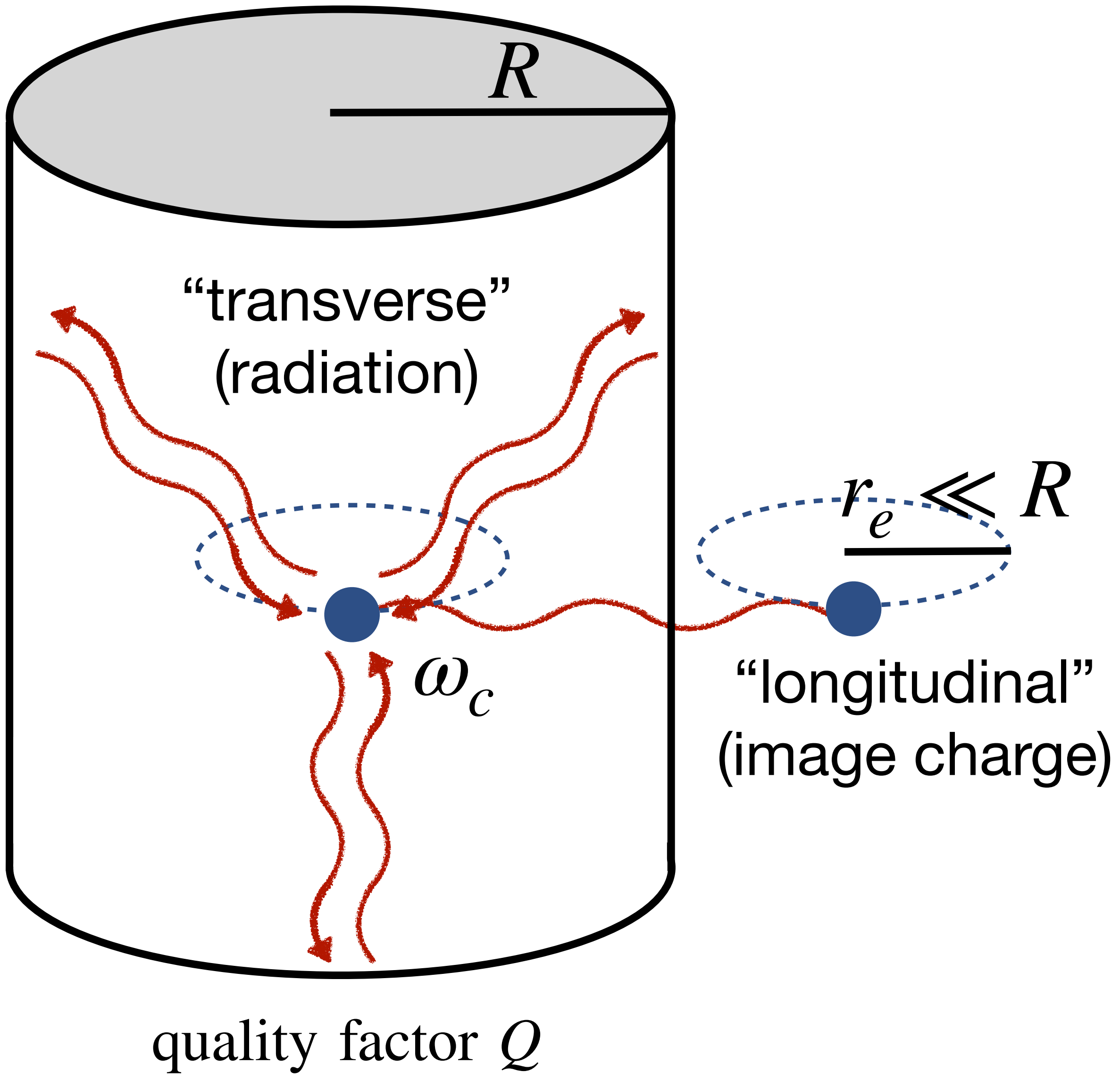


Self-energy in B: $\dfrac{\Delta\omega_c}{\omega_c} \sim \alpha \left(\dfrac{\omega_c}{m}\right)^2 \log(m/\omega_c) \sim 10^{-19}$

Finite lifetime from radiation emission: $\dfrac{\delta\omega_c}{\omega_c} \sim \dfrac{\Gamma}{\omega_c} \sim \dfrac{\alpha\omega_c}{m} \sim 10^{-11}$ **uncertainty**, not shift!

Effects of the Cavity

Enclosing electron in a cavity increases lifetime if ω_c between cavity modes, but it also yields a "cavity shift" of ω_c itself!



Estimating the longitudinal effect ($\omega_c R \sim 10$):

$$\frac{\Delta\omega_c}{\omega_c} \sim \frac{F_{\text{image}}}{F} \sim \frac{q^2 r_e/(4\pi R^3)}{m\omega_c^2 r_e} \sim \frac{\alpha}{m\omega_c^2 R^3} \sim 10^{-14}$$

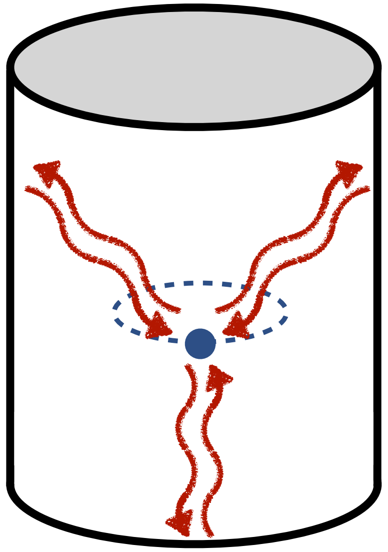
Negligible, though dominant for ion traps

Estimating the transverse effect on resonance:

$$\frac{\Delta\omega_c}{\omega_c} \sim \frac{F_{\text{rad}}}{F} \sim \frac{q^2 a_e Q/(4\pi R)}{m a_e} \sim \frac{Q\alpha}{mR} \sim 10^{-12}\,Q$$

Already observable in the 1980s on resonance, leading systematic for modern experiments

Computing the Cavity Shift

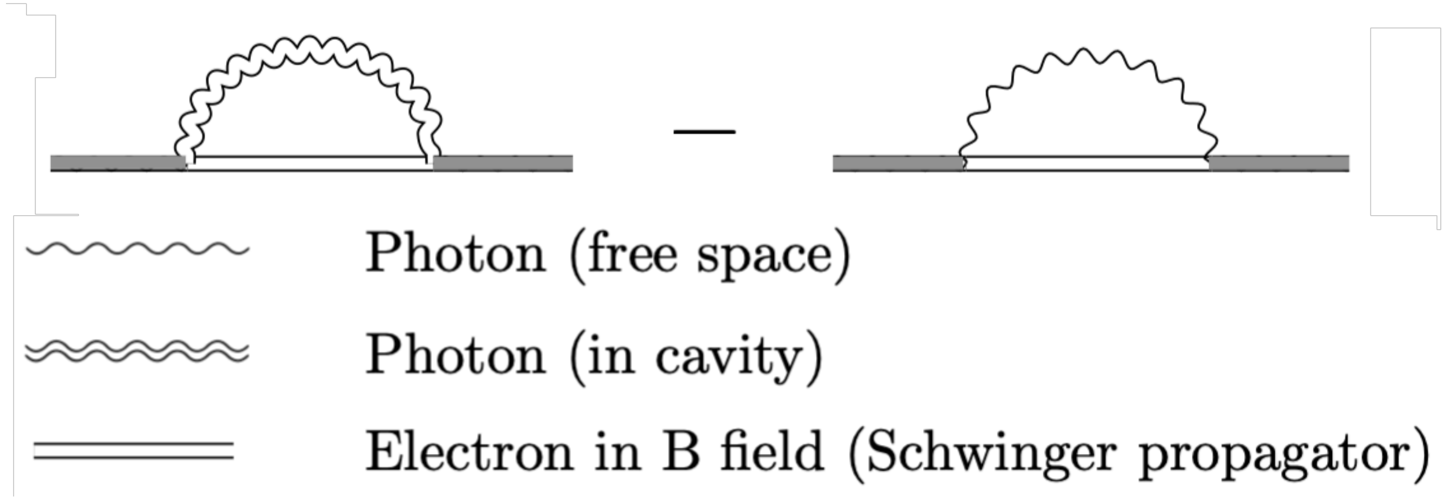


classical: radiation self-field

must remove divergent self-field

done for plates, spherical and cylindrical cavities in 1980s

hard to generalize to realistic cavities



quantum: modifies self-energy diagram, which must be renormalized

QED between conducting plates: Corrections to radiative mass and $g-2$

Magnetic effects in non-relativistic quantum electrodynamics: image corrections to the electron moment

QUANTUM ELECTROMAGNETICS OF AN ELECTRON NEAR MIRRORS

Mass and magnetic moment of localised electrons near conductors

The interaction of an atom with electromagnetic vacuum fluctuations in the presence of a pair of perfectly conducting plates

ON THE APPARATUS DEPENDENCE OF THE ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON

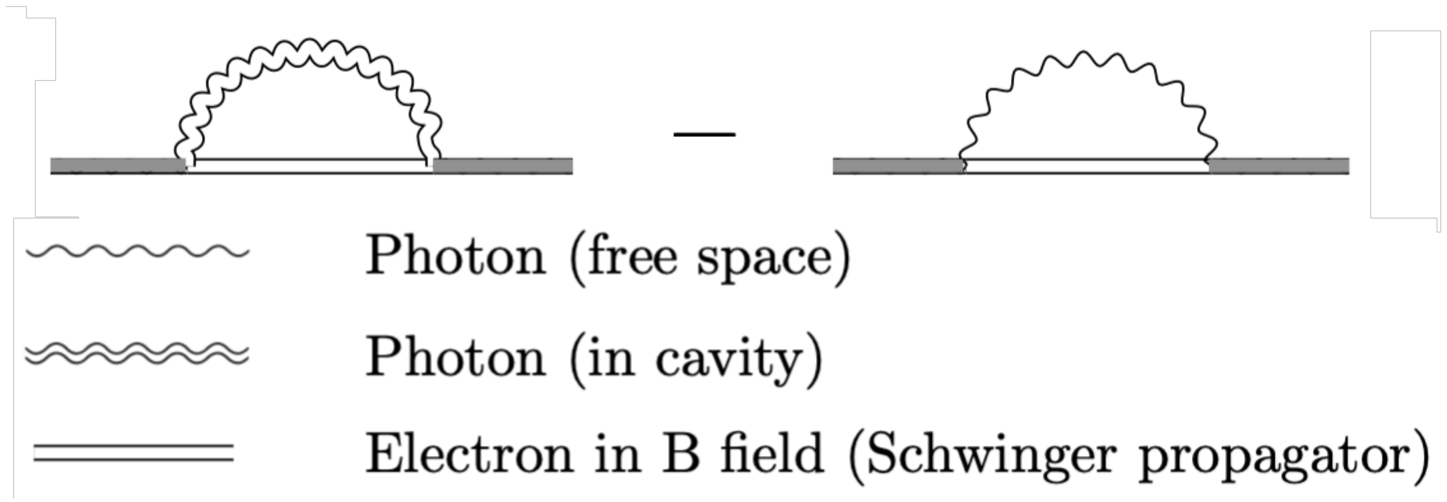
Apparatus-dependent contributions to $g-2$ and other phenomena

Mass and Anomalous Magnetic Moment of an Electron between Two Conducting Parallel Plates

many papers initially disagreed for plate, but eventually matched classical result

never attempted for closed cavity — too hard?

Cavity Shift in Relativistic QED



Electron energy levels can be extracted from poles of self-energy diagrams

(usual $g-2$ can also be found this way)

$$\delta E_n = \int d\mathbf{x}\, d\mathbf{x}'\, \overline{u}_n(\mathbf{x}) \Sigma_A(\mathbf{x}, \mathbf{x}') u_n(\mathbf{x}') \qquad \Sigma_A(\mathbf{x}, \mathbf{x}'; E) \sim e^2 \int d\tau\, \gamma^\mu S_A(\mathbf{x}, \mathbf{x}'; \tau) \gamma^\nu D_{\mu\nu}(\mathbf{x}, \mathbf{x}'; \tau) e^{iE\tau},$$

electron state

Schwinger's exact propagator in external $\mathbf{B}$

photon propagator

$$D^{ij}(\omega, \mathbf{k}) \sim \sum_s u_s^i(\mathbf{0})\, u_s^j(\mathbf{0})^* \frac{\delta(\mathbf{k})}{\omega^2 - \omega_s^2 + i\epsilon}.$$

To account for mass renormalization, must also subtract self-energy at $B = 0$

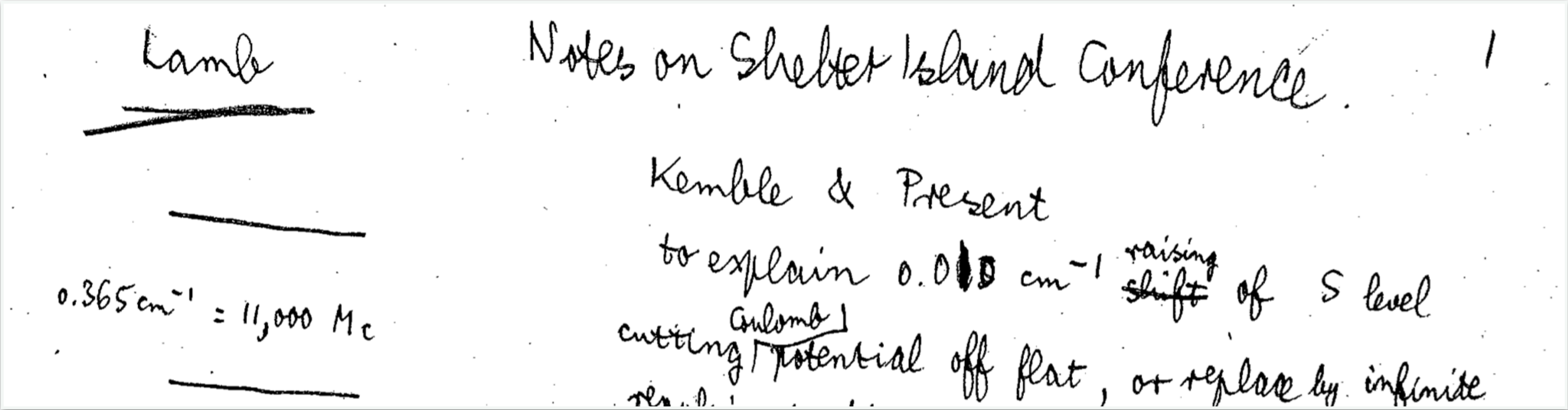
approximates electron as localized

sum over cavity modes s

Divergences in the Nonrelativistic Limit

Delicately taking NR limit recovers NRQM result: $\delta E_n \simeq \dfrac{e^2}{m^2} \sum_{s,n'} \dfrac{|\langle n', 1_s | \mathbf{A} \cdot \pi | n, 0 \rangle|^2}{E_n - (E_{n'} + \omega_s)}$

Result is linearly divergent; what does this mean?


Lamb

0.365 cm^{-1} = 11,000 Mc

Notes on Shelter Island Conference. 1

Kemble & Present

to explain 0.0[illegible] cm^{-1} raising ~~shift~~ of S level

cutting Coulomb potential off flat, or replace by infinite

NR free-space self-energy is linearly divergent, cannot even be estimated in NRQM

But Lamb shift is log divergent, estimable in NRQM (first success of renormalization)

Cavity shift is **convergent** after subtracting free-space part, so calculable in NRQM

Divergences in a Spherical Cavity of Radius a

$$\delta E_n \simeq \frac{e^2}{m^2} \sum_{s,n'} \frac{|\langle n', 1_s | \mathbf{A} \cdot \pi | n, 0 \rangle|^2}{E_n - (E_{n'} + \omega_s)} \qquad \Delta\omega_c = \delta E_{n+1} - \delta E_n$$

In cavity: only TM_{11p} modes are nonzero at cavity center

$$\frac{\Delta\omega_c^{\mathrm{cav}}}{\omega_c} = -\frac{8}{3\pi}\frac{\alpha}{ma} S \qquad S = \sum_{p=1}^{\infty} f(p) = \sum_{p=1}^{\infty} \frac{c_p^3}{c_p^2 - 2} \frac{1}{J_{3/2}^2(c_p)} \frac{1}{c_p^2 - z^2}$$

In free space: integrate over plane waves $(\omega, \mathbf{k})$

$$\frac{\Delta\omega_c^{\mathrm{free}}}{\omega_c} = -\frac{8}{3\pi}\frac{\alpha}{ma} I \qquad I = \frac{1}{2}\int_0^{\infty} dc \, \frac{c^2}{c^2 - z^2}$$

where $c_p = \omega_p a$, $z = \omega_c a$, $c = \omega a$

Smooth regulator should treat c_p and c the same way

How can we subtract the divergent sum and integral analytically?

Subtracting the Sum and Integral

$$S = \sum_{p=1}^{\infty} f(p) = \sum_{p=1}^{\infty} \frac{c_p^3}{c_p^2 - 2} \frac{1}{J_{3/2}^2(c_p)} \frac{1}{c_p^2 - z^2} \qquad I = \frac{1}{2} \int_0^{\infty} dc\, \frac{c^2}{c^2 - z^2} \qquad \begin{aligned} c_p &= \omega_p a \\ c &= \omega\, a \end{aligned}$$

First trick: define a continuous analogue of the spherical Bessel root c_p

$$\frac{d}{dx}\left(\frac{\sin x}{x} - \cos x\right)_{x=c_p} = 0 \longleftrightarrow c(p) + \arctan\left(\frac{c(p)}{c(p)^2 - 1}\right) = \pi p$$

after Bessel identities: $f(p) = \frac{1}{2}\frac{dc}{dp}\frac{c^2}{c^2 - z^2}$

Second trick: relate both the regulated sum and integral to the same contour integral

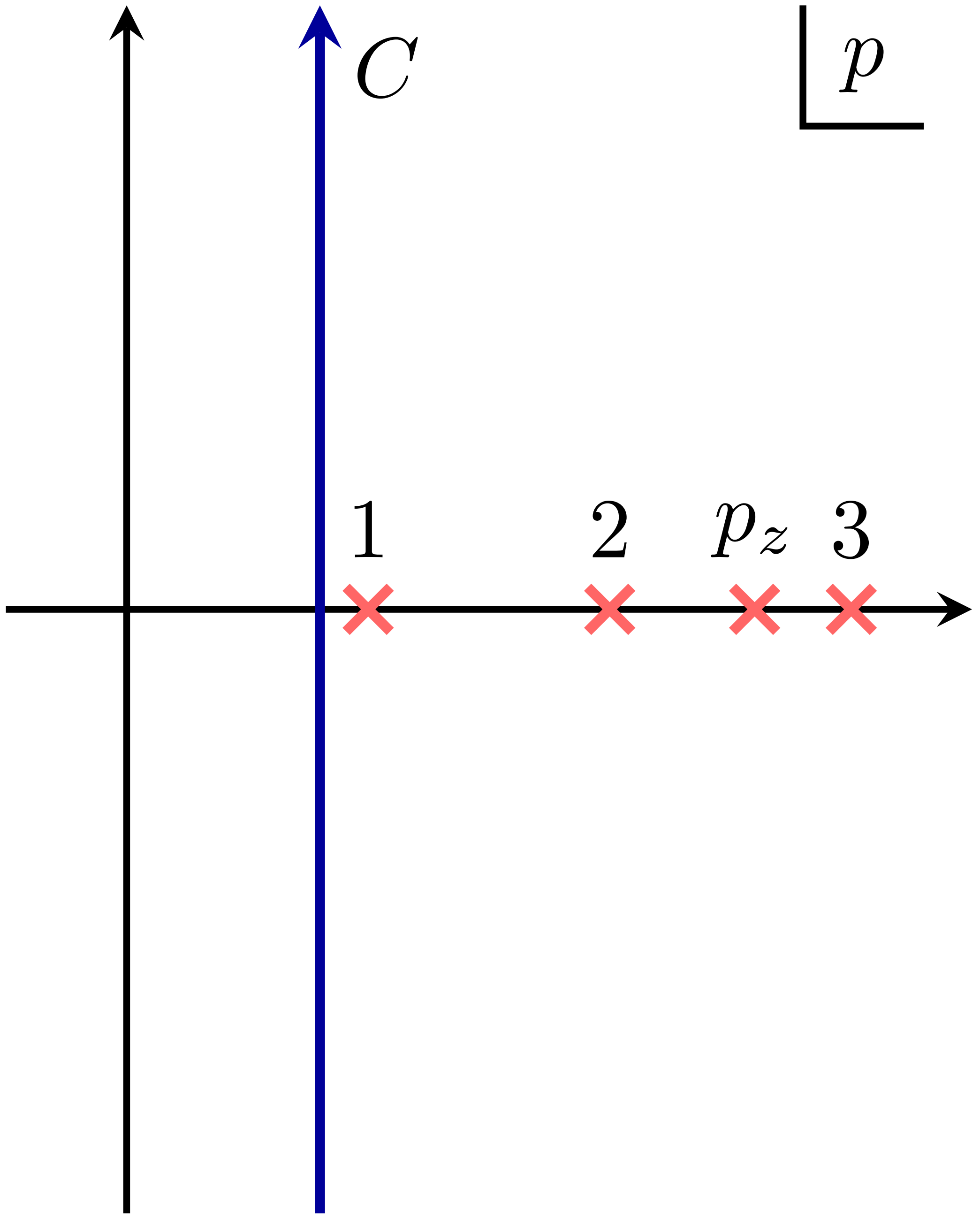
Will keep the regulator (damping at large c) implicit

Physical difference arises at low c, regulator independent

Contour Integration for the Spherical Cavity

Consider contour integral $A = \int_C \frac{f(p)}{e^{-2\pi i p} - 1} dp$

Analytic regulator lets us close at infinity, $A = S + (\text{pole})$



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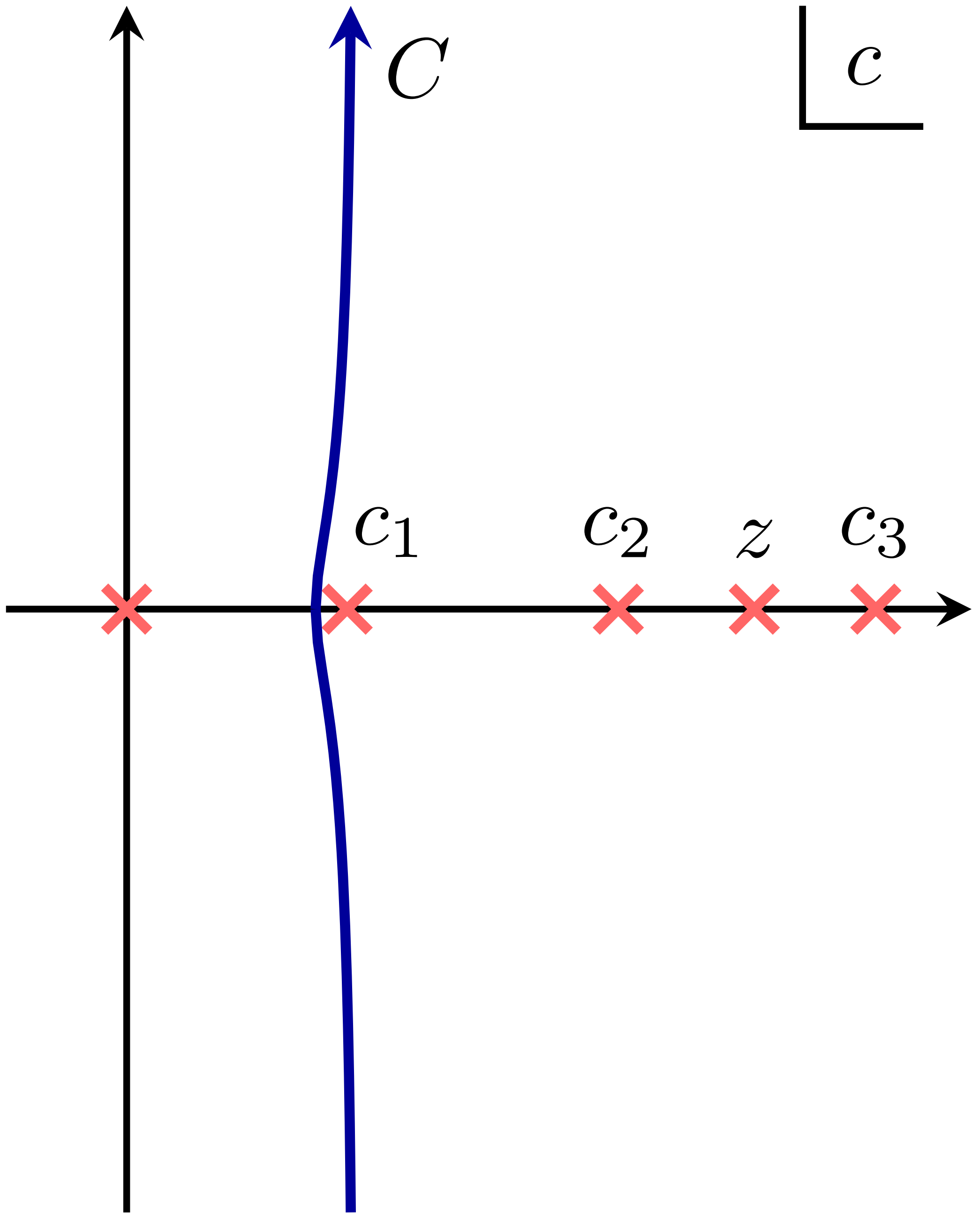
Changing variables to c gives $A = \int_C \frac{1}{2} \frac{c^2}{c^2 - z^2}\, g(c)\, dc$

$$g(c) = \frac{1}{2}\left(-1 + \frac{(c^2-1)\,\cos c - c\,\sin c}{c\,\cos c + (c^2-1)\,\sin c}\, i\right)$$



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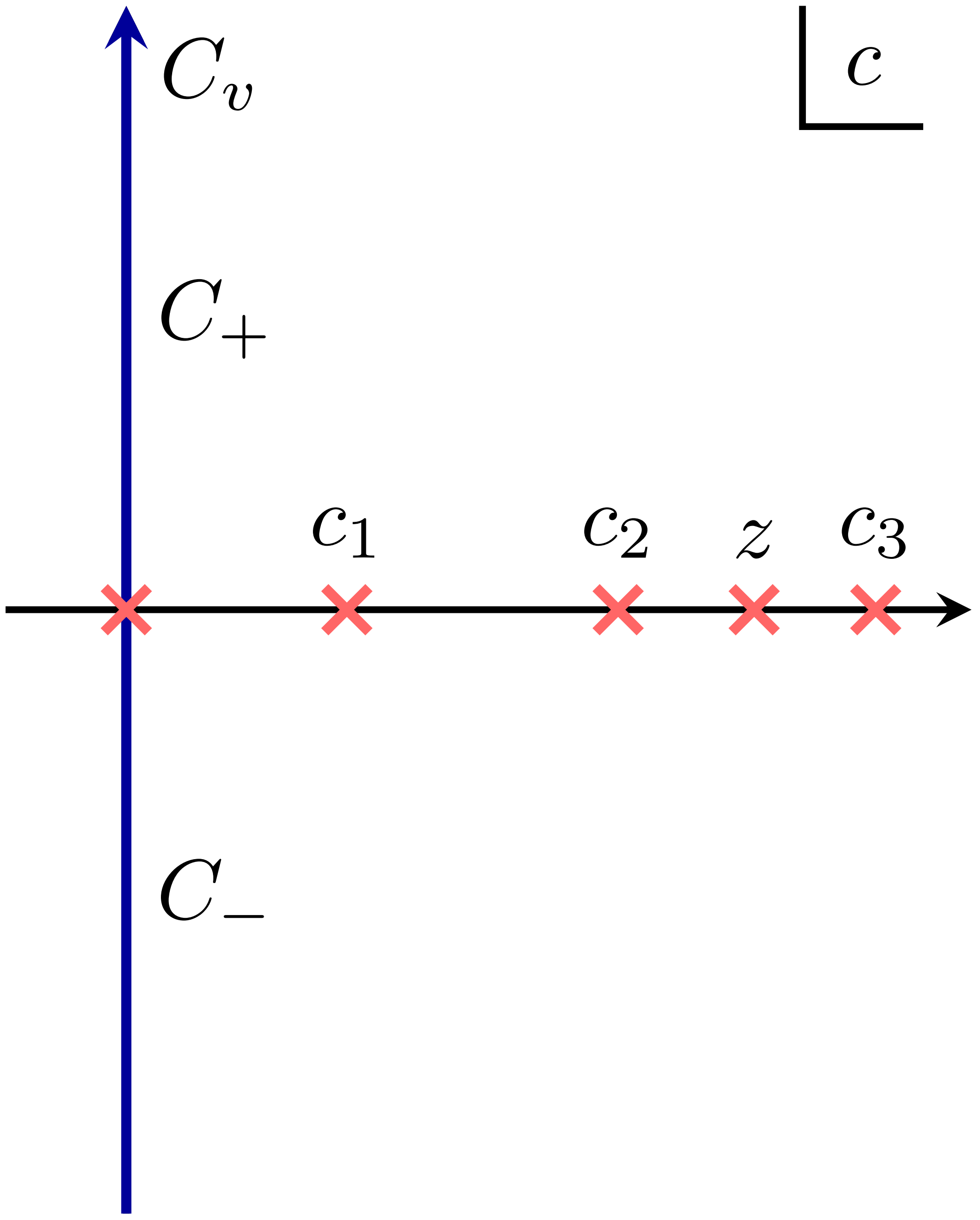
Push contour to imaginary axis, $g(-iy) + 1 = -\,g(iy)$

Pull out $+1$ and cancel, $A = \mathrm{pole} - \int_{C_-} \frac{1}{2} \frac{c^2}{c^2 - z^2} dc$



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Combining results:

$$I - S = \frac{\pi z}{4} \times \left(\frac{(1-z^2)\cos z + z\sin z}{(1-z^2)\sin z - z\cos z} + \frac{3}{2z^3} \right)$$

perfectly matches
classical answer!

Sketch: Divergences in a Realistic Cylindrical Cavity

$$\frac{\Delta\omega_c^{\text{cav}}}{\omega_c} = -\frac{\alpha}{ma}\sum_{\nu=1}^{\infty}\frac{4a}{L}\frac{1}{J_2^2(\chi_{1n})}\sum_{p=1}^{\infty}\frac{\sin^2(p\pi/2)}{\omega_{\text{TM},1np}^2 a^2}\frac{(p\pi a/L)^2}{\chi_{1n}^2 - z^2 + (p\pi a/L)^2} + \text{TE terms}$$

both length L and radius a matter

sum over p can be done exactly

both TM_{1np} and TE_{1np} modes contribute

Subtract against integral of plane waves in cylindrical coordinates

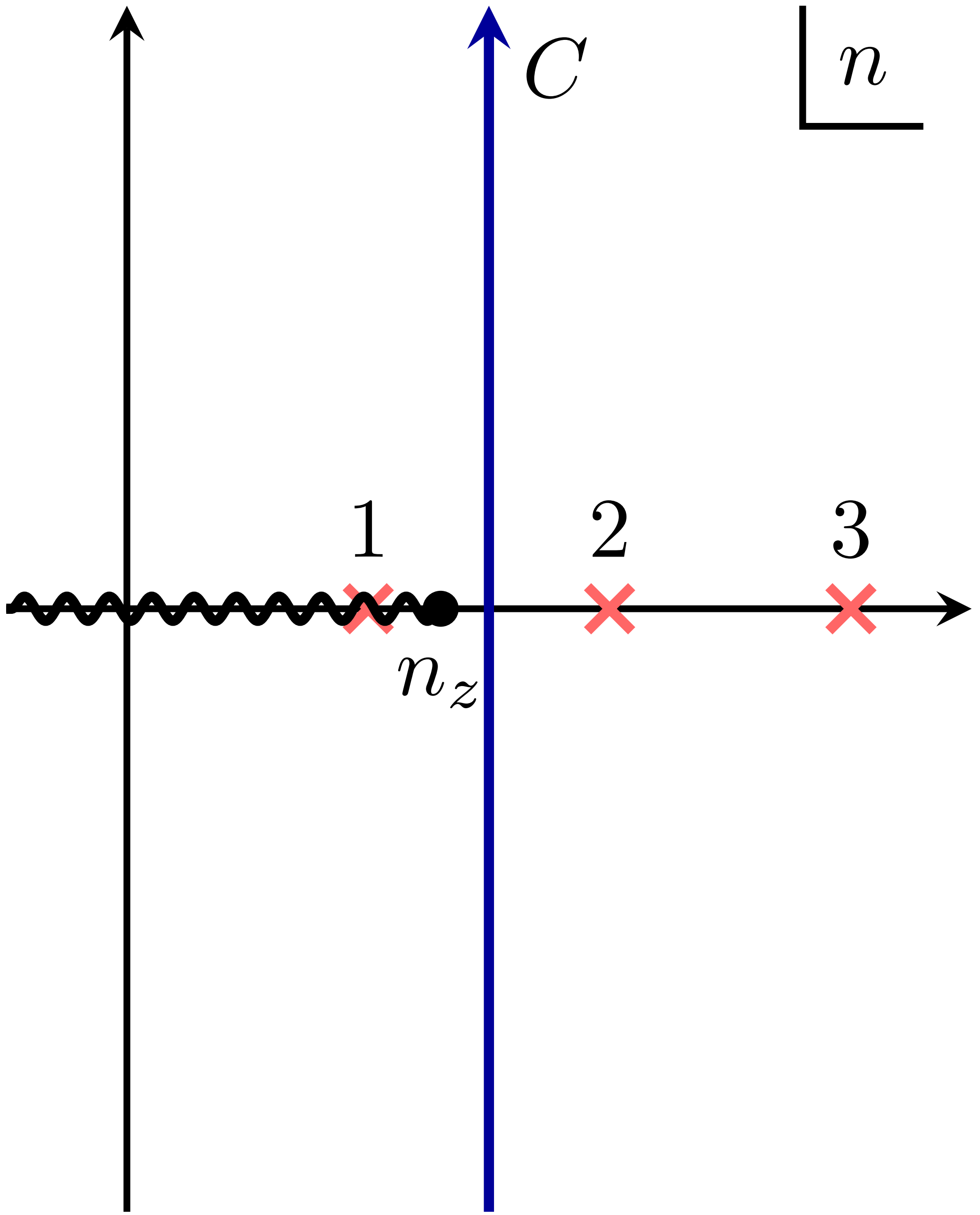
$$\frac{\Delta\omega_c^{\text{free}}}{\omega_c} = -\frac{\alpha}{ma}\frac{a}{2\pi}\int_0^{\infty} dk_\perp\, k_\perp \int_{-\infty}^{\infty} dk_z \left(1 + \frac{k_z^2}{k_\perp^2 + k_z^2}\right)\frac{1}{k_\perp^2 + k_z^2 - \omega_c^2}$$

Make summand analytic in n using $\tan(\pi n) + \dfrac{J_1(\chi_{1n})}{Y_1(\chi_{1n})} = 0$

Must split sum and integral into 3 parts, with different pole and branch cut structure

Sketch: Contour Integration for the Cylindrical Cavity

For TE modes, let $A_{\mathrm{TE}} = \int_C \frac{f_{\mathrm{TE}}(n)}{e^{-2\pi i n} - 1} dp = S_{\mathrm{TE}} - (\text{poles})$



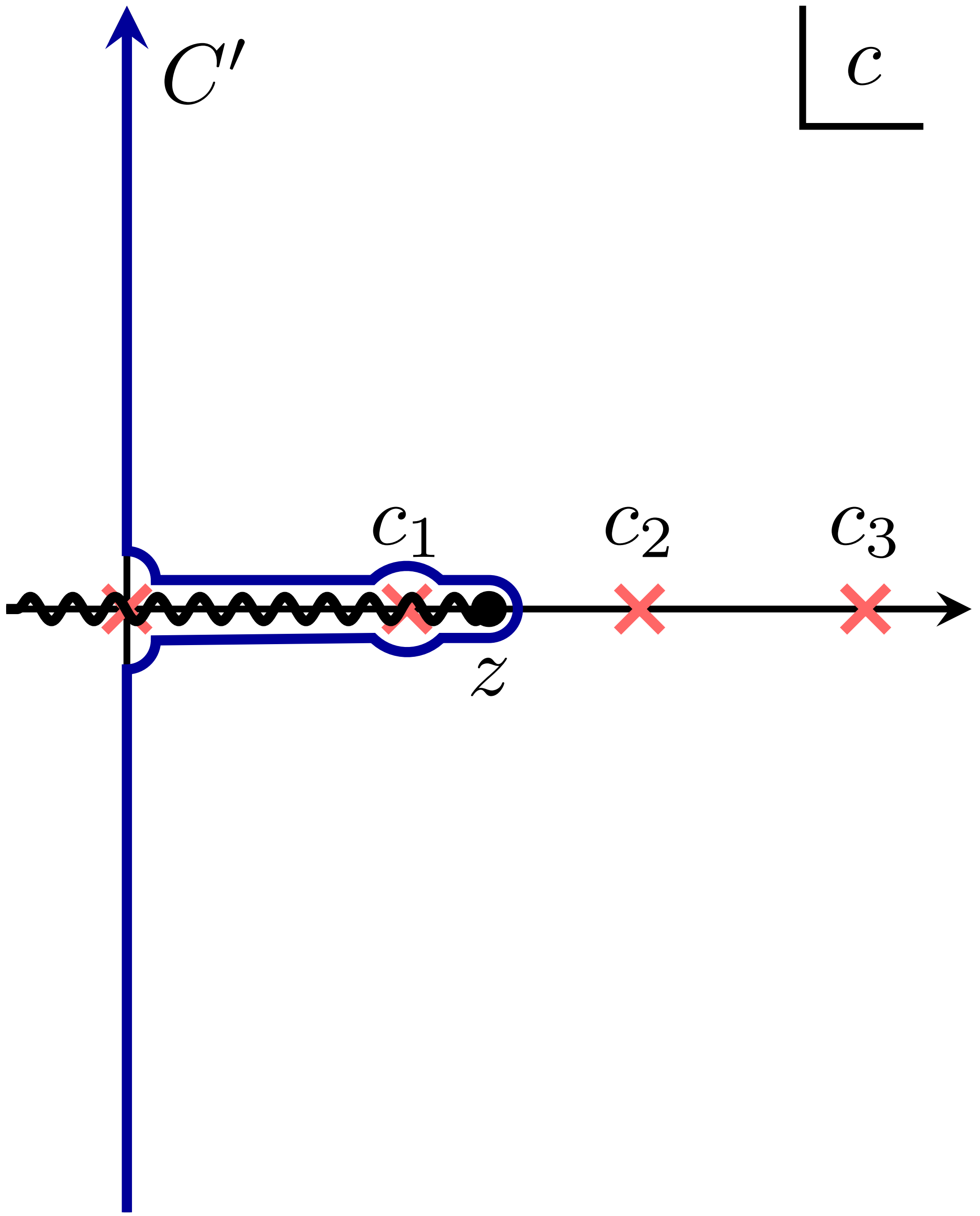
Sketch: Contour Integration for the Cylindrical Cavity

For TE modes, let $A_{\text{TE}} = \int_C \frac{f_{\text{TE}}(n)}{e^{-2\pi i n} - 1} dp = S_{\text{TE}} - (\text{poles})$

Changing variables to c gives $A_{\text{TE}} = \frac{1}{2} \int_C \frac{c}{\sqrt{c^2 - z^2}} g(c)\, dc$

$$g(c) = -\frac{1}{2}\left(1 + \frac{Y_1'(c)}{J_1'(c)} i\right) \qquad g(-iy) + 1 = g(iy)$$

Deform contour left, extract $+1$, use symmetry



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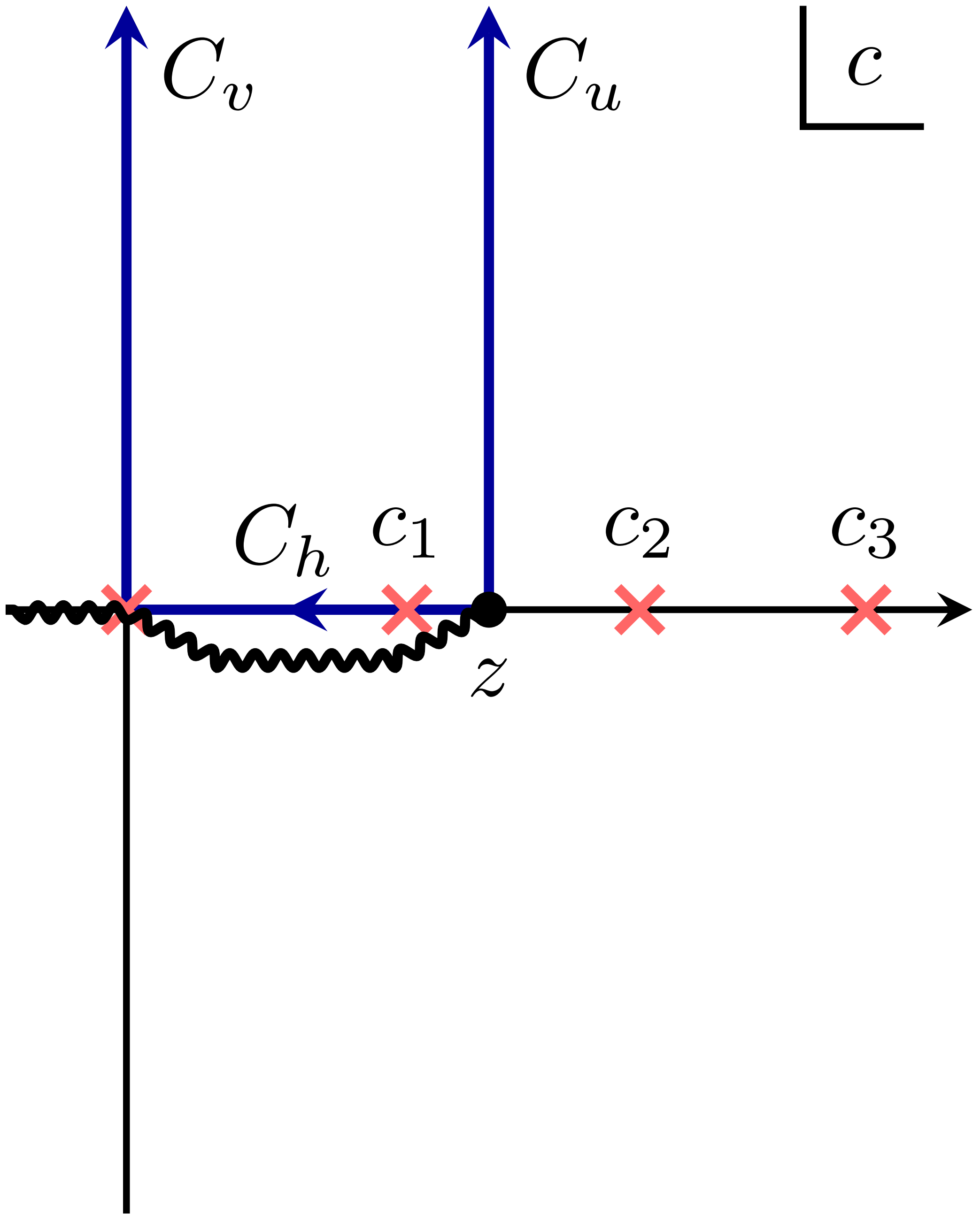
Deform contour left, extract $+1$, use symmetry

Point contour up for exponentially damped remainder

$$A_{\mathrm{TE}} = I_{\mathrm{TE}} + \mathrm{Re}\int_{C_u} \frac{c}{\sqrt{c^2 - z^2}}\, g(c)\, dc$$



Final Results for the Cylindrical Cavity

New quantum result

$$\frac{\Delta\omega_c}{\omega_c} = -\frac{\alpha}{ma}\left(\Delta S + J\right)$$

$$J = \frac{1}{\pi}\,\mathrm{Re}\int_0^\infty \frac{(y-iz)\sqrt{y^2-2iyz}}{z^2}\frac{K_1(y-iz)}{I_1(y-iz)} - \frac{K_1(y)}{I_1(y)}\frac{y^2}{z^2} + \frac{y-iz}{\sqrt{y^2-2iyz}}\frac{K_1'(y-iz)}{I_1'(y-iz)}\,dy$$

exponentially damped

$$\Delta S = \sum_{n=\bar{n}^*+1}^{\infty}\left(\tanh\left(\frac{L\sqrt{\bar{c}_n^2-z^2}}{2a}\right)-1\right)f_{\mathrm{TE}}(n) + \sum_{n=1}^{\bar{n}^*}\tan\left(\frac{L\sqrt{z^2-\bar{c}_n^2}}{2a}\right) i f_{\mathrm{TE}}(n) + (\text{TM terms})$$

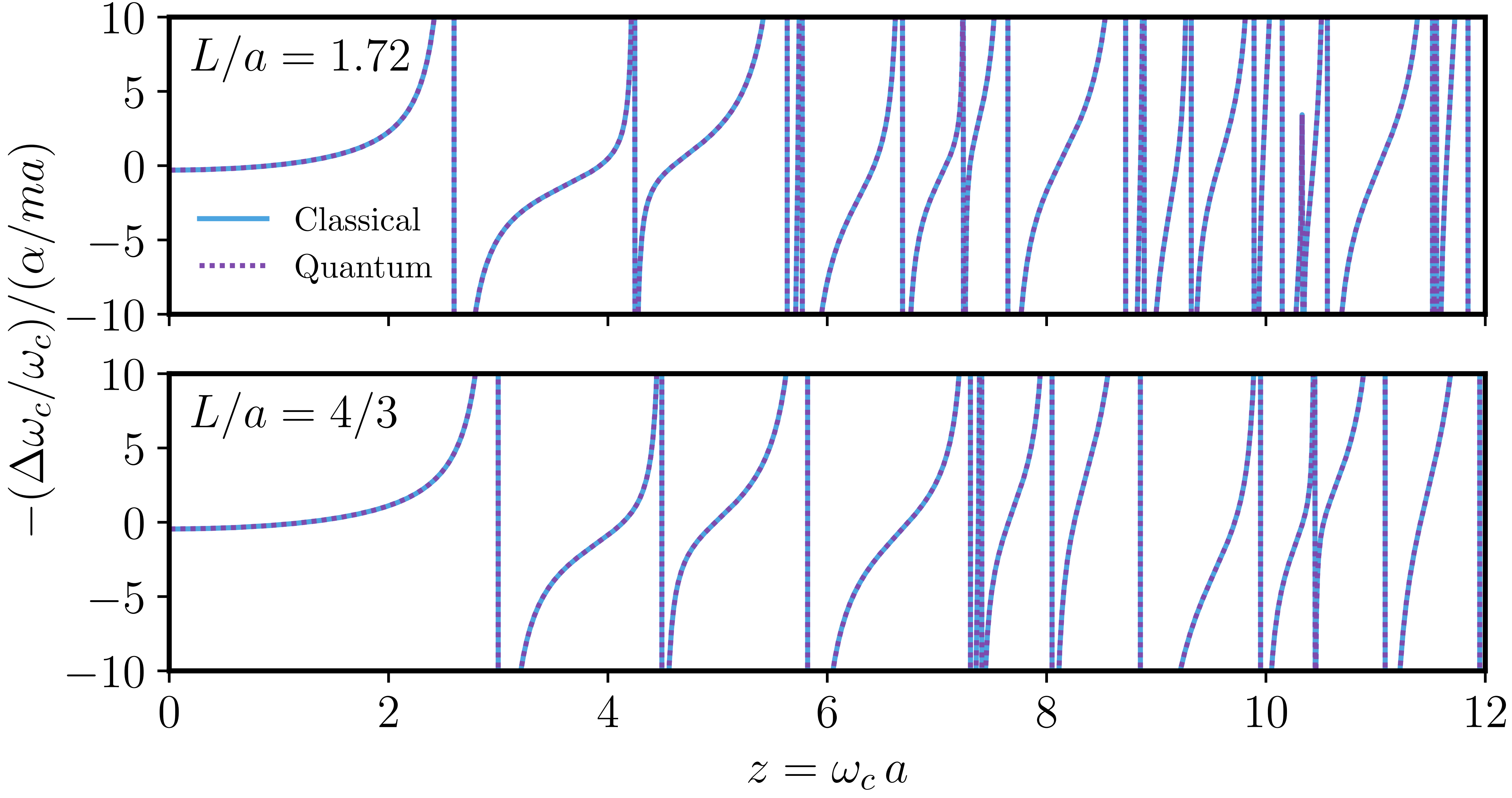
finite + damped

Existing classical result

$$\frac{\Delta\omega_c}{\omega_c} = \frac{2\alpha}{mL}\left[\frac{1}{2}\log(4\cos^2(\xi/2)) + \sum_{n=1}^{\infty}(-1)^n\left(\frac{\sin(n\xi)}{n^2\xi} + \frac{\cos(n\xi)-1}{n^3\xi^2}\right) - \mathrm{Re}\sum_{p=0}^{\infty}\left(\frac{K_1'(\mu_p a)}{I_1'(\mu_p a)} + \frac{k_p^2}{\omega_c^2}\left(\frac{K_1(\mu_p a)}{I_1(\mu_p a)} - \frac{K_1(k_p a)}{I_1(k_p a)}\right)\right)\right]$$

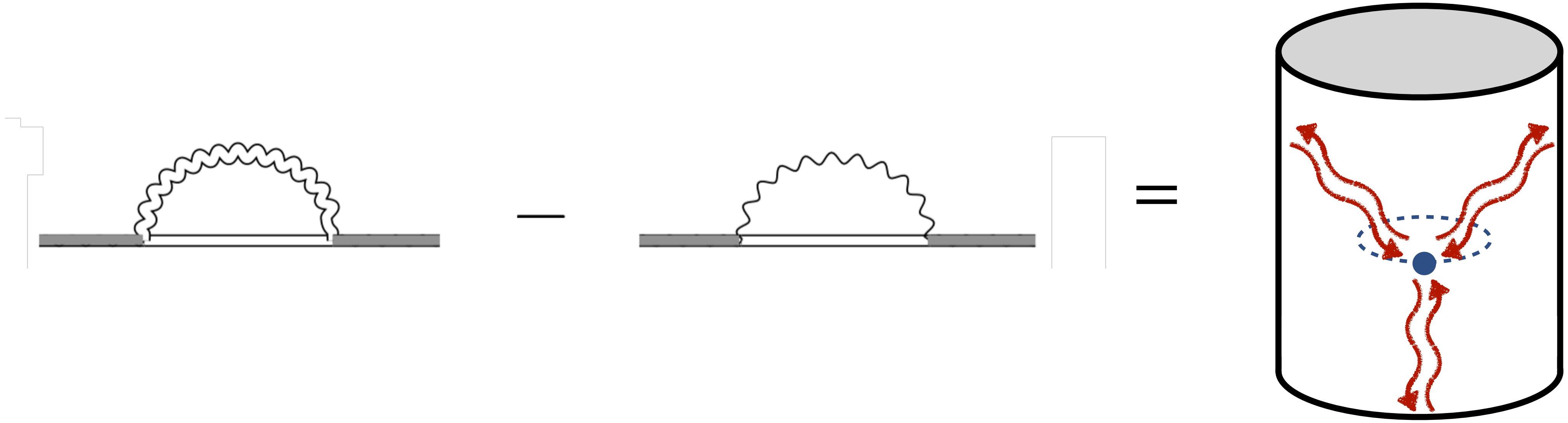
$$\xi = \omega_c L \qquad k_p = (2p+1)\pi/L \qquad \mu_p = \sqrt{k_p^2-\omega_c^2}$$

Quantum and Classical Comparison



Analytic match possible in special cases; in general, perfect numeric match

What Have We Learned?



The quantum scattering amplitude knows everything!

(but only when set up right; textbook amplitudes only for free electrons in free space)

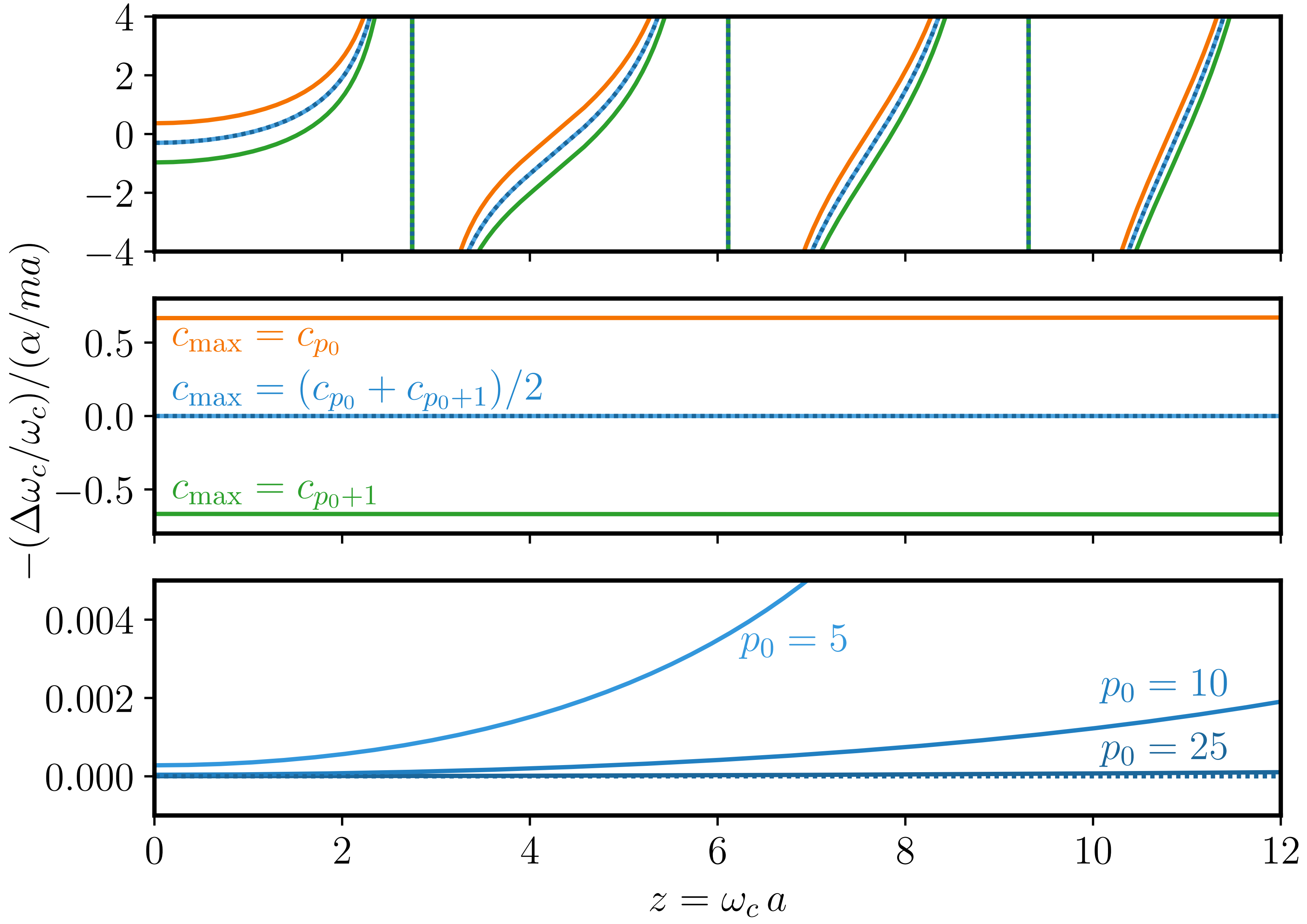
Classical physics captures some effects that sound highly quantum

Classical and quantum calculations can provide complementary insights

Rest of this talk: future $g-2$ experiments, and effects of ultralight dark matter

Sphere with Concrete Cutoffs

Now that we know the quantum answer (with frequency-dependent cutoff) matches the classical one, we can efficiently evaluate it with specific cutoffs



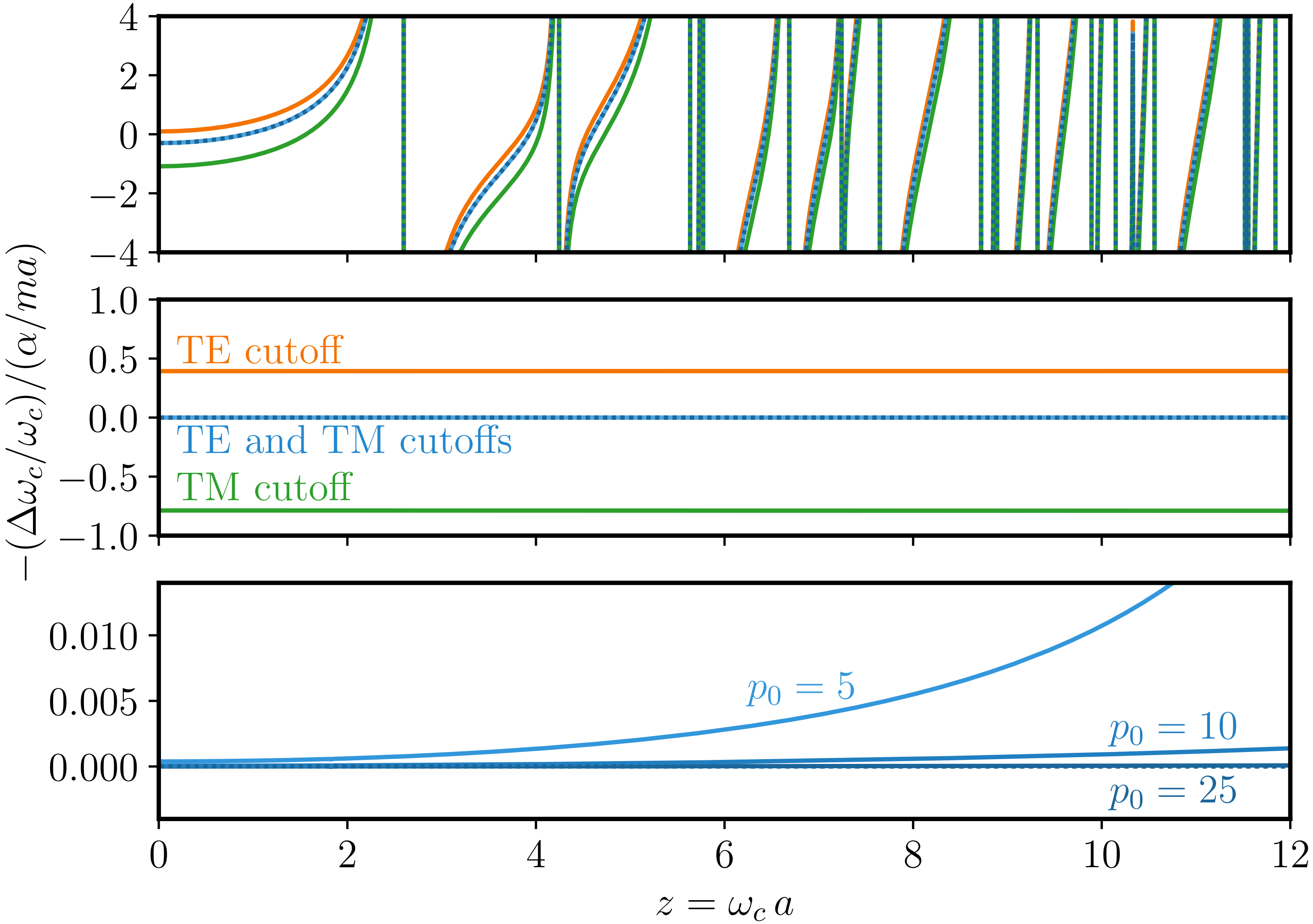
$\sim 10^{-12}$ variation for incorrect cutoff

$\sim 10^{-15}$ variation with ~ 10 terms

For sphere, hard cutoff in the right place gives highly accurate result with few terms

Cylinder with Concrete Cutoffs

For the cylinder, need to cut off parts of the integral differently, depending on if they correspond to TE or TM modes



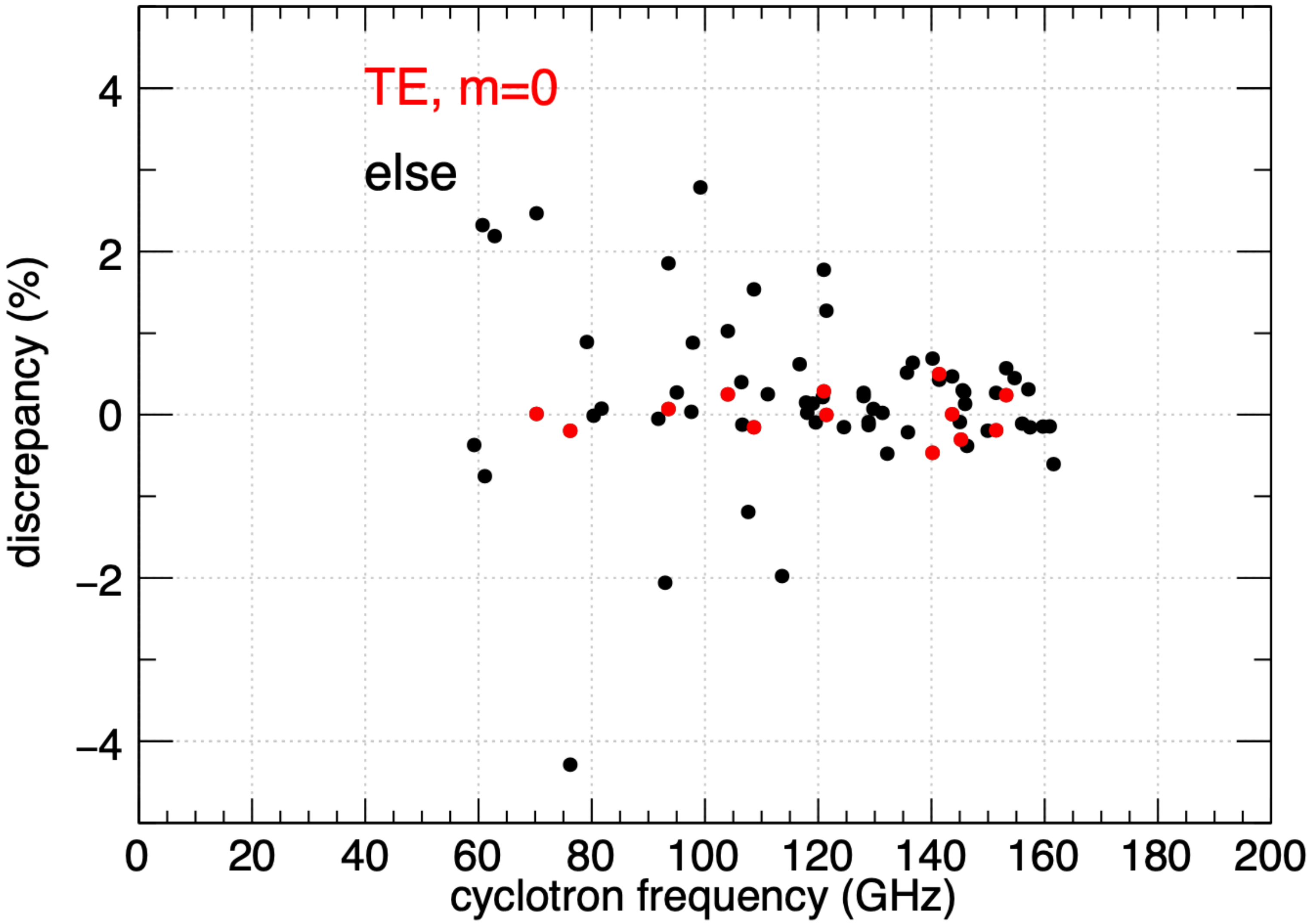
$\sim 10^{-12}$ variation for incorrect cutoffs

$\sim 10^{-15}$ variation with ~ 10 terms

Again, highly accurate results with small number of terms

Next Steps for Experimental Calibration

Results with low cutoff are still very accurate, because the exact $S - I$ only depends on exponentially damped quantities



Xing Fan, PhD thesis (2022)

Opens door to treating the real cavity, which is **not** an ideal cylinder

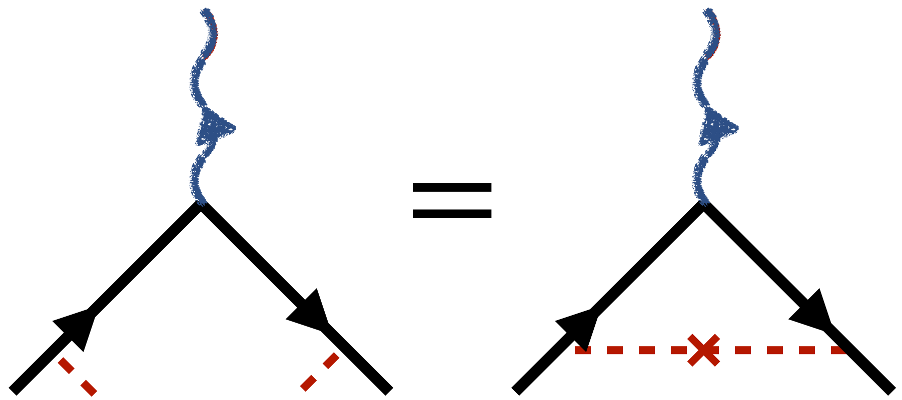
Mode frequencies and coupling strengths deviate by few %, and quality factors vary

Already one of the most important systematics

Mode-based calculation naturally accommodates these features!

Dark Matter "Background-Induced" $g-2$ Shifts

DM background claimed to yield large shifts of electron $g-2$ and EDMs



$$\Delta g_e \sim \frac{\rho_{\rm DM}}{m_{\rm DM}^2 m_e^2}\, g^2$$

2302.08746, PRL (2024)
2308.05375, JHEP (2025)
2410.10715
2412.14664
2509.12869

Described as intrinsically quantum, but again can be derived classically!

KZ, JHEP (2025)

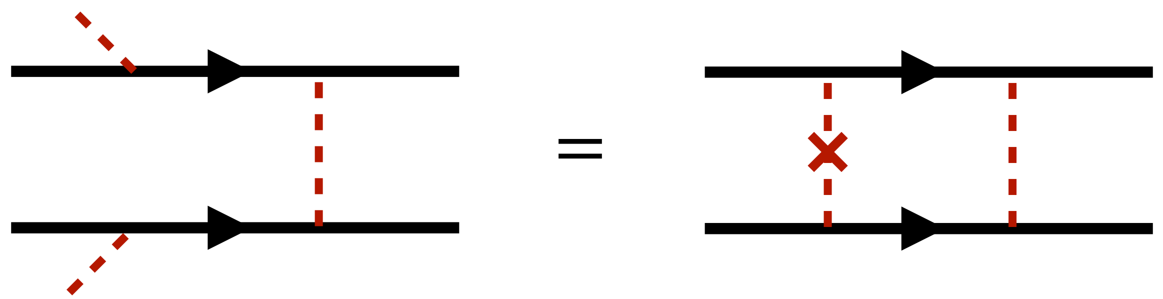
$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \frac{d\mathbf{S}}{dt} = \mathbf{S}\times\left(\frac{q g_e}{2m_e}\left(\mathbf{B} - \frac{\gamma}{\gamma+1}(\mathbf{v}\cdot\mathbf{B})\mathbf{v} - \mathbf{v}\times\mathbf{E}\right) + \frac{\gamma^2}{\gamma+1}\mathbf{v}\times\mathbf{a}\right).$$

Like the ponderomotive force or gravitational wave memory,
derived by carefully solving for classical motion at second order

Classical derivation reveals IR cutoff neglected in amplitude, making effect negligible

Dark Matter “Background-Induced” Forces

Tree-level coupling to external axions is formally a background-corrected loop diagram



Quantum calculation: summing 5 diagrams, potential can be $1/r$ and spin independent

Again, short classical calculation gives same result!

$$(\partial^2 + m_a^2)\, a = J = \partial_t(\mathbf{v} \cdot \hat{\mathbf{s}})\, \delta^{(3)}(\mathbf{x} - \mathbf{r}(t)) \qquad \frac{d\mathbf{v}}{dt} = \frac{\mathbf{F}}{m} = \frac{g\, \ddot{a}\, \hat{\mathbf{s}}}{m}$$

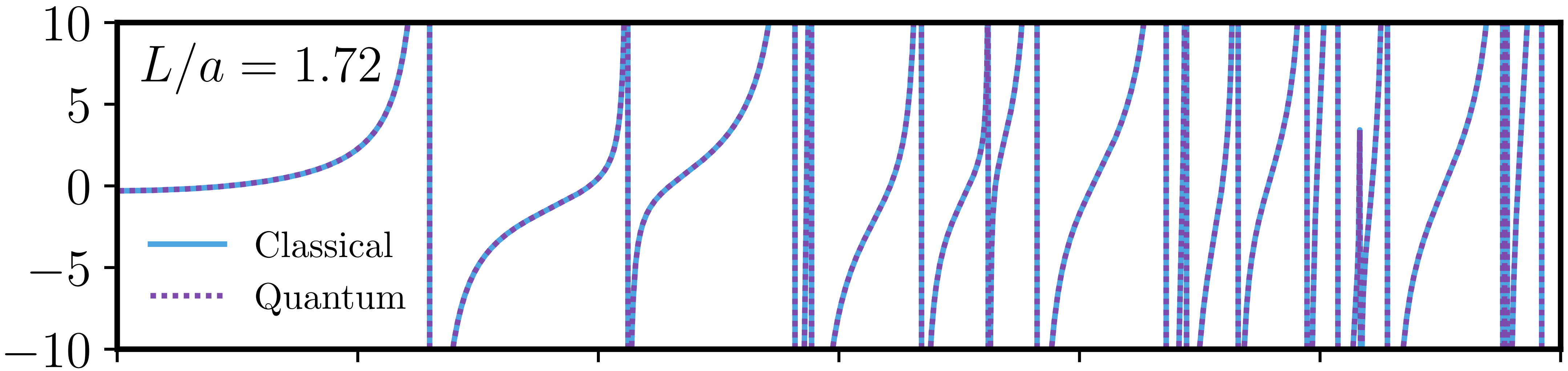
Classical analogues for ultralight DM effects essentially always exist

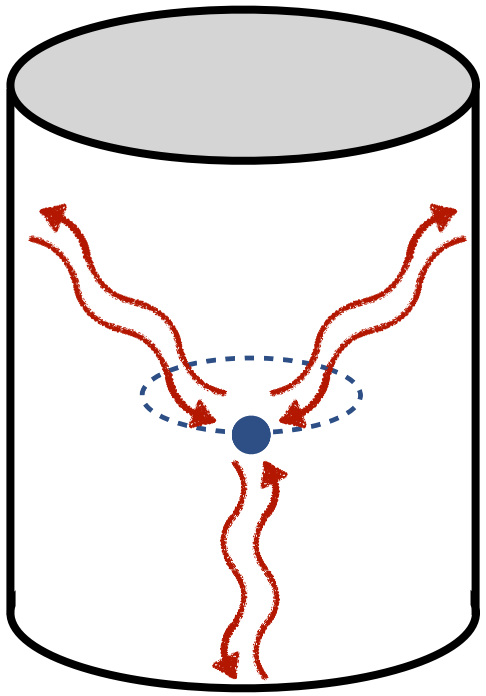
Relative to classical effects, intrinsically quantum effects always suppressed by **additional** powers of the weak coupling, and practically unobservable

Y. Bao, D. Y. Cheong, N. Rodd, J. Takach, L.-T. Wang, KZ, Phys. Rev. Lett. 136, 171601 (2026)

Y. Bao, D. Y. Cheong, N. Rodd, J. Takach, L.-T. Wang, KZ, arXiv:2607.xxxxx

Conclusion





When treated properly, quantum electrodynamics knows about subtle infrared effects

Classical calculations can make scaling and cutoffs more transparent

Quantum mode calculation easier to adapt to imperfect cavities

From the Lamb shift to the cavity shift: 75 years of the rich physics of $g-2$